NONPOINT SOURCE POLLUTION MANAGEMENT MODELS
FOR REGIONAL GROUNDWATER QUALITY CONTROL

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FOR REGIONAL GROUNDWATER QUALITY CONTROL

A Dissertation Presented

by

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FOR REGIONAL GROUNDWATER QUALITY CONTROL
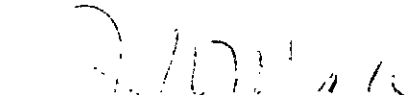

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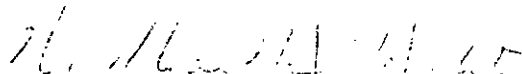Chairperson of Committee


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DEDICATION


To my loving wife

Jean

# ACKNOWLEDGMENTS

ABSTRACT

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FOR REGIONAL GROUNDWATER QUALITY CONTROL

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Nonpoint source pollution threatens the quality of enormous
reservoirs of groundwater.  Limits on distributed source densities,
effluent quality standards for sources, and regulations on land use
activities can be effective components of strategies to protect
groundwaters.  To facilitate the formulation or evaluation of strategies
to protect groundwaters from nonpoint source pollution, groundwater
quality management models have been developed.  Decision makers can use
these models to obtain estimates of regional subsurface wasteload
allocations that can in turn be used to obtain tangible estimates of
desirable distributed source densities (i.e., septic tank densities) or
identify land use patterns that will maintain acceptable subsurface
water quality.

Several steady-state groundwater quality management models useful
for investigating regional groundwater wasteload allocation from
nonpoint sources are presented.  These management models are constructed
as linear programming optimization models.  Equations from a finite
difference, steady-state, two-dimensional horizontal, unconfined,
advective contaminant transport model are used as part of each

optimization problem constraint set.  The management models were applied

over the Sole Source aquifer of Barnstable County, Massachusetts.

Barnstable County is incurring widespread nitrate contamination from

distributed septic systems which serve 88 percent of the population.

The modeling approach requires general data normally available through

state geological surveys, regional planning commissions, and the census

bureau.  The optimal regional nonpoint source groundwater wasteload

allocations are generated from this data as are resultant contaminant

distributions, boundaries of critical recharge areas, and the associated

water quality tradeoffs for changes in existing and proposed land use

(or source) management schemes.  The optimal wasteload allocations were

translated into estimates of distributed source densities and land use

development patterns.  The results of these groundwater quality

management models, used in conjunction with additional field and

planning information, provide additional insight that is essential for

consideration of groundwater resource protection in the planning of

regional land use activities.

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$A_{1,2}$ = a linear algebraic function obtained from the finite difference approximation for the groundwater flow (23) at element (i=1,j=2)

$B$ = saturated thickness which may equal h for an unconfined aquifer with a horizontal impermeable bottom boundary, (L)

$C$ = the vertically averaged concentration of dissolved chemical species, $(M/L^3)$

$C_{E_{i,j}}$ = effective nitrate concentration in element i,j for sewer exfiltration recharge, $(M/L^3)$

$C_i$ = concentration of contaminant at node i, $(M/L^3)$

$C_I$ = nitrate concentration in the recharge I, $(M/L^3)$

$C_{i,j}$ = a variable for contaminant concentration in element (i,j), $(M/L^3)$

$C_o$ = background nitrate concentration, $(M/L^3)$

$C^P$ = concentration of dissolved chemical at the source, $(M/L^3)$

$C_i^P$ = concentration of waste injected in element i, $(M/L^3)$

$C_q$ = observed background nitrate concentration in groundwaters (for convenience precipitation was treated as the source, or generically contaminant concentration in natural recharge flow, $(M/L^3)$

$C_{s_{i,j}}$ = effective nitrate concentration in all recharge flows in element i,j derived from the combined domestic and commercial use of municipal water, sewers, and lawn fertilizers, $(M/L^3)$

$C_{ss}$ = effective concentration of nitrate in recharge from domestic and commercial use of septic systems, $(M/L^3)$

$C_w$ = nitrate concentration in septic system effluent, $(M/L^3)$

$C_{w_{i,j}}$ = effective nitrate concentration in all recharge flows in element i,j derived from the combined domestic and commercial use of on-site wells, septic systems, and lawn fertilizers, $(M/L^3)$

$C_{z_{i,j}}$ = effective nitrogen concentration in all recharge flows in element i,j derived from the combined septic systems, and lawn fertilizers, $(M/L^3)$

D = decision variable corresponding to the maximum deviation occurring to the maximum deviation occurring between

approximate simulated contaminant concentrations in
groundwater and allowable limits in operational areas
(a = 1,2,...n)

$D_a$ = approximate positive deviation resulting from the
management scheme as defined through the optimum values
of $q_K$

$D_a^o$ = initial positive deviation from the water quality
standard in operational area 'a' under the $q_o$ conditions

$D_{ij}$ = vertically averaged coefficient of hydrodynamic
dispersion which is descriptive of the combined effects
of Fickian diffusion and dispersion caused by
microscopic variations in fluid velocities within
individual pores, $(L^2/t)$

$D_{v_{i,j}}$ = the reciprocal of the per capita generation of recharge
flow $V_{i,j}$, $(t \cdot person/L^3)$

$D_{w_{i,j}}$ = the reciprocal of the per capita generation of recharge
flow $W_{i,j}$, $(t \cdot person/L^3)$

$D_{z_{i,j}}$ = the reciprocal of the per capita generation of recharge
flow $Z_{i,j}$, $(t \cdot person/L^3)$

$F_i$ = flow rate of groundwater at land use section i, $(L/t)$

$F_{i,j}$ = effective per capita nitrate load from lawn fertilizer, (M/t·person)

$G$ = water quality goal, $(M/L^3)$

$G_{x_{i,j}}$ = the error in simulated hydraulic gradient in the x direction at element i,j

$G_{y_{i,j}}$ = the error in simulated hydraulic gradient in the y direction at element i,j

$H_{i,j}$ = observed head at element i,j, (L)

$H_L$ = elevation of water table at the coast, (L)

$I$ = recharge in the circle defined by $r_w$, $(L^3/t)$

$K_C$ = ratio of commercial flows to domestic flows

$K_{c_{i,j}}$ = ratio of commercial flows to domestic flows in element i,j

$K_{i,j}$ = vertically averaged hydraulic conductivity tensor which is a physical parameter indicating the ease with which water passes through porous material in the direction i,j, (L/t)

$K_u$ = ratio of unaccounted water loss for the water distribution system to domestic flows

$K_{u_{i,j}}$ = ratio of unaccounted water loss for the water distribution system to domestic usage in element i,j

$L$ = the radial distance from the center of the well to the coast, $(L)$

$(L)$ = length, (meter)

$L_{s_{i,j}}$ = supply of resource in element $i,j$

$L_v$ = the resource requirement per unit flow of $V_{i,j}$

$Lw$ = the resource requirement per unit flow of $W_{i,j}$

$L_z$ = the resource requirement per unit flow of $Z_{i,j}$

$(M_{in})_i$ = decision variable corresponding to the injection flux at proposed injection site $i$, $(L/t)$

$P$ = volumetric flux of withdrawal per unit surface area of aquifer, $(L/t)$

$P_{i,j}$ = constant corresponding to the total groundwater withdrawal per unit area in element $(i,j)$, $(L/t)$

$P_s$ = average daily service population for a sewered area

$P_{w_{i,j}}$ = average daily population in element $i,j$ using on-site wells

$P_z$ = average daily population served by municipal water

$P_{z_{i,j}}$ = average population in element $i,j$ using municipal water and septic systems

$Q$ = natural recharge, $(L/t)$

$Q_{i,j}$ = natural recharge flow in element $(i,j)$, $(L/t)$

$Q_{in}$     = volumetric flux of recharge per unit surface area of aquifer, (L/t)

$Q_{ini}$     = assumed volumetric flux of recharge into element i, (L/t)

$Q_{out}$     = volumetric flux of withdrawal per unit surface area of aquifer, (L/t)

$Q_{outi}$     = assumed volumetric flux of withdrawal from element i, (L/t)

$R$     = nitrate production rate, (M/person·t)

$R_i^b$     = known boundary conditions in element i (e.g., constant flux conditions), (L/t)

$R_s$     = resource supply

$R_v$     = the unit resource requirement per unit flow $V_{i,j}$

$R_w$     = the unit resource requirement per unit flow $W_{i,j}$

$R_z$     = the unit resource requirements per unit recharge flow $Z_{i,j}$

$S$     = combined recharge from sewer exfiltration and water distribution leakage, $(L^3/t)$

$s$     = total number of possible contaminant reactions

$S_y$     = vertically averaged specific yield (dimensionless), which physically corresponds to the percent of saturated porosity which drains under the force of gravity

$H_{i,j}^s$     = specified hydraulic head, (L)

$T_{i,j}^r$     = transfer coefficient defining the ratio of resultant

concentration (at year 2000) at surveillance point j to

the peak concentration at land use sector:

$U_{i,j}$     = the recharge in element i,j from land application of

secondary sewage collected from elements where

underlying sewers convey flows to site i,j

$V_i$     = the vertically averaged specific discharge or the mass

average flux of fluid flow in the i direction, (L/t)

$V_k$     = chemical, biological, or physical reaction k, negative

for the addition of solute and positive for the removal

of solute, $(M/L^3 \cdot t)$

$V_x$, $V_y$     = horizontal Darcian fluid velocities in the x and y

directions respectively, (L/t)

$W_{i,j}$     = recharge in element i,j from septic system effluent

derived from domestic and commercial use of on-site well

water, (L/t)

$W_L$     = maximum recharge rate from septic systems in a mature

residential development, (L/t)

$X_i$     = the decision variable for the unsewered population of

land use sector i; i = 1,2...n

$x_i$, $x_j$     = horizontal coordinate axis i,j, (L)

$Y_{i,j}$ = one of the contaminant concentration in element i,j is the target of interest (i.e., an element containing a municipal well)

$Z$ = combined recharge from septic system effluent derived from domestic and commercial activities plus recharge from water distribution system leakage, $(L^3/t)$

$Z_{i,j}$ = recharge in element i,j from septic system effluent derived from domestic and commercial use of municipal well water plus recharge from water distribution system leakage, $(L/t)$

$r_{i,j}$ = the combined parameters generated from the algebraic approximation of the governing groundwater flow equation at given element i in terms of neighboring element j, for i,j = 1,...n, $(1/t)$

$D_j^+$ = the decision variable for the positive deviation from the water quality goal at surveillance point j, j=1,2...m

$d_j^-$ = the decision variable for the negative deviation from the water quality goal at surveillance point j, j = 1,2...m

$b_i$ = known boundary conditions for element i (e.g., contributions of contaminant through natural sources), $(M/L^2t)$

$e_{i,j}$ = the combined parameters generated from the algebraic approximation of the governing mass transport model at given element i in terms of neighboring elements j, for i,j = 1,...n, (L/t)

h = elevation of the water table above the bedrock, (L)

$h_i$ = variable corresponding to the hydraulic head in element i, (L)

$h_{i,j}$ = a variable corresponding to the hydraulic head in element (i,j), (L)

i,j = the respective y and x coordinates of an element in a grid superimposed over an aquifer being modeled

$n_e$ = vertically averaged effective porosity (dimensionless)

n = the number of elements in the x direction in a two dimensional field or n equals the number of land use sectors

m = the number of elements in the y direction in a two dimensional field or m equals the number of surveillance points

$q_c$ = per capita domestic usage rate, $(L^3/t)$

$q_{c_{i,j}}$ = elemental per capita domestic usage, $(L^3/t)$ in element i,j

$q_k$ = p decision variables corresponding to either the quantity of water from supply wells or the quantity of

wastewater transported, or the quantity of water recharged in the various regions $(k = 1, 2, ..P)$ of the basin

$q_k^o$ = is the initial condition of the k decision variable, this could be a water supply pumping rate, wastewater transmission flow rate or recharge rate for $k = 1, 2...P$

$r_w$ = radius of the well, (L)

$S_k$ = a constant, (1/L)

$\Delta x, \Delta y$ = the x and y dimensions of the numerical element, (m)

$\omega_{i,j}$ = integer variable which is equal to one if sewers underlying an element convey flows to land application site i,j

[R] = nxn vector of coefficients generated from the algebraic approximation of the governing flow equation

[A] = $(n \cdot m) \times (n \cdot m)$ vector of coefficients generated from the algebraic manipulation of the finite difference approximation of the governing flow equation

{C} = $(n \cdot m) \times 1$ vector of variables for contaminant concentrations at every element or in Chapter 2 a nx1 column vector of decision variables defining solute concentrations throughout the system

$\{C^e\}$ = nx1 column vector of decision variables corresponding to the concentrations of contaminant $e = 1, 2...z$, for each element

$[C_s]$ = (n·m) x (n·m) diagonal matrix of nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of municipal well waters, sewers, and lawn fertilizers

$[C_u]$ = (n·m) x (n·m) diagonal matrix of effective nitrate concentrations in the elemental recharge from land application of secondary sewage

$[C_w]$ = (n·m) x (n·m) diagonal matrix of effective nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of waters from on-site wells, septic systems and lawn fertilizers

$[C_z]$ = (n·m) x (n·m) diagonal matrix of nitrate concentrations in elemental recharges from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers

$[G]$ = (n·m) x (n·m) vector of coefficients generated from the algebraic manipulation of the finite difference approximation of the governing contaminant transport equation

$[I]$ = (n·m) x (n·m) identity matrix or in Chapter 2 nxn identity matrix

$\{M_{min}\}$ = a nx1 vector of decision variables defining the contaminant injection fluxes (each flux equal to $Q_{in}C^P$,

the solute concentration in the injected waste times the flow rate

$[P]$     = a nxn diagonal matrix with values of one for entries that correspond to the injection sites and values equal to zero for all other entries

$\{P\}$     = (n·m) x 1 vector of known pumping fluxes in every element

$\{Q\}$     = (n·m) x 1 vector of elemental natural recharge flows

$\{Q_{in}\}$     = nx1 vector of assumed or known recharge rates

$\{Q_{in}c^P\}$     = nx1 vector of known waste injection fluxes

$\{Q_{out}\}$     = nx1 vector of assumed or known pumping or withdrawal rates

$\{R^b\}$     = nx1 vector of known boundary conditions

$\{S\}$     = (n·m) x 1 vector of elemental recharge flows from the combined domestic and commercial use of municipal well water, sewers, and lawn fertilizers

$\{U\}$     = (n·m) x 1 vector of elemental recharge flows from land application of secondary sewage

$\{W\}$     = (n·m) x 1 vector of elemental recharge flows from the combined domestic and commercial use of waters from on-site wells, septic systems and lawn fertilizers

$\{Z\}$     = (n·m) x 1 vector of elemental recharges from the combined domestic and commercial use of municipal well water, septic systems, and lawn fertilizers

{b}     = nx1 right-hand side vector reflecting boundary
        conditions (i.e., existing disposal fluxes)

{f}     = nx1 column vector defining boundary conditions and input
        fluxes of contaminant p as a nonlinear function of the
        integer decision variable $X_i$, the chosen treatment
        received before subsurface injection and the decision
        variable D, dilution water flows

[g]     = a nxn matrix of coefficients derived from a known
        constant velocity field and finite difference
        approximation of the mass transport equation

{h}     = (n·m) x 1 vector of variables for hydraulic heads at
        every element or in Chapter 2 nx1 vector of variables
        corresponding to the hydraulic heads at every element

CHAPTER 1


INTRODUCTION


The purpose of this research is to develop models which can be used in the evaluation of strategies for managing land surface activities so that the long term quality of groundwater is protected from the nonpoint source pollution associated with those activities.

The long term groundwater quality impacts of nonpoint pollution are significant because the groundwater systems receiving contaminated waters derived from various land use activities are at the same time functioning as sources of drinking water. Thus current decisions on land surface activities must take into account the long term impacts of such activities on groundwater. If the groundwater resources are expected to continue to function as drinking water sources, then some management of the land surface activities is necessary to ensure that desired groundwater quality standards are met over the long run. This is a particularly difficult problem on two accounts:

1) management decisions must be made now even though the total groundwater quality impacts of these decisions will not be fully evidenced for decades, due to the long response time of the groundwater,

2) managing activities requires a feedback loop be added to the usual cause and effect analysis framework so that the land use activities being managed can be limited to levels which will not violate desired future water quality levels. These difficulties can be overcome

with existing simulation models (Robson and Saulnier, 1981, Konikow and Bredehoeft, 1974, Gelhar and Wilson, 1974, Mercado, 1976), but they require multiple simulations with trial and error combinations of various land use patterns until a satisfactory land use management strategy is found. Such an approach is neither efficient nor does it give any indication of the merit of the managemant strategy selected.

Presented in this work are several steady-state regional groundwater quality management models. These management models are also optimization models which appear as variations on a basic linear program. The models are used to evaluate regional land use development patterns which cause acceptable changes in groundwater quality.

## 1.1  Nonpoint Source Pollution of Groundwater

The integrity of our groundwater resources is threatened on a local and regional scale by numerous point and nonpoint sources delivering an array of organic, inorganic and biological substances to the subsurface. Unlike surface water pollution, subsurface contamination is more persistent, complex, and expensive to reverse; consequently, groundwater pollution may exact lasting restrictions on water resource availability where comparable contamination of surface waters may not.

Over 117 million people in the United States depend on groundwater reservoirs for potable water (U. S. Environmental Protection Agency, 1984). As demands for groundwater increase so have the threats placed on supplies from the continual expansion of urban and other land uses. Considering the national dependence on subsurface waters and the

persistence of groundwater contamination, nonpoint source pollution is particularly nefarious because it endangers enormous reservoirs of water. Nonpoint source pollution (which includes areally distributed point sources) from septic tanks, buried pipelines and storage tanks, various agricultural activities, and highway deicing salts is creating regional groundwater quality problems across the United States (Miller, DeLuca, and Tessier, 1974, and U. S. Environmental Protection Agency, 1984). The resultant groundwater contamination is evidenced by small contaminant concentration gradients and homogeneity of pollutant levels over large areas (Gormly and Spalding, 1979, and Robertson, 1979).

Groundwater pollution from on-site domestic waste disposal systems (septic tanks, cesspools, etc.) is particularly prevalent in the eastern half of the country. Of the 27 counties across the United States which have over 50,000 on-site domestic waste disposal systems, 21 are in the east, and 8 are in Massachusetts and Connecticut alone (U.S. Environmental Protection Agency, 1977). Septic systems have been found to be the most frequent cause of groundwater contamination in the United States (Perkins, 1984). Work is need to ascertain proper methods of septic system placement, installation, and operation because these systems are currently installed in approximately 25 percent of all newly constructed houses (Canter and Knox, 1985).

Agricultural activities are also significant contributors of nonpoint source groundwater pollution. Poultry farms and intensive crop production have contaminated coastal wells in Sussex County, Delaware, where 32 percent of 210 wells sampled have nitrate levels above the U.S.

Environmental Protection Agency's drinking water standard (Ritter and Chirnside, 1984). On Long Island, New York, the combined use of lawn and garden fertilizers and septic systems has brought increases in nitrate over the last 30 years (Flipse et al., 1984). Other forms of nonpoint source pollution have been identified as threatening vast quantities of groundwater in other parts of the country (U. S. Environmental Protection Agency, 1984, Peters and Turk, 1981, and U. S. Environmental Protection Agency, 1977); however, few regulations have been implemented on the local, state, or federal level to protect subsurface reservoirs from such diffuse, areally distributed sources (Devine and Ballard, 1983).

1.2  Protection of Groundwater from Nonpoint Source Pollution

Protection of underlying aquifers against nonpoint source pollution is intimately tied to the control of overlying regional land use activities. Regulating land use activities is an expedient approach to controlling nonpoint source pollution, because most states, counties, and local governments have the necessary institutional structures and administrative bodies to readily invoke land use controls. Barnstable, Massachusetts, Dade County, Florida, and the State of Connecticut are among several local, county, and state governments which have incorporated land use controls as components of larger groundwater protection strategies. The focus of their strategies has been to manage land use activities in critical recharge areas through local zoning ordinances, the purchase of land or land rights, state-wide groundwater

classification systems, source performance standards, and source design standards. Various land use controls have been implemented through permitting systems to restrict or prohibit the placement of selected sources in sensitive recharge areas or to regulate contaminant loading through source density restrictions and effluent quality requirements.

Aside from political, economic, and technological considerations, a strategy for protecting groundwater from point and nonpoint source pollution must deliver protection where protection is most needed and to a degree which ensures the long term preservation of groundwater availability. Deliberate protection requires knowledge of the boundaries of critical recharge areas and the relationships defining the impacts of land use activities on local and distant groundwater quality. The boundaries of critical recharge areas must be located before land or land rights can be purchased and before regulations and zoning ordinances can be written and enforced to restrict or prohibit specific land use activities in those sensitive areas. How much protection is necessary to secure the long term availability of subsurface water resources, depends on the relationship between overlying land use activities and the underlying local and regional groundwater quality; this relationship must be understood and modeled before the minimum levels of groundwater quality protection can be achieved through, 1) adequate source effluent quality standards (design or performance) and 2) restrictions placed on source loadings and densities. Unless the location and extent of groundwater protection is determined, the effectiveness of existing and proposed regulations remains uncertain.

## 1.3  Research Objectives

Nonpoint source pollution groundwater quality management models can facilitate formulation, evaluation, application, and justification of groundwater protection strategies that incorporate land use controls and water quality goals.  To this end the overall objectives of this research are:

1) To develop regional groundwater quality management models to:

    -evaluate the long term water quality impacts of nonpoint source pollution,

    -delimit boundaries of critical recharge zones,

    -estimate regional nonpoint source groundwater wasteload allocations,

    -determine the optimum pattern of land use development over a region given a development objective and the constraints to be imposed on that development (for instance, a model that will facilitate maximum growth potential but minimize deleterious groundwater quality impacts).

2) To calibrate the management models to a region now experiencing a developing nonpoint source pollution problem.

3) To generate insight into how existing hydrologic and distributed anthropogenic stresses affect regional groundwater quality.

The material presented in the following chapters is arranged to present a lucid picture of the development and application of groundwater quality management models.  The general field of groundwater

quality management modeling is reviewed in Chapter 2. Chapter 3
presents a conceptual picture of an aquifer receiving sustained nonpoint
source contamination. The nature of information available through
management models is discussed in terms of a two dimensional horizontal
aquifer. The general components of a management model constructed as a
linear program are presented. Chapters 4 and 5 are devoted to the
development and validation of models which define the groundwater flow
field, estimate aquifer parameters, or serve as components in the
groundwater quality management models. Numerical models, used to
estimate aquifer parameters and describe the groundwater flow field, are
both developed and validated against analytical models in Chapter 4.
Chapter 5 presents the development and validation of equations (a
numerical contaminant transport model) which will serve as part of the
constraint set of a groundwater quality management model.

Chapters 6 through 9 are devoted to the application and
construction of four management models. The groundwater fluid velocity
field must be defined before a groundwater management model is
constructed and applied over a study area. In Chapter 6 the validated
groundwater flow model is calibrated over a defined study area on Cape
Cod, Massachusetts. All applications of the groundwater quality
management models are made within the hydrologically defined area of
Cape Cod. The first two management models are applied in Chapter 7.
The first model elucidates areas overlying an aquifer which are salient
to maintaining regional groundwater quality goals. A different
application is made with the second model to identify critical

groundwater recharge zones surrounding municipal water supplies. In Chapter 8 a third management model is constructed and applied over the town of Falmouth, Massachusetts. The third model ascertains patterns of maximum residential/commercial development for a population committed to the long term application of lawn fertilizers and use of septic systems for domestic/commercial waste disposal. Ultimate development is limited by requirements to maintain groundwater nitrate concentrations within specified water quality standards and housing development within zoning restrictions. The construction and application of the last management model is accomplished in Chapter 9. This model determines residential/commercial development patterns that cause a minimum regional impact on groundwater quality. Finally, Chapter 10 reviews results and presents research conclusions and recommendations for further work in this area.

CHAPTER 2


LITERATURE REVIEW


Concerns over the impacts of groundwater pollution have stimulated considerable research to characterize and control the fate and transport of dissolved contaminants. Most investigators have focused on the local impacts of visually acute point sources (e.g., hazardous waste sites), while few have researched the less obvious regional groundwater degradation from distributed and nonpoint sources of pollution. Groundwater quality simulation models have been used in most efforts to characterize regional and local subsurface contamination. A few investigators have used simulation models or groundwater quality management models (simulation models coupled to optimization models) to investigate strategies of controlling the extent and rate of groundwater pollution.

Groundwater quality simulation models approximate changes in water quality through the separate mathematical description of fluid flow and solute transport. A deterministic groundwater quality simulation model is usually composed of a solute transport model coupled to a groundwater flow model. The models are employed sequentially. Initially a flow model is constructed to reproduce an observed phreatic surface. Next Darcy's equation is used to translate the phreatic surface into a fluid velocity field. Finally a dissolved contaminant mass transport model is exercised over the defined fluid velocity field to estimate solute

exercised over the defined fluid velocity field to estimate solute concentrations in groundwater.

Two common types of deterministic mathematical groundwater quality simulation models are empirical lumped parameter models and conceptual distributed parameter models. Lumped parameter modeling treats the aquifer as a single cell or compartment; spatial variations in physical, chemical and biological characteristics of the groundwater system are ignored. These models yield a representative regional average contaminant concentration (with the mass transport model) and water table elevation (with the hydraulic model) for the entire aquifer system being modeled. Steady-state or temporal variations in regional average groundwater quality or phreatic surface elevation are investigated.

The most common approach to regional groundwater quality modeling has been through the use of distributed parameter models. Unlike lumped parameter models, distributed parameter models attempt to reproduce observed spatial and transient variations in the phreatic surface elevation and contaminant concentrations. Distributed parameter models incorporate the physical, chemical, and biological mechanisms which induce spatial variations in fluid flow and solute transport; as such, data on the locations and magnitudes of groundwater recharge and discharge, the locations and magnitudes of contaminant loading, the location of occurring boundary conditions (e.g., impermeable boundaries), the spacewise variation of aquifer characteristics (e.g., porosity and permeability), and contaminant reactions are incorporated in the model. Selection of a lumped parameter or a distributed

parameter model depends primarily on the knowledge being sought from the modeling effort; however, the amount of data available and the complexity of the groundwater system determines whether a chosen model can be calibrated and implemented in the field (Balek, 1983).

Predictions of regional aquifer responses to contaminant loadings have been attempted with both lumped and distributed parameter simulation models. These models have contributed significantly to the characterization of regional groundwater contamination from nonpoint source pollution. In addition, these models have served directly or indirectly as components of more complex attempts to explore strategies of managing nonpoint source pollution. Examples of both the distributed and the lumped parameter groundwater models are discussed in detail in the first half of the literature review.

The second half of the literature review presents several groundwater quality management models. These groundwater quality management models are much more efficient at evaluating groundwater quality management schemes than simple simulation models. Several transient and steady-state models will be reviewed which have yielded information useful to the development of groundwater quality management models for nonpoint source pollution.

2.1 Lumped Parameter Simulation Modeling of Regional Groundwater Quality

One of the earliest attempts to evaluate the groundwater quality impacts of nonpoint source pollution was through lumped parameter

models. Here the aquifer was treated as a completely mixed compartment. Sources of pollution were assumed uniformly distributed over the region being modeled. The relative importance of contaminant sources in recharge areas versus sources near discharge areas was ignored because sources were combined and assumed to pose a uniform hazard to the subsurface environment.

Lumped parameter models are simple and easy to calibrate with a limited amount of water quality and hydrologic data; however, too few or too many samples from highly contaminated areas may bias the representativeness of estimates of the mean regional contaminant concentrations. As tools for evaluating point source groundwater pollution problems, lumped parameter models are inappropriate because major point sources can constitute local rather than regional water quality threats; hence, including major point sources can artificially elevate estimated regional contaminant levels for a system modeled as completely mixed. As a result, application of lumped parameter models has been limited to estimating transient changes in regional groundwater quality effected by nonpoint source pollution.

Gelhar and Wilson (1974) proposed a lumped parameter model suitable for an aquifer bounded by a groundwater divide at one end and a stream at the other. For their model they assumed a phreatic aquifer could be described as a completely mixed linear reservoir. Recharge from precipitation flowed in a path perpendicular to the stream. Contaminant concentrations were constant throughout the groundwater system. Gelhar and Wilson (1974) used their model to study the transient effects of

highway deicing salts on groundwater in Massachusetts. Several highway deicing policies, involving different salt applications, were reviewed. Their work clearly demonstrated that years could elapse before continuous source loading would be reflected in changes in ambient groundwater quality; consequently, Gelhar and Wilson (1974) concluded that groundwater quality monitoring alone would not adequately reflect the true magnitude of developing groundwater contamination from land use activities.

Mercado (1976) developed a lumped parameter model of a coastal aquifer underlying 87 square kilometers of agricultural land. The whole aquifer was represented mathematically as a complete mix compartment. Contaminant concentrations at natural points of groundwater discharge and at pumping wells were the same for the entire system. The model consisted of a simple water balance equation and an equation expressing the conservation of solute mass. Mercado (1976) studied the regional chloride and nitrate pollution from irrigation, fertilizers, and land application of treated waste water. The model was calibrated to reproduce an historical water quality record, and then used to evaluate 13 alternative groundwater protection measures.

Gelhar and Wilson (1974) and Mercado (1976) found that their lumped parameter models could deliver useful information on temporal changes in regional groundwater quality, if sources of comparable magnitude were uniformally distributed, and if sufficient water quality data were available. In addition, these investigators showed that lumped parameter models could be used to facilitate the evaluation of land use

management strategies for the protection of groundwater quality from nonpoint source pollution. However, spatial heterogeneities in the intensity of nonpoint source pollution exist in the field and erroneous results and conclusions could be gathered from this form of modeling if used to formulate or evaluate management strategies to preserve subsurface water quality. This is because the loss of spatial resolution resulting from the use of lumped parameter models may preclude determination of where and to what extent groundwater protection should be undertaken through land use controls. Mercado (1976) and Gelhar and Wilson (1974) employed their models to investigate the merits of multiple regional nonpoint source groundwater protection strategies. However, spatial variations of nonpoint source pollution intensity were supressed, leaving it possible that their models could have obscured the true merits of the several groundwater protection strategies reviewed.

2.2 Distributed Parameter Simulation Modeling of Regional Groundwater Quality

Unlike lumped parameter models, which yield steady-state or transient changes in the regional average phreatic surface elevation and contaminant concentration, distributed parameter models attempt to reproduce observed spatial and temporal or just spatial changes in the phreatic surface elevation and contaminant concentrations. These models are conceptual in that they are predicated on differential equations descriptive of the conservation of mass, energy, and momentum.

Distributed parameter models are used if a more detailed or more realistic depiction of the groundwater system is desired. As in the lumped parameter models, the components of distributed parameter models are mathematical descriptions of groundwater flow or contaminant transport. Because distributed parameter models attempt to reproduce that which is observed in both space and time, the equations for solute transport and fluid flow are considerably more complex than the simple mass balance equations used in the lumped parameter models.

## 2.2.1  Governing Flow and Mass Transport Equations

In most cases the aquifers being modeled are characterized by natural spatial heterogeneities in the physical and chemical characteristics (e.g., porosity, permeability, chemical adsorptive capacity) of the porous medium which determines the ease of fluid flow and contaminant movement in different directions through the solid matrix. An isotropic medium allows fluid to flow with equal ease in all directions, while an anisotropic medium exhibits directional variability in the properties of the solid matrix to transmit water. If vertical movement of water is minor, equation (1) describes the transient, two-dimensional, areal flow of a homogeneous fluid through an unconfined, horizontal, nonhomogeneous, anisotropic aquifer (Pinder and Bredehoeft, 1968).

$$\frac{\partial}{\partial x_i} \left( K_{i,j} B \frac{\partial h}{\partial x_i} \right) = S_y \frac{\partial h}{\partial t} + Q_{out} - Q_{in} \qquad i,j=1,2 \qquad (1)$$

where  $K_{i,j}$ = vertically averaged hydraulic conductivity tensor (L/t),

which is a physical parameter indicating the ease with

which water passes through porous material in the

direction i,j;

$S_y$ = vertically averaged specific yield (dimensionless),

which physically corresponds to the percent of saturated

porosity which drains under the force of gravity;

h = hydraulic head, (L);

B = saturated thickness which may equal h for an unconfined

aquifer with a horizontal impermeable bottom boundary,

(L);

$Q_{in}$ = volumetric flux of recharge per unit surface area of

aquifer, (L/t);

$Q_{out}$ = volumetric flux of withdrawal per unit surface area of

aquifer, (L/t);

$x_i$, $x_j$ = horizontal coordinate axis i,j, (L);

L = length;

t = time.

The unconfined groundwater flow equation is solved analytically or

numerically to reproduce observed areal and transient changes in the

water table elevation.  Several numerical models have been developed by

Prickett and Lonnquist (1971), Trescott, et al. (1976), etc.  Assuming

the flow equation has been solved by whatever method is deemed

appropriate, Darcy's equation is used to translate spatial variations in

the phreatic surface elevations into a horizontal fluid velocity field for two-dimensional groundwater flow. Darcy's equation is written as

$$v_i = K_{i,j}\left(\frac{\partial h}{\partial x_j}\right) \qquad i,j=1,2 \tag{2}$$

where $v_i$ = the vertically averaged specific discharge (L/t) or the

mass average flux of fluid flow in the i direction;

$K_{i,j}$, h, i, j, and $x_j$ are defined above.

Once the constant velocity field is defined a third and final equation is solved to estimate solute concentrations in space and time. Equation (3) describes transient, two-dimensional areal, advective-dispersive transport of a miscible contaminant through an unconfined, horizontal, nonhomogeneous, anisotropic aquifer (Konikow and Bredehoef, 1978).

$$\frac{\partial}{\partial x_i}\left(BD_{i,j}\left(\frac{\partial c}{\partial x_j}\right)\right) - \frac{\partial}{\partial x_i}\left(Bv_iC\right) = n_e\left(\frac{\partial(BC)}{\partial t}\right) - Q_{in}C^p + Q_{out}C +$$

$$n\cdot B\cdot \sum_k V_k^s \qquad i,j=1,2 \tag{3}$$

where C = the vertically averaged concentration of dissolved

chemical species, $(M/L^3)$;

$D_{i,j}$ = vertically averaged coefficient of hydrodynamic

dispersion $(L^2/t)$ which is descriptive of the combined

effects of Fickian diffusion and dispersion caused by

microscopic variations in fluid velocities within

individual pores;

$n_e$     = vertically averaged effective porosity (dimensionless);

$c^p$     = concentration of dissolved chemical at the source, $(M/L^3)$;

$V_k$     = chemical, biological, or physical reaction k, negative for the addition of solute and positive for the removal of solute, $(M/L^3t)$;

s     = total number of possible contaminant reactions;

$x_i$, $x_j$ = horizontal coordinate axis i and j, (L);

L     = length;

t     = time;

$v_i$, h, B, $Q_{in}$, and $Q_{out}$ are defined above.

Like the groundwater flow equation, the above equation for advective-dispersive contaminant transport has been solved numerically (Konikow, 1977) and analytically (van Genuchten and Alves, 1982). The first term in equation (3) approximates contaminant transport due to hydrodynamic dispersion. Dispersion is important wherever steep contaminant concentration gradients occur in the groundwater system, such as along the edge of a plume. The dispersion coefficients are often described in the literature as functions of the fluid velocity and the longitudinal and lateral dispersivity coefficients (Anderson, 1979 and Mercer and Faust, 1981), where the dispersivity coefficients operate as convenient calibration parameters. The effects of dispersive transport are difficult to replicate in simulation without considerable

data to calibrate the dispersivity coefficients. The second term appearing in the solute transport equation describes advective transport, if it is assumed that dispersion can be ignored, the first term appearing in equation (3) is dropped leaving a simple equation for two-dimensional horizontal advective transport of a miscible contaminant.

### 2.2.2  Numerical Approximations

Unless the groundwater flow field is simple or predefined, a distributed parameter model for simulating transient or steady-state changes of groundwater quality in space contains three governing equations, a groundwater flow equation, Darcy's equation to translate results from the flow model into a constant velocity field, and a contaminant transport equation. Numerical as opposed to analytical solutions to the governing equations have permitted modellers to evaluate more complex transient and steady-state groundwater quality problems involving multiple sources and boundary conditions in two and three dimensional flow regimes in anisotropic aquifers.

To obtain a numerical solution for a steady-state two-dimensional horizontal groundwater flow model, the aquifer is first discretized into elements. Within each element aquifer characteristics are defined and assumed to be spacewise constant. From the partial differential equation for groundwater flow an algebraic equation is derived (by way of finite difference approximations of the partials) for each element. Each algebraic equation defines a mathematical relationship between the

water table elevation in the center of each element relative to the water table elevations in neighboring elements. An aquifer discretized into 100 elements will yield 100 algebraic equations written in terms of 100 variables corresponding to the discrete water table elevation in the center of every element in the aquifer domain. The set of 100 equations which numerically approximate the groundwater flow equation over a defined aquifer can be written in the form

$$r_{1,1}h_1 + r_{1,2}h_2 + r_{1,3}h_3 \cdots \cdots \cdots \cdots r_{1,n}h_n = Q_{out1} - Q_{in1} + R^b_1$$

$$r_{2,1}h_1 + r_{2,2}h_2 + r_{2,3}h_3 \cdots \cdots \cdots \cdots r_{2,n}h_n = Q_{out2} - Q_{in2} + R^b_2$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \qquad (4)$$

$$r_{n,1}h_1 + r_{n,2}h_2 + r_{n,3}h_3 \cdots \cdots \cdots \cdots r_{n,n}h_n = Q_{outn} - Q_{inn} + R^b_n$$

where $h_i$ = variable corresponding to the hydraulic head in element

i, (L);

$r_{i,j}$ = the combined parameters generated from the algebraic

approximation of the governing groundwater flow equation

at given element i in terms of neighboring element j,

for i,j = 1,...n, (1/t);

$Q_{outi}$ = assumed volumetric flux of withdrawal from element i,

(L/t);

$Q_{ini}$ = assumed volumetric flux of recharge into element i, (L/t);

$R^b_i$ = known boundary conditions in element i (e.g., constant flux conditions), (L/t);

n = number of elements.

Using vector notation:

$$[R]\{h\} + [I]\{Q_{out}\} - [I]\{Q_{in}\} = \{R^b\} \tag{5}$$

where {h} = nx1 vector of variables corresponding to the hydraulic heads at every element;

[R] = nxn vector of coefficients generated from the algebraic approximation of the governing flow equation;

[I] = nxn identity matrix;

$\{Q_{out}\}$ = nx1 vector of assumed or known pumping or withdrawal rates;

$\{Q_{in}\}$ = nx1 vector of assumed or known recharge rates;

$\{R^b\}$ = nx1 vector of known boundary conditions.

A solution to the expanded steady-state groundwater flow model is obtained by a simultaneous solution of the 100 algebraic equations. If a numerical solution were desired for the above problem but for transient conditions, then a solution for the 100 algebraic equations would have to be obtained for each time step in a series of steps taken through a desired time period. In simulating observed aquifer behavior the magnitudes of $Q_{ini}$, $Q_{outi}$, and $R_i$ would vary in time; consequently

the values of these terms would be specified and held constant for each time step.

A numerical solution to the steady-state two-dimensional horizontal advective-dispersive contaminant transport equation is obtained in a manner similar to the groundwater flow equation, yielding a set of 100 algebraic equations corresponding to the same 100 discrete elements in the hypothetical aquifer.

$$e_{1,1}C_1 + e_{1,2}C_2 + e_{1,3}C_3 \cdots\cdots\cdots e_{1,n}C_n = -Q_{in1}C_1^p + b_1$$

$$e_{2,1}C_1 + e_{2,2}C_2 + e_{2,3}C_3 \cdots\cdots\cdots e_{2,n}C_n = -Q_{in2}C_2^p + b_2$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \qquad (6)$$

$$e_{n,1}C_1 + e_{n,2}C_2 + e_{n,3}C_3 \cdots\cdots\cdots e_{n,n}C_n = -Q_{inn}C_n^p + b_n$$

where $C_i$ = concentration of contaminant at node i, $(M/L^3)$;

$e_{i,j}$ = the combined parameters generated from the algebraic approximation of the governing mass transport model at given element i in terms of neighboring elements j, for i,j=1,...n, (L/t);

$C_i^p$ = concentration of waste injected in element i, $(M/L^3)$;

$b_i$ = known boundary conditions for element i (e.g.,

contributions of contaminant through natural sources),

$(M/L^2t)$;

$Q_{ini}$ is defined above.

Using vector notation:

$$[e]\{C\} = - [I]\{Q_{in}C^p\} + \{b\} \tag{7}$$

where {C} = nx1 vector of variables corresponding to the

concentration of contaminant in every element;

[e] = nxn vector of coefficients generated from the algebraic

approximation of the governing mass transport equation;

[I] = nxn identity matrix;

$\{Q_{in}C^p\}$ = nx1 vector of known waste injection fluxes;

{b} = nx1 vector of known boundary conditions.

Again a solution to the expanded steady-state mass transport model is

obtained by simultaneous solution of the 100 algebraic equations.

The validity of model simulation results depends on the extent of

model calibration attainable with existing data. Calibration of

distributed parameter groundwater quality models is a process of

adjusting parameters (e.g., porosities, hydraulic conductivities,

storage coefficients, dispersivity coefficients, and reaction

coefficients) and boundary conditions (e.g., constant flux) in the

hydraulic and the contaminant transport models until observed phreatic

surface elevations and contaminant concentrations are reproduced in the

mathematical simulation. Calibration requires data from transient and steady state conditions of flow and contaminant transport. Simulation results are only as accurate as the data which describe aquifer properties, water table elevations, and contaminant concentrations in the elements of a discretized aquifer (Reddell, 1970). As the scale of modeling increases from local to regional levels, the availability of data to describe groundwater quality changes decreases. In regional modeling, the elements of discretization are often increased to accommodate decreases in available data; to do otherwise yields detailed simulations which are generally unsupportable and perhaps deceptive. Therefore, the resolution of regional simulations should be restricted to the same order of detail exhibited from available data.

### 2.2.3 Applications

Distributed parameter models have been used primarily though not exclusively as simulation models. Many modeling efforts have focused on simulating groundwater pollution under transient conditions. Models have been employed to trace the movement of contaminant plumes (Konikow, 1977 and Dasqupta et al., 1984), explain historical changes in groundwater quality (Bredehoeft and Pinder, 1973), and predict transient groundwater quality impacts of various land use activities (Robson and Saulnier, 1981 and Konikow and Bredehoeft, 1974).

Several examples of groundwater simulation modeling can be cited; most of these studies address groundwater problems originating from point source pollution, while only a few have detailed water quality

implications of nonpoint source pollution. Robson and Saulnier (1981),
used a three-dimensional distributed parameter advective mass transport
model coupled to a three-dimensional flow model to simulate the
potential transient nonpoint source water quality impacts of dewatering
operations at a proposed oil shale mine in Northwestern Colorado. They
predicted that changes in groundwater flows induced from mine dewatering
operations would alter the chemical quality of ground and surface waters
in the area. In a different study Konikow and Bredehoeft (1974) studied
the effects of irrigation practices and strategies on the distribution
of dissolved solids in an alluvial aquifer in Colorado, where serious
nonpoint source pollution has resulted from a long history of crop
irrigation.

Other examples in which groundwater quality simulation models were
used to characterize or evaluate schemes of controlling nonpoint source
pollution can be presented; however, more effective use of simulation
models has been with their application in groundwater quality management
models as direct or indirect optimization models.

## 2.3 Groundwater Quality Management Models

Groundwater quality management models are simply optimization
models which have been coupled by any one of several methods to the
response surface of a calibrated mass transport model. These models can
operate as efficient tools for generating strategies of coordinating
water supply demands and subsurface disposal needs in groundwater
systems which function as both sources of water supply and receptacles

of waste waters. Several models are reviewed below which have potential
application in the formulation or evaluation of transient and steady-
state schemes of managing point and nonpoint source groundwater
pollution. Unfortunately, because most applications of management
models have been with hypothetical test aquifers, the true utility of
these models remains to be demonstrated in the field.

### 2.3.1 Steady-state Models for Point Source Pollution

Willis (1976) recognized groundwater systems as multipurpose
resource systems used conjunctively as sources of potable water and as
sites for the treatment and disposal of wastes. He examined a
hypothetical regional wastewater treatment system comprised of surface
waste water treatment, imported dilution water, and the waste
assimilative capacity of the underlying aquifer. The hypothetical
aquifer had two wastewater injection wells and two water supply wells.
Willis (1976) formulated a nonlinear mixed integer programming model to
select a cost effective combination of unit wastewater treatment
processes to produce an injectable effluent which would satisfy water
quality constraints at the injection wells and at supply wells. The
decision variables were Q (the flow rate of the treatment plant which
included the initial wastewater flows and the dilution water D), D (the
flow rate of the imported dilution water), plus integer variables
$X_i$ (corresponding to affirmative or negative decisions on available
choices of unit treatment options i=1,...17). The nonlinear cost
objective function incorporated transmission costs of imported dilution

water (based on pipe capacity, distance, and method of transmission), and annual treatment plant costs (based on flow). The objective function was minimized subject to linear constraints on flow capacities of treatment plants, nonlinear water quality constraints on allowable contaminant levels at injection wells and water supply wells, and finally a linear constraint limiting construction to one treatment plant.

The water quality constraints were derived from matrix manipulations of a finite difference approximation of steady-state two-dimensional horizontal advective contaminant transport model incorporating first order biochemical reactions and linear adsorption. A steady-state model was used because management decisions could be based upon the ultimate response of the groundwater systems to a policy of continuous contaminant injection and sustained demand for potable water. It was necessary to know the location and rates of groundwater recharge and discharge to define in advance the contaminant transport simulation model over a desired constant velocity field (meaning that a steady-state solution to a groundwater flow model was obtained external to the mixed-integer programming model so that the Darcian velocity coefficients appearing in the solute transport model could be defined). Recalling from page (21) that a numerical finite difference approximation of the two-dimensional horizontal mass transport equation is a set of algebraic equations

$$[e]\{c^e\} = \{f\} \tag{8}$$

where [e]    = nxn finite difference coefficient matrix which is
             derived from the numerical discretization for which
             spacewise constant Darcian velocity coefficients,
             kinetic reaction parameters, and dispersivity
             coefficients have been defined. However, if injection
             rates are unknown, this vector will contain expressions
             defining the Darcian velocity coefficients as functions
             of the decision variable D (the dilution water flow
             rate) and the constant initial flow rate of the waste
             stream;

$\{C^e\}$    = nx1 column vector of decision variables corresponding to
             the concentrations of contaminant e=1,2...z, for each
             element;

$\{f\}$      = nx1 column vector defining boundary conditions and input
             fluxes of contaminant p as a nonlinear function of the
             integer decision variable $X_i$, the chosen treatment
             received before subsurface injection and the decision
             variable D, dilution water flows;

n            = the number of elements of discretizaton.

Willis (1976) computed the inverse of the coefficient matrix $[g]^{-1}$ to
obtain a new set of algebraic equations.

$$\{C^e\} = [e]^{-1}\{f\} \tag{9}$$

This new set of equations yielded a vector of decision variables
$\{C^e\}$ corresponding to the contaminant concentrations in each element

expressed as a function of decision variables corresponding to treatment plant design $(X_i)$ and dilution water flows (D). A small subset of equations was selected to serve as water quality constraints. This subset of equations described the concentration of constituent e in the elements containing water supply wells and injection wells; the remaining equations which described contaminant concentrations in the other elements were ignored. Willis (1976) looked at several contaminants (BOD, nitrates, metals, etc.) simultaneously; hence, small subsets of water quality constraints were obtained for each contaminant.

A solution to the programming problem would yield a cost effective combination of unit processes and dilution water necessary to preserve a minimum level of water quality at injection wells and supply wells. Willis (1976) was not able to obtain a solution directly but studied each of the 17 possible wastewater treatment plant designs individually by decomposing the mixed integer model into 17 separate continuous variable optimization problems. Each of the 17 optimization problems was further simplified by ignoring the changes imposed on the subsurface hydraulics through the use of imported dilution water. This permitted linear approximations of the nonlinear water quality constraints through a simple elimination of decision variable D from the vectors {f} and $[e]^{-1}$ used to derive these constraints. In their final form the 17 optimization problems had nonlinear objective functions subject to linear constraints. Willis (1976) used a nonlinear optimization technique to solve each problem.

The greatest contribution of this work was the development of water quality constraints from the distributed parameter model. However, his real intent was to develop a management model which could be used as an efficient accurate instrument to determine a cost effective treatment plant design and the necessary dilution water flows to maintain groundwater quality at injection wells and water supply wells. The model failed to be efficient since every feasible treatment plant designed had to be evaluated in separate optimizations. Willis (1976) was able to obtain a linear approximation of the water quality constraints by ignoring the hydraulic effects of injecting unknown volume of dilution water. But this approach may have severely compromised the accuracy of the model because 14 of the 17 optmization models (one for each plant design) selected optimum dilution water flows which were 6 times the flow rate of the initial waste stream.

Gorelick and Remson (1982a) developed two efficient steady state management models for siting point source subsurface waste disposal facilities and determining the associated groundwater wasteload allocations. The groundwater quality management models were simple linear programming models. A finite difference approximation of a steady-state two dimensional areal advective-dispersive mass transport model expressed as

$$[e]\{C\} + [P]\{M_{in}\} = \{b\} \tag{10}$$

where $[e]$ = a nxn matrix of coefficients derived from a known constant velocity field and finite difference approximation of the mass transport equation;

[P]     = a nxn diagonal matrix with values of one for entries that
        correspond to the injection sites and values equal to
        zero for all other entries;

{C}     = a nx1 column vector of decision variables defining solute
        concentrations throughout the system;

$\{M_{in}\}$ = a nx1 vector of decision variables defining the

        contaminant injection fluxes (each flux equal to

        $Q_{in}C^p$, the solute concentration in the injected waste

        times the flow rate);

{b}     = nx1 right-hand side vector reflecting boundary conditions
        (i.e., existing disposal fluxes);

n       = the number of nodes or elements.

was directly embedded as part of the constraint set of a linear

programming model. Decision variables corresponded to unknown

contaminant concentrations at each element (C) of a discretized aquifer

and the waste disposal fluxes $(M_{in})$ occurring in predetermined elements.

Because the solute transport model was a component of the linear

programming formulation of each groundwater management model, a solution

to each management model led directly to simultaneous estimates of the

maximum disposal fluxes and the associated steady-state response

(contaminant concentrations in every element) of the groundwater system

to the disposal activities.

Each management model required a mass transport model and a

groundwater flow model which were both calibrated to the aquifer being

managed. Both management models, however, were tested over hypothetical aquifers; hence, selection of dispersivity coefficients, porosity, transmissivities and boundary conditions used in the hydraulic and the mass transport models was purely arbitrary. Steady-state modeling results were desired because Gorelick and Remson (1982a) felt such groundwater system responses often represent worst case scenarios. Although contaminant reactions were ignored, management model formulations were sufficiently general that reactions could be easily incorporated.

Before the solute transport simulation model was embedded into the optimization model, it was necessary to define Darcian velocity coefficients and dispersion coefficients (recall that the dispersion coefficients are a function of velocity components) appearing in the mass transport equation; consequently, information on the location of existing waste disposal sites and water supply wells and their respective injection and withdrawal rates was needed to permit advance definition of the hydraulic regime. Finally data were needed on the concentration of contaminant in wastes at existing and proposed subsurface disposal sites to permit calculation of injection volumes from the optimum values of decision variables representing disposal fluxes.

Two different groundwater management problems were presented and investigated using the different models. In the first problem, a hypothetical aquifer contained one existing disposal site discharging a 1000 mg/l chloride waste at a rate of 200 l/s and two potential disposal

sites upgradient from three water supply wells pumping at known rates.
The management problem was to maximize waste loading to the aquifer
while maintaining water quality standards (250 mg/l Cl) at water supply
wells. The objective function of the first groundwater quality
management model was simply

$$\text{Maximize} \quad \sum_{i=1}^{3} ( M_{in} )_i \tag{11}$$

where $(M_{in})_i$ = decision variable corresponding to the injection flux at

proposed injection site i.

The above objective function was subject to constraints generated from
the embedded solute transport model, constraints imposing maximum
allowable values on decision variables corresponding to solute
concentrations at water supply wells, constraints defining existing and
permissible disposal activities in the various discrete elements
throughout the aquifer, and nonnegativity constraints.

A solution to the first management model led to estimates of the
allowable subsurface injection fluxes at each proposed waste disposal
site which did not violate groundwater quality standards at supply
wells. Parametric programming was used to investigate disposal
tradeoffs between the existing and the proposed disposal sites. It was
found that a slight reduction in the disposal flux at the existing site
would permit an overall increase in the total allowable waste load
delivered from all three sites.

With a second management model, Gorelick and Remson (1982a)
addressed a different problem of siting new subsurface waste disposal

facilities. The goal was to seek out sites suitable for the disposal of a liquid waste at a known constant flux. In another hypothetical aquifer, 56 potential disposal sites were identified in a delineated zone upgradient from two water supply wells. Any one site was considered suitable if a constant disposal flux of 500 g/s chloride could be delivered without violating a 250 mg/l chloride standard at either of the two water supply wells. Gorelick and Remson (1982a) constrained the waste discharge fluxes for each of the 56 sites to a value of one. They then maximized contaminant concentrations first at one water supply well and then for the other. The resultant optimum values of the dual variables were interpreted as 'unit source impact multipliers' and used to predict the effect of a per unit change in the disposal flux at any of the 56 potential facility sites on water quality at each of the two supply wells. Six sites were identified where waste disposal could be conducted without endangering the potable water supply.

The groundwater quality management models presented by Willis (1976) and Gorelick and Remson (1982a) were solved by different programming techniques seeking to optimize single objective functions subject to various constraints on water quality and quantity, and contaminant source loading fluxes. The water quality constraints were derived directly (through an embedding technique) or indirectly (through complex matrix inversions) from steady-state areal finite difference contaminant transport models. The models developed by Gorelick and

Remson (1982a) generated more information and were easier to construct and solve than the model formulated by Willis (1976).

## 2.3.2  Transient Models for Point Source Pollution

Several transient groundwater quality management models have been developed for optimizing the management of aquifers conjuctively used as sources of potable water and as subsurface waste disposal systems. Most of these models have been used to evaluate the continuous transient disposal at injection wells over specified management time horizons and were derived from transient one or two-dimensional horizontal solute transport models. In these management models the constraints defined groundwater quality at specified points in space and time as a function of sustained mass flux loadings or constant waste concentrations at disposal wells for specified management periods. The constraints were composed of influence coefficients descriptive of the unit change in water quality at an observation well resulting from a unit change in disposal flux at each disposal well, and decision variables corresponding to the disposal flux for each injection well at each time increment. Influence coefficients were derived from multiple simulation of the mass transport models; one simulation for each source.

In transient management models, the size of the mathematical problem is a function of the number of disposal wells and observation wells, the size of the area under management, and the length of the planning time horizon. With every model either the number of sources, the size of the  aquifer, or the length of the planning horizon were

curtailed to keep the size of problem tractable (sufficiently small that a solution could be obtained). Willis (1979) had to restrict the planning time horizon to 480 days because the aquifer being managed was 50 $km^2$. Gorelick and Remson (1982b) developed a one-dimensional horizontal transient management models for a confined aquifer 5 km long, but they limited their study to the management of 3 sources and 3 water supply wells and a management horizon of 600 days. In a more complex two-dimensional model with seven injection wells and eight observation wells, Gorelick (1982) studied the transient disposal policies for a nine year planning horizon over a small hypothetical aquifer of 2 $km^2$.

The utility of the transient management model for evaluating the optimal control of nonpoint source pollution may be limited in light of the regional nature of the groundwater quality modeling problem, the large number of sources, and the fact that long term analysis is needed since many sources are by nature semipermanent (i.e., agricultural activities and septic systems). One model (Louie, et al., 1984) will be reviewed because it was formulated to view groundwater quality impacts from multiple sources over a large basin at only specified times and not continuously as in the models above; consequently the complexity of the model does not expand as rapidly with increases in the aquifer managed, the number of injection wells, or the management horizon.

Louie, et al. (1984) constructed a linear multiobjective water resources planning model to view aspects of allocating water resources, groundwater quality control, and prevention of undesirable groundwater overdraft simultaneously. Water quality constraints served to link the

multiobjective optimization model to external finite difference approximations of equations for transient two-dimensional horizontal groundwater flow and mass transport. The model was designed for application over a large basin and was tested on a hypothetical basin containing three water supply sources (two sources are groundwater, and one is imported water), municipal, industrial, and agricultural water supply users, three wastewater treatment plants (having different capacities and treatment efficiencies), two treatment plants which recharge agricultural return water, and six wastewater disposal sites (one outside the basin). Louie, et al. (1984) simplified the problem of managing a large basin by subdividing the basin into smaller areas called "operational areas". The occurrence of wastewater treatment, wastewater disposal, and groundwater pumping and recharge was predetermined within each operational area.

The first of three objective functions was an economic function defining the costs of supplying water to multiple users and the costs of wastewater transmission to and from treatment plants to ultimate sites of disposal. This objective function was assumed linear with respect to decision variables corresponding to $q_{ij}^s$ (the quantity of water delivered from source i to user j), $q_{jk}^t$ (the quantity of wastewater transported from user j to treatment plant k), and $q_{kl}^d$ (the quantity of wastewater effluent from treatment plant k transported to disposal site l). A second objective function was used to search out management plans to minimize groundwater overdraft. This objective function was simply a

summation of decision variables corresponding to the quantities of groundwater pumped from multiple supply wells less the summation of decision variables corresponding to the quantities of water recharged at various points throughout the basin. Finally aspects of groundwater quality control objectives were expressed through a third objective function consisting of a single decision variable 'D' representing the deviation between the maximum simulated contaminant concentrations in predetermined operational areas and water quality standards for the operational areas.

All three objective functions were minimized subject to several different sets of constraints. The first series of constraints defined available water supplies, water demands, treatment plant capacities, ultimate disposal site capacities, and transmission losses to and from wastewater treatment plants. These constraints were expressed in terms of decision variables representing quantities of water transferred through a known network of pipe connections between sources, users, treatment plants, and disposal sites. Upper and lower limit constraints specified the maximum and minimum quantities of water transported though each pipe. Finally there was a set of constraints included which attempted to capture the response surface of the external simulation models for groundwater quality and quantity. These water quality constraints expressed the maximum deviation between approximate simulated contaminant concentrations in groundwater and allowable limits in predetermined operational areas. Each water quality constraint was

constructed from predetermined influence coefficients (Becker and Yeh, 1972); typical constraints took the form

$$D_a^O + \sum_{k=1}^{P} \left. \frac{\partial D_a}{\partial q_k} \right|_{q_O} \Delta q_k \leq D_a \qquad \forall~a \tag{12}$$

$$D_a \leq D \qquad \forall~a \tag{13}$$

where   $D$         = decision variable corresponding to the maximum

deviation occurring between approximate simulated

contaminant concentrations in groundwater and

allowable limits in operational areas $(a = 1,2,...n)$;

$D_a^O$        = initial positive deviation from the water quality

standard in operational area 'a' under the $q_O$

conditions;

$D_a$         = approximate positive deviation resulting from the

management scheme as defined though the optimum

values of $q_k$;

$\left. \dfrac{\partial D_a}{\partial q_k} \right|_{q_O}$      = is the influence coefficient defined as the change in

$D_a$, the positive contaminant deviation in operational

area 'a', due to a unit change in the kth decision

variable around the initial condition $q_O$;

$q_k$ = p decision variables corresponding to either the quantity of water from supply wells or the quantity of wastewater transported, or the quantity of water recharged in the various regions (k=1,2,...P) of the basin;

$\Delta q_k$ = $q_k - q_k^o$;

$q_k^o$ = is the initial condition of the k decision variable, this could be a water supply pumping rate, wastewater transmission flow rate or recharge rate for k=1,2...P.

The influence coefficients were derived through exhaustive groundwater flow and mass transport simulations external to the management model. One simulation of the coupled groundwater flow and mass transport model was required for each decision variable to obtain all the necessary influence coefficients. Though it was not explicitly stated, it was believed that the influence coefficients represented long-term or steady-state changes in water quality in the operational areas resulting from unit perturbations of decision variables $q_k$.

Minimizing the water quantity objective function alone (without considering groundwater quality or excessive overdraft) generated cost effective plans for meeting water demands, allocating and treating wastewaters and siting ultimate disposal of wastewater effluent. Minimizing the third objective (the water quality objective) alone, subject to all of the the constraints described above yielded the best

feasible groundwater quality regardless of the allocation costs or the extent of overdraft. To view the tradeoffs among multiple objectives the constraints method was used to develop trade-off curves of the noninferior solution sets for cost versus water quality, cost versus overdraft, and cost versus water quality versus overdraft. The trade off curves provided an understanding of the interactions among the water resource planning objectives.

### 2.3.3  Steady-state Models for Nonpoint Source Pollution

No steady-state management models for nonpoint source pollution have been found in the literature; however, several reasons can be stated as to why steady-state modeling is particularly suited for deriving management schemes to protect groundwater from nonpoint source pollution. Consider first, that many sources contributing to distributed groundwater contamination are semi-permanent or are expected to persist for prolonged but indefinite periods of time (e.g., agricultural activities, on site septic systems, etc.); consequently, resource protection through transient controls (structural and nonstructural) would not be as reliable or as easy to enforce as long term restrictions on the presence or intensity of various activities in critical recharge areas (Devine and Ballard, 1983). Secondly, contaminated aquifers are slow to show the full impact of continuous discharges and are also slow to recover, both of which necessitate long term water resource planning. Finally, steady-state conditions often

represent the worst case pollution scenario, which makes consideration of the long term water quality impacts a conservative approach.

Steady-state management models can readily operate as efficient tools to evaluate the long term water quality impacts of nonpoint source pollution, estimate subsurface waste assimilative capacity, and determine the optimum pattern of long term land use development that minimizes the degradation of subsurface water quality. The development and demonstration of steady-state models for the management of nonpoint source groundwater pollution will be seen in Chapter 7, 8 and 9.

## 2.3.4  Transient Models for Nonpoint Source Pollution

The groundwater quality management models discussed thus far have directly or indirectly incorporated the response surface generated from distributed parameter models of groundwater flow and contaminant transport. These management models were used to examine local (Gorelick and Remson, 1982) and regional (Willis, 1976) groundwater contamination from point source pollution alone and then only through hypothetical case problems. Helweg and Labadie (1976 and 1977) were among the first investigators to develop and apply a distributed parameter groundwater quality management model for nonpoint source pollution. Working on a nonpoint source pollution problem in the Bonsall Subbasin of the San Luis Rey River basin in California, Helweg and Labadie sought to manage groundwater salinity levels by controlling the distribution of waters pumped from various wells (sources) possessing different water quality

to various irrigated fields (destinations) having different underlying groundwater quality.

Normally irrigation water is applied to land surrounding the pumping well, where it drains back into the aquifer only to be recaptured by the well and reused as irrigation water. The concentration of dissolved salts increases as the applied water percolates through the soil. If any cyclic reuse of water occurs, this brings about increasingly higher levels of salinity in the ground waters.

Helweg and Labadie (1976) proposed that contamination from irrigation return flows could be flushed out of the aquifer at an accelerated rate if irrigation waters were applied at locations down gradient from their origin of subsurface withdrawal. Irrigating down gradient from the source would short circuit the continuous reuse of applied water and accelerate the down gradient movement of contaminated groundwaters.

The water allocation and water quality problem was solved using a linear programming model and ancillary simulation models for transient groundwater flow and mass transport. The linear cost objective function of the linear programming model was written in terms of decision variables $q_{ij}$ (the average annual amount of water transported from source i to demand location j); only transportation costs were considered. The value of the objective function was minimized subject to source supply constraints, water demand constraints, water mass continuity constraints, and water quality constraints.

The water quality constraints operated to couple the linear programming model to the external unsteady areal finite difference groundwater flow and contaminant transport models, which were calibrated for the entire basin. The water quality constraints placed upper limits on the allowable flow averaged salinities of irrigation drainage waters produced in each of four sections of the discretized groundwater basin. Iterations between solutions of the linear programming model (which estimated optimum pumping and recharge fluxes throughout the aquifer) and solutions to the external finite difference models (which estimated resultant salinities from the water allocation policy) were necessary to converge upon a water allocation policy which would meet irrigation demands and assure that average salinities of groundwaters underlying irrigated fields fell within desired levels.

The Helweg and Labadie model (Helweg and Lebadie, 1976) searched out cost effective management alternatives of meeting irrigation demands that would simultaneously effect desirable salinity levels for groundwaters underlying irrigated croplands. Feasible water allocation strategies would deliver needed water to meet irrigation demands and attain desirable groundwater salinities. The best feasible strategy was the one identified as costing the least. Though their model was simple, it was however, limited to managing groundwater salinities in stream aquifer systems, where sufficient water external to the basin could be imported to balance evaporation losses and outflows. A more generalized model formulation would permit broader application to the analysis of other various forms of nonpoint source pollution.

More recently, Mooseburner and Wood (1980) formulated a groundwater quality management model to identify land use patterns which would minimize impacts on groundwater quality. Information from the model could be used to design land use regulations to control or prevent groundwater degradation. The management model incorporated the response surface of a transient two-dimensional horizontal analytical mass transport model (Cleary, 1978) into a multiobjective goal programming optimization model. Mooseburner and Wood (1980) were particularly interested in the impacts of a rapidly growing residential population on groundwater quality; consequently, their management model was used to identify patterns of unsewered residential development which would permit the attainment of desired groundwater quality goals.

Jackson Township of the New Jersey Pine Barrens was chosen as the study site because 50 percent of the existing homes use septic systems and because the population is expected to quadruple between the years 1970 and 2000. On-site domestic waste disposal systems deliver nitrates to the underlying aquifer which poses a health hazard to the growing population of Jackson Township which depends on the aquifer as a source of potable water.

Based on present and projected land use patterns, 17 discrete land use sectors were found within the 100 square mile area of Jackson Township. Spaced between the 17 identifiable land use sectors were several equally large parcels of land which were ignored in the investigation because they were sparsely populated or were expected to exhibit little or no population growth.

The intensity of pollution in one sector affects water quality in other sectors. The 17 defined land use sectors, were treated as sources of nitrate, generated from undetermined populations of unsewered residents. A transient two dimensional areal analytical mass transport model (Cleary, 1978) was used to determine "transfer coefficients" $T^r_{i,j}$ (similar to the influence coefficients used by Louis, et al., 1984) which define the associated change in nitrate concentrations at surveillance point j (j=1,2...n) resulting from per unit change in the concentration of nitrates from the unsewered population $(X_i)$ in land use sector i (i=1,2...n). The physical location of surveillance points corresponded to the population centers of land use sectors. To use an analytical solution (Cleary, 1978), pollution from land use sectors was posed as contaminant plumes originating from the population centers of offending land use sectors; within the boundaries of offending land use sectors, contaminant was distributed in a Gaussian fashion along the horizontal axis perpendicular to the direction of groundwater flow. The analytical mass transport model incorporated first order decay and was calibrated for the area under investigation using a constant and uniform flow field and constant dispersion coefficients.

Decision variables used in the management model were $X_i$ (the unsewered population residing in each land use sector i), $d^+_j$ and $d^-_j$ (the positive deviation and the negative deviation of simulated nitrate levels at each surveillance point from desired goals), and $C_j$ (the total

nitrate concentration at surveillance point j). The objective function was composed from a summation of decision variables corresponding to the positive deviations of simulated nitrate concentrations above surveillance point goals.

$$\text{Min} \sum_{j=1}^{m} d_j^+ \tag{14}$$

Knowing the projected population growth and the desired water quality goals at each surveillance point, the model identified the optimum pattern of unsewered population growth which would minimize the positive deviation of the resultant nitrate levels from the specified goals. The objective function was minimized subject to contaminant mass balance constraints, land use sector population constraints, regional population constraints and water quality constraints.

Two types of mass balance constraints were used to couple the multiobjective goal programming optimization model to the response surface of the solute transport model. Equation (15) represents the first type of constraint in which the total concentration of nitrate at any surveillance point j ($C_j$) was determined using linear superposition and was defined simply as the background concentration ($C_o$) plus the summation of appropriate transfer coefficients ($T_{i,j}^r$) from all the other sectors i multiplied by the contributed concentration of nitrates resulting from the unsewered population in each sector. Nitrate concentrations contributed from septic systems in each sector i were

expressed as function the decision variable $X_i$, the unsewered population in sector i.

$$R \sum_{i=1}^{n} X_i F_i^{-1} T_{i,j}^r + C_o = C_j \qquad \text{for } j=1,2...m \qquad (15)$$

where $X_i$ = the decision variable for the unsewered population of land use sector i; i=1,2...n;

$R$ = nitrate production rate (mg/person-day);

$F_i$ = flow rate of groundwater at land use section i (1/day);

$T_{i,j}^r$ = transfer coefficient defining the ratio of the resultant concentration (at year 2000) at surveillance point j to the peak concentration at land use sector i;

$C_o$ = background nitrate concentration (mg/l);

$m$ = number of surveillance points;

$n$ = number of land use sectors.

The second type of mass balance constraint (Eq. 16) essentially equates the total nitrate concentration at surveillance point j to a desired water quality goal plus any positive or negative deviations from the goal. A set of these water quality constraints simply establishes absolute limits (standard) on nitrate concentrations at every surveillance point.

$$R \sum_{i=1}^{n} X_i F_i^{-1} T_{i,j}^r + C_o - d_j^+ + d_j^- = G \qquad \text{for } j=1,2...m \qquad (16)$$

where $d_j^+$ = the decision variable for the positive deviation from

the water quality goal at surveillance point j;

j=1,2...m;

$d_j^-$ = the decision variable for the negative deviation from

the water quality goal at surveillance point j;

j=1,2...m;

G = water quality goal (mg/l).

Results of minimizing the objective function subject to the above constraints showed the optimum pattern of residential development which would ensure minimum positive deviation from water quality goals for the year 2000. The model attempted to concentrate development in land use sectors near the boundaries where groundwaters discharge from Jackson Township into adjacent downgradient municipalities. The tendency of concentrating pollution near sites of groundwater discharge was a serious problem inherent in model application. The problem could be handled through either strict water quality standards for waters discharging from one region and entering another or through regional modeling of complete hydrologic units. The best possible residential development plan was not necessarily represented in their problem solution because much of the vacant land in Jackson Township was not considered available for development and because the region was discretized in rather large and nonuniform elements which could have resulted in a loss of model sensitivity and resolution.

The Mooseburner and Wood model (Mooseburner and Wood, 1980) could be applied only where an aquifer was assumed homogeneous and horizontal and where groundwater flows were approximately constant and uniform. A more general formulation would have permitted greater flexibility of application over a broader set of hydrogeologic regimes. In addition, this model (and the other management models which used transfer or influence coefficients to couple the response surface of the groundwater quality simulation model to the optimization model) required individual external groundwater water quality simulations for each contaminant source to identify values of transfer coefficients. For regional groundwater quality management problems involving numerous sources, the number of simulations necessary to define the transfer coefficients would render this management modeling approach cumbersome if not infeasible; hence, this management model design would remain limited to management problems involving a small number of sources.

## 2.4  Conclusions

Groundwater quality simulation models tied to optimization models are unequivocally more efficient at identifying plans of optimal groundwater management than simulation models alone. Because of the regional nature of nonpoint source pollution (extended over large areas of an aquifer), the large number of sources, or the long time horizon, transient management models are not suited for evaluating or formulating strategies of managing semipermanent nonpoint source pollution on a regional scale. Steady-state management models, however, appear to be

promising tools for ascertaining where and to what extent nonpoint
source subsurface pollution should be controlled to preserve ground
water resource availability, but no models have been developed.
Gorelick and Remson (1982a) have developed an efficient approach of
tying the water quality response surface of a steady contaminant
transport model to an optimization model. Their approach of embedding
the finite difference mass transport model as part of the constraint set
of an optimization model is a feasible means of formulating a water
quality management model for nonpoint source pollution generated from
numerous sources distributed over a large expanse of aquifer. Using the
embedding approach, in a management model will yield simultaneous
estimates of optimal nonpoint source disposal fluxes and the associated
steady-state response (contaminant concentrations in every discrete
element) of the groundwater system to the disposal activities.

CHAPTER 3


GENERAL FORMULATION OF A

NONPOINT SOURCE GROUNDWATER POLLUTION MANAGEMENT MODEL


This chapter introduces the conceptual framework of viewing an

aquifer under sustained nonpoint source pollution. Next, the basic

components of a management model are presented. Finally, an outline is

given for model application in the field.


3.1  Conceptual Framework of an Aquifer Receiving Nonpoint Source

     Pollution

In the research presented here groundwater flow and groundwater

quality changes will always be regarded in the conceptual frame of

regional changes occurring in a two-dimension horizontal aquifer. To

facilitate the modeling of subsurface flow and contaminant transport

processes, the aquifer is discretized into elements or cells. Spatial

changes in nonpoint source groundwater pollution and groundwater quality

impacts are approximated over the discretized two-dimensional horizontal

aquifer. Contaminant concentrations, hydraulic stresses (i.e., pumping

and recharge), and all activities contributing to the pollution of

groundwaters are perceived piecewise constant within each element.

Figure 1 illustrates a hypothetical aquifer discretized into elements

which are identified by an i,j coordinate system. The management models

are used to simultaneously select total elemental flows of pollutants

from various nonpoint source subsurface disposal activities (e.g. septic tank densities), and calculate the consequent steady-state contaminant concentrations in each element, i,j.

In the case of a distributed source such as pollution from on-lot septic tanks discharging into a descretized aquifer, the waste source can be characterized as either the total volume of septic tank effluent entering the aquifer within that element or the density of septic tanks within that element. The two perspectives are equivalent since all septic tanks are assumed to have the same strength and flow rate. In fact, the model works with discharge flows (at constant concentration) and the interpretation of the model is in terms of development density (number of houses per unit area).

The elemental regional nonpoint source groundwater wasteload allocations are calculated from the land use activities and the known contaminant concentration in subsurface disposal flows. From the discrete contaminant mass loadings, decision makers can obtain tangible estimates of desirable elemental source densities (i.e., septic tank densities or agricultural land use densities in each element), which are compatible with stated groundwater quality goals and land use controls.

## 3.2 General Management Model Components and Formulation

The various management models presented in later chapters are used to identify patterns of nonpoint source pollution that would be acceptable since they would be compatible with water quality goals and subsurface disposal needs. All of these management models are linear

programs. A linear program is a mathematical model comprised of a set of linear functions which include an objective function and a set of linear constraints. The objective function and constraints are written in terms of variables. Values of model variables are found which lead to an optimum value to the objective function under a set of satisfied constraints.

As linear programming formulations, the management models can simultaneously locate multiple sources, set each source contaminant flux, and predict groundwater impacts. In addition, postoptimal analysis can provide information about waste disposal and water quality tradeoffs associated with relaxing constraints on source densities (land use activities) and water quality standards.

The Management models presented in this work usually have the following five components described in greater detail in the next sections:

     1) Decision Variables;

     2) Objective Function;

     3) Management Constraints;

     4) Continuity Constraints;

     5) Nonnegativity Constraints.

## 3.2.1 Decision Variables

Two categories of decision variables are used in the management models. The first represents the flow of recharge from land use activities polluting groundwaters within each node. For a simple

contamination problem these recharge decision variables are represented as $Z_{i,j}$ for all nodes corresponding to all values of i and j. The other category of decision variables represent the steady-state depth-averaged concentrations of a dissolved constituent in each element resulting from the nonpoint source pollution. In this work the variable symbol $C_{i,j}$ is used to represent dissolved contaminant concentrations in node i,j.

### 3.2.2  Objective Function

The objective function is an algebraic representation of a criterion used to judge the quality of linear program solutions. A solution is a set of values for decision variables used in a management model. To solve a linear program is to identify a set of values for decision variables which maximizes or minimizes the value of objective function. In this research, the values of objective functions used in the various management models represented desirable groundwater pollutant distributions or feasible subsurface disposal flows. An example objective function is the summation of decision variables representing subsurface disposal flows. This objective function is written

$$\text{Max} \sum_{j=1}^{m} \sum_{i=1}^{n} Z_{i,j} \tag{17}$$

where nxm is the number of elements.

The optimum value of the objective function represents the maximum allowable recharge from land use activities polluting groundwaters in all elements within a study area.

### 3.2.3 Management Constraints

Management constraints place upper and/or lower boundaries on the numerical values of decision variables. Water quality and source density constraints are two major types of management constraints which appear in the nonpoint source groundwater pollution management models. Water quality constraints specify upper limits on values of decision variables representing dissolved contaminant concentrations. The upper limits for dissolved contaminant concentrations are either specified water quality standards or water quality goals. One water quality constraint is written for each element within a discretized aquifer. Typical water quality constraints assume the form

$$C_{i,j} \leq \text{(water quality standard or goal)} \qquad \forall \ i \ \text{and} \ j \qquad (18)$$

Water quality constraints ensure that the optimal values selected for the subsurface disposal flows meet water quality standards or goals for those elements.

Source density constraints specify upper and lower limits on subsurface disposal flows. Two types of source density constraints are used. The first type specifies minimum discharge flows in each element that reflect present day nonpoint source pollution activities. A simple form of this constraint is

$$Z_{i,j} \geq \text{(present day disposal flows)} \qquad \forall \ i \ \text{and} \ j \qquad (19)$$

The second form of source density constraint places upper limits on allowable nonpoint source pollution activity in each element, for example:

$Z_{i,j} \leq$ (upper limit or restriction on disposal flows in element

i,j) $\quad \forall$ i and j $\qquad$ (20)

The source density constraints ensure that the optimal pattern of polluting activities will fall with existing and allowable discharge or density limits.

### 3.2.4 Continuity Constraints

One continuity constraint is written for each element of a discretized study area. The continuity constraints tie subsurface recharge variables with variables representing contaminant concentrations. These constraints operate as an expressed approximation of the relationship between subsurface disposal activities and the resultant groundwater contamination. Each continuity constraint is an algebraic approximation of the partial differential equation governing contaminant transport in a specified element i,j. The form of these constraints is

$$G_{i,j} \left( C_{1,1}, \; C_{1,2} \ldots C_{1,m}, \; C_{2,1}, \; C_{2,2} \ldots C_{n,m}, \; W_{i,j}, Z_{i,j} \right) =$$

$$+ C_q Q_{i,j} + C_s S_{i,j} + C_u U_{i,j} \qquad \forall \text{ i and j} \qquad (21)$$

where $G_{ij}$ is a linear algebraic function composed of dissolved contaminant variables from all elements in the study area, plus variables representing nonpoint source subsurface disposal flow rates in element i,j. Terms on the right hand side are actually constants corresponding to known background contamination from existing sources

not subject to control (i.e, contamination from natural recharge, $C_q$ $Q_{i,j}$ and artificial recharge, $C_s$ $S_{i,j}$ and $C_u$ $U_{i,j}$).

### 3.2.5 Nonnegativity Constraints

Nonnegativity constraints are the final component of a management model. These constraints impose a restriction that values of all decision variables must be greater than or equal to zero. Beyond the fact that negative concentration and disposal flows have no physical meaning, it is a restriction of linear programming that decision variables not assume negative values. The form of these contraints is

$$Z_{i,j} \geq 0$$

$$C_{i,j} \geq 0 \qquad \forall \text{ i and j}$$

The nonnegativity constraints are imposed implicitly in the solution algorithm.

### 3.2.6 General Formulation

The management models presented in this work were always formulated with decision variables, an objective function, continuity constraints, and nonnegativity constraints. Depending on the model, management constraints were not always included.

An example formulation is presented below. This model incorporates all five of the components described in the preceding sections. Variation on the general formulation can be made with changes in the objective function and the constraint set. A solution obtained with

this model corresponds to values of decision variables which yield a maximum value to the objective function without violating any constraints. The objective function represents the total recharge from a spatially varying land use activity which pollutes groundwaters. For each element the optimal recharge flux and the steady-state contaminant concentration are evaluated.

The management constraints perform two tasks. First, values of recharge decision variables are restricted to fall between existing flows and the maximum recharge allowed by regulation. The second task is to ensure selection of recharge flows which produce a contaminant distribution which satisfies water quality standards.

The continuity constraints serve to tie recharge decision variable with contaminant variables in a manner consistent with algebraic functions describing the cause-effect relationship between subsurface disposal and groundwater contamination. Decision variables are precluded from taking on negative values because of the nonnegativity constraints. The complete formulation of this model is

Objective function:

$$\text{Max} \sum_{j=1}^{m} \sum_{i=1}^{n} Z_{i,j}$$

s.t.

Continuity Constraints:

$$G_{i,j} \left( C_{1,1}, \ C_{1,2} \ldots C_{1,m}, C_{2,1}, \ C_{2,2} \ldots C_{n,m}, \ W_{i,j}, Z_{i,j} \right) =$$

$$+ C_q \ Q_{i,j} + C_s S_{i,j} + C_u \ U_{i,j} \qquad \forall \ i \text{ and } j$$

Management Constraints:

$Z_{i,j} \leq$ (upper limit or restriction on disposal flows in element

i,j)     ∀ i and j

$Z_{i,j} \geq$ (present day disposal flows)     ∀ i and j

$C_{i,j} \leq$ (water quality standard or goal)     ∀ i and j

Nonnegativity constraints:

$Z_{i,j} \geq 0$

$C_{i,j} \geq 0$     ∀ i and j

## 3.3 Construction and Application of a Nonpoint Source Groundwater Quality Management Model

The combined use of numerical simulation models, numerical inverse models and linear programming will allow the construction and application of several nonpoint source groundwater pollution management models. Management model construction and application occurs over three phases. The first phase details the nature of the nonpoint source groundwater pollution problem and specifies site specific management information needs (see Chapter 6). In the second phase a validated numerical inverse model and a validated numerical groundwater flow model are used to define the subsurface fluid velocity field within the boundaries of the study area (see Chapter 6). Once the groundwater flow field is defined, phase three, the creation of the site specific management model, begins (see Chapters 7, 8, and 9). This last phase is initiated with the selection of decision variables. Next, the

continuity and management constraints and the objective function are
constructed.  Finally the linear program is solved and the results are
evaluated.

CHAPTER 4


GROUNDWATER FLOW MODELS


The construction of subsurface water quality management models is

predicated on a governing equation for unconfined solute transport

coupled to a mathematical description of unconfined groundwater flow.

This chapter presents the governing equation for steady-state two-

dimensional horizontal, unconfined groundwater flow. The equation is

used to construct a subsurface groundwater flow model and a simple

numerical inverse model. The inverse model is used to obtain estimates

of aquifer parameters required by the groundwater flow model. Both the

numerical flow model and the numerical inverse model are validated

against an analytical model for steady, two-dimensional horizontal,

unconfined, groundwater flow.


4.1   Development of the Numerical Groundwater Flow Model

For purposes of identifying long term management schemes an

approach is taken to construct several groundwater quality management

models from information derived from a steady-state groundwater flow

regime. This approach is acceptable as long as actual deviations from

the projected volumes of water exported and consumed do not induce

significant changes in the original piezometric surface used in the

construction of the groundwater quality management model. Subsurface

flow is described with the governing equation for steady-state, two-

flow is described with the governing equation for steady-state, two-dimensional horizontal, isotropic, unconfined, density independent groundwater flow which is

$$\frac{\partial}{\partial x}\left( T \cdot \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y}\left( T \cdot \frac{\partial h}{\partial y} \right) = +P - Z - S - U - Q \tag{22}$$

where  h      =  hydraulic head, (L);

       T      =  transmissivity of an isotropic aquifer, note, transmissivity is calculated as the product of the saturated thick and the hydraulic conductivity of the aquifer, (L/t);

       P      =  volumetric flux of withdrawal per unit surface area of aquifer, (L/t);

       Z,S,U    =  volumetric flux of recharge per unit area of aquifer from three sources, (L/t);

       Q      =  natural recharge, (L/t);

       x,y     =  horizontal coordinate axes;

       L      =  length, (meter);

       t      =  time, (day).

Assuming pumpage (P), total recharge (Z + S + Q + U) and the aquifer transmissivity (T) are known, a solution to equation (22) can be obtained analytically or numerically to yield hydraulic head a function of location in the x and y coordinate system.

4.1.1  Formulation of Finite Difference Groundwater Flow Equations

     Finite difference approximations of the equation for steady, two-dimensional horizontal groundwater flow (equation 22) permit the

development of a numerical groundwater flow model. The numerical model

can be applied over discretized hypothetical or real aquifers which

encounter simple or complex boundary conditions. Several numerical

formulations of the flow equation are available (Trescott, Pinder, and

Larson, 1976; Pinder and Bredehoeft, 1968, Prickett and Longquist,

1971). The finite difference formulation developed by Trescott, Pinder,

and Larson (1976) is used because it is well documented, simple, stable,

and accurate. The discrete numerical flow equation is

$$T_{i,j-.5} \cdot \left( \frac{h_{i,j-1} - h_{i,j}}{(\Delta x)^2} \right) + T_{i,j+.5} \cdot \left( \frac{h_{i,j+1} - h_{i,j}}{(\Delta x)^2} \right)$$

$$+ T_{i-.5,j} \cdot \left( \frac{h_{i,1-j} - h_{i,j}}{(\Delta y)^2} \right) + T_{i+.5,j} \cdot \left( \frac{h_{i+1,j} - h_{i,j}}{(\Delta y)^2} \right)$$

$$= P_{i,j} - Z_{i,j} - S_{i,j} - U_{i,j} - Q_{i,j} \tag{23}$$

where $i,j$ = the respective y and x coordinates of an

element in a grid superimposed over an aquifer

being modeled;

$h_{i,j}$ = a variable corresponding to the hydraulic head

in element $(i,j)$, (L);

$T_{i,j+.5}$ = a constant corresponding to the transmissivity

in the x direction between element $(i,j)$ and

element $(i,j+1)$;

$Z_{i,j}, S_{i,j}$ and = constants corresponding to the total

$U_{i,j}$ recharge occurring in element $i,j$ from three

sources, (L/t);

$P_{i,j}$          = constant corresponding to the total groundwater

withdrawal per unit area in element (i,j),

(L/t);

$Q_{i,j}$          = constant corresponding to the total natural

recharge per unit area in element i,j; (L/t);

$\Delta x, \Delta y$        = the discretization scale which is the distance

between the center of elements, (L);

i,j           = the y and x location coordinates of an element.

With the transmissivity $(T_{i,j})$ specified, hydraulic head $(h_{i,j})$ is

the only variable in equation (23). If only the hydraulic conductivity

is known, then the transmissivity is estimated to obtain an initial

solution to the set of equations. Subsequent solutions to the set of

equation update the transmissivity estimate to correspond with the

product of the known hydraulic conductivity and the predicted saturated

thickness of the aquifer. An aquifer modeled numerically is discretized

into n·m elements. Equation (23) is used to approximate the governing

equation for groundwater flow (22) over each element; hence, n·m

equations are generated for n·m unknown water table elevations at the

n·m discrete elements. If the elemental transmissivities $(T_{i,j})$ and the

discretization scales ($\Delta x$ and $\Delta y$) are known, a system of equations are

created and may be expressed as

$$A_{1,1}\left(h_{1,1},h_{1,2},\ldots h_{1,m},h_{2,1},\ldots h_{n,m}\right) = P_{1,1}{-Z_{1,1}}{-S_{1,1}}{-U_{1,1}}{-Q_{1,1}}$$

$$A_{1,2}\left(h_{1,1},h_{1,2},\ldots h_{1,m},h_{2,1},\ldots h_{n,m}\right) = P_{1,2}{-Z_{1,2}}{-S_{1,2}}{-U_{1,2}}{-Q_{1,2}}$$

$$\cdot \qquad\qquad \cdot$$
$$\cdot \qquad\qquad \cdot$$
$$\cdot \qquad\qquad \cdot$$

$$A_{n,m}\left(h_{1,1},h_{1,2},\ldots h_{1,m},h_{2,1},\ldots h_{n,m}\right) = P_{n,m}{-Z_{n,m}}{-S_{n,m}}{-U_{n,m}}{-Q_{r,m}}$$

$$(24)$$

where $A_{1,2}$ = a linear algebraic function obtained from the finite

difference approximation for the groundwater flow (23) at

element (i=1, j=2).

This system of equations with appropriate boundary conditions represent the groundwater flow model. Alternatively, the above system of equations will be expressed more conveniently in vector notation as

$$[A] \{h\} = [I] \{P\} - [I] \{Z\} - [I] \{S\} - [I] \{U\} - [I] \{Q\}$$

where $\{h\}$ = (n·m) x 1 vector of variables for hydraulic heads at every element;

$[A]$ = (n·m) x (n·m) vector of coefficients generated from the algebraic manipulation of the finite difference approximation of the governing flow equation;

$[I]$ = (n·m) x (n·m) identity matrix;

$\{Z\},\{S\}$ and = (n·m) x 1 vector of known artificial recharge
$\{U\}$ fluxes from three sources in every element;

$\{Q\}$ = (n·m) x 1 vector of known natural recharge fluxes in every element.

## 4.1.2 Boundary Conditions

A solution to a numerical groundwater flow model must incorporate boundary conditions that reflect groundwater flow conditions at or near the physical boundaries of a study area. The groundwater flow model can accommodate three boundary conditions.

-Specified head boundaries reflect flow conditions near a large
    body of water or along a coastline. Here the hydraulic head
    is treated as a known constant,

-Impermeable boundaries appear in regions where groundwater flow is
    assumed negligible because the porous media is impermeable.
    This boundary condition is addressed by specifying zero
    transmissivities for nodes within an impermeable region of a
    groundwater study area.

## 4.1.3 Model Solution Technique

The Gauss-Seidel solution technique is employed as an iterative method of solving the system of equations comprising the groundwater flow model. Solution convergence is identified when successive iterations no longer produce significant changes in simulated heads at each of the n·m elements.

## 4.2   Validation of the Steady-state Numerical Groundwater Flow Model

The numerical solution for steady-state, two-dimensional

horizontal, unconfined groundwater flow was validated against an

analytical model.  The effects of element size (numerical

discretization) on the accuracy and precision of the numerical model

were elucidated.  Finally a mass balance check was performed to test for

the conservation of mass through the hypothetical aquifer.

The numerical solution to the governing equation is coded in

Fortran.  For a given groundwater flow problem the program would request

data on elemental recharge rates, pumping rates, element areas,

horizontal element dimensions, and boundary conditions.  Finally,

elemental transmissivities are needed; however, if transmissivities were

not known, depth averaged horizontal hydraulic conductivities are used

in conjunction with elemental depths to the aquifer floor.


### 4.2.1   The Hypothetical Aquifer

The accuracy and precision of the numerical groundwater flow model

was checked against an analytical model derived for a hypothetical

horizontal circular island aquifer.  The hypothetical aquifer receives

constant and uniform surface recharge plus constant recharge from a

fully penetrating well located in the center of the island.  The aquifer

permeability is uniform and isotropic.  Figure 2 illustrates a general

plan view of the island and a profile of the hypothetical aquifer

where  h    =  elevation of the water table above the bedrock, (L);

$H_L$    =  elevation of the water table at the coast, (L);
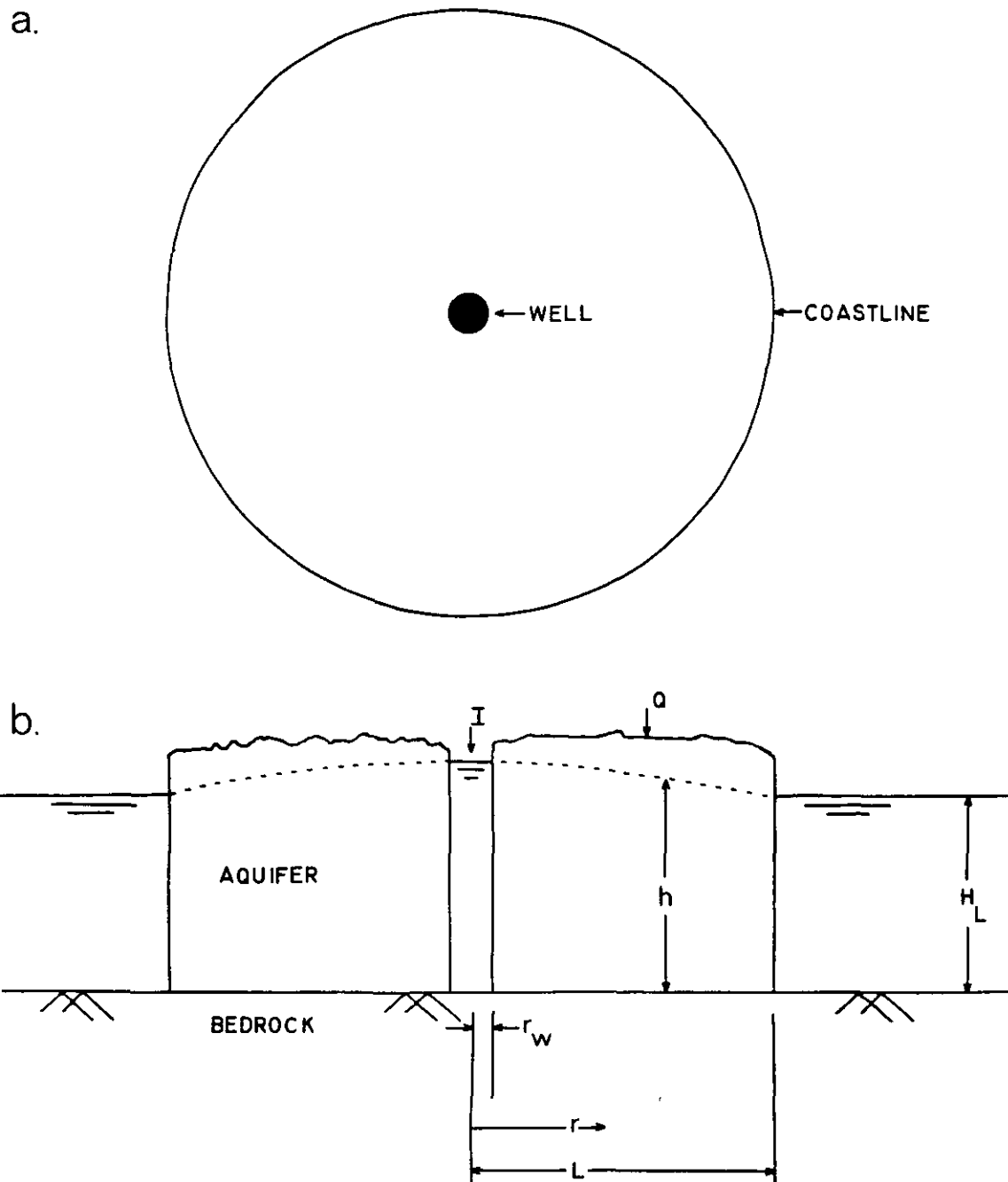
Figure 2. Plan view (a) and Profile view (b) of the hypothetical, two-dimensional, horizontal, and unconfined island aquifer.

I    =   recharge rate at the well, $(L^3/t)$;

L    =   the radial distance from the center of the well to the coast, (L);

r    =   radial distance from the center of the well, (L);

Q    =   surface recharge rate, (L/t);

$r_w$   =   radius of the well, (L).


## 4.2.2   The Analytical Model

The equation which describes groundwater flow over the radial distance from $r_w$ to L is

$$\frac{d}{dr} \left( K2\pi rh \cdot \frac{dh}{dr} \right) = - 2\pi rQ \tag{26}$$

s.t. boundary conditions:   1)  $h \cdot \frac{dh}{dr} = - \frac{I}{2\pi r_w K}$ at $r = r_w$

2)  $h = H_L$ at $r = L$

where  K = hydraulic conductivity, (L/t).

The solution to the above differential equation is

$$h^2 = H_L^2 + Q (L^2 - r^2)/2K + r_w^2 \cdot [\ln(r/L)]/K + I \cdot [\ln(L/r)]/K\pi$$

$$\tag{27}$$

The specific conditions of the test problem are

$H_L$   =   75 m;

I    =   4303 $m^3$/day;

K    =   60 m/day;

L    =   14000 m;

$$Q \quad = \quad .00137 \text{ m/day};$$

$$r_w \quad = \quad 1000 \text{ m.}$$

### 4.2.3  Validation Results

The groundwater flow model was applied over a discretized version of the hypothetical island aquifer shown in Figure 3. Specified for each element were the hydraulic conductivity (K), depth to the aquifer floor, and the total recharge. Discretization in the x and y directions was the same. The boundary condition for elements comprising the coastline were specified hydraulic heads. The water table elevations for interior nodes were calculated by the model.

Analytical and numerical estimates of the water table elevation are plotted as contours in Figure 4; the numerical results correspond to the discretization scale of two kilometers. Comparisons were made between numerical and analytical estimates of the water table elevation at every interior node. Model accuracy and precision were investigated over three discretization scales. The numerical error in water table estimates was calculated as the difference between between numerical and analytical values for hydraulic head.

$$\text{Error} = \text{numerical head} - \text{analytical head} \qquad (28)$$

The relative error in numerical water table estimates was expressed as the ratio of the numerical error over the analytical water table elevation (expressed as meters above mean sea level).

$$\text{Relative Error} = \frac{\text{Error}}{\text{Analytical Head}} \qquad (29)$$
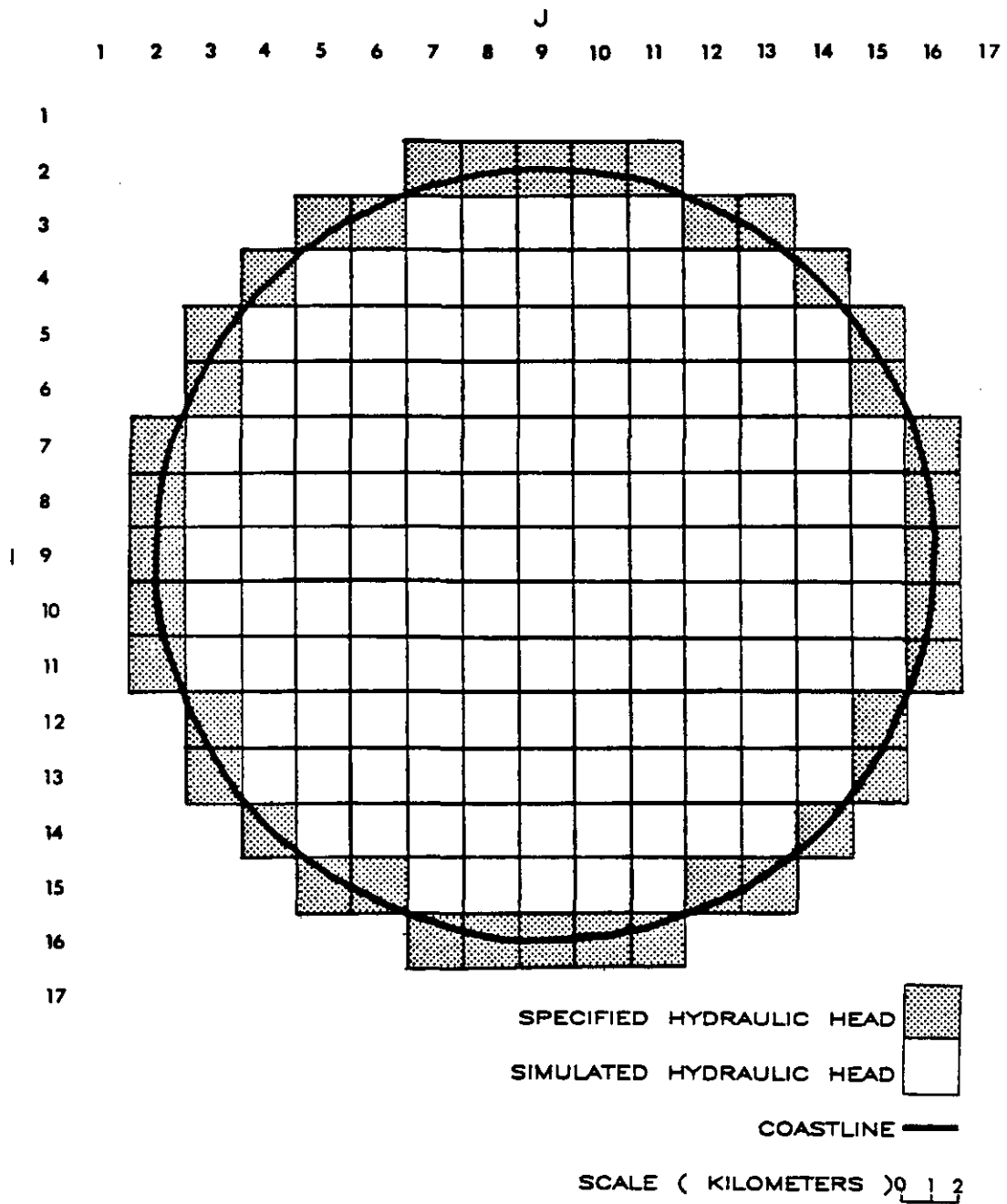
Figure 3.    Two-dimensional and horizontal numerical discretization of the hypothetical island aquifer with illustrated boundary conditions for the groundwater flow model.
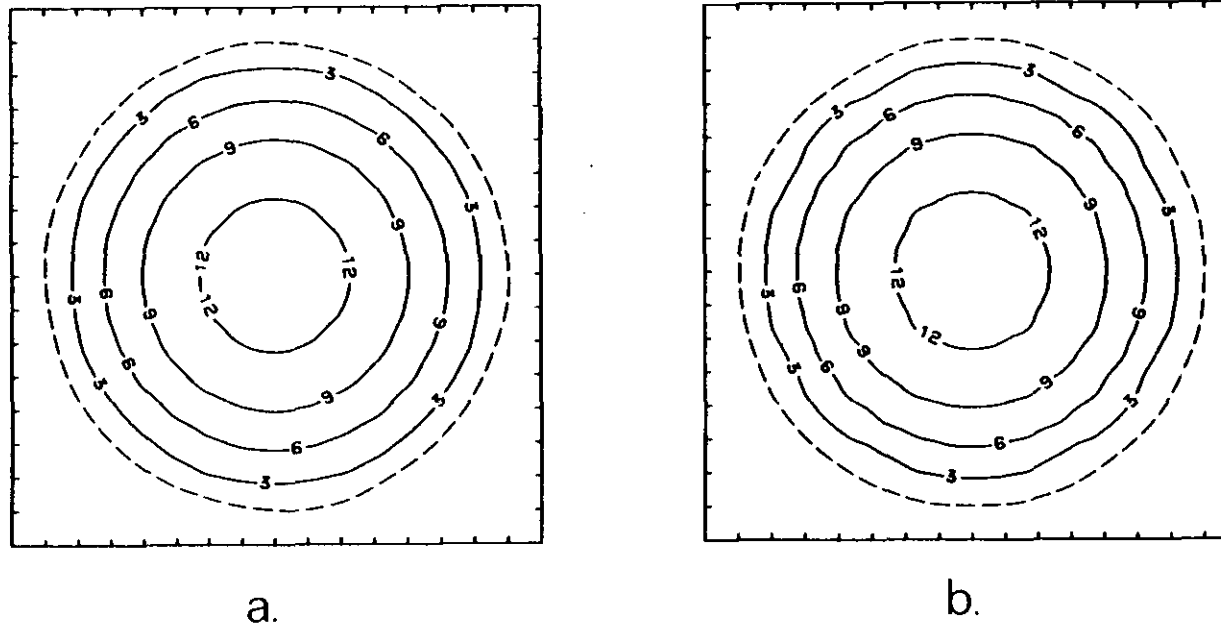
Figure 4.    Contour plots of predicted water table elevations (meters above mean sea level) from an analytical model (a) and from a numerical model using a two kilometer discretization scale (b).  Dashed lines represent the coastline, while solid lines correspond to the water table elevation contours.

Figures 5 and 6 show the relationship of the numerical model accuracy to discretization scale. Figure 5 shows the average error overall n·m elements and Figure 6 plots the average relative error (expressed as percent) in simulated water table elevations. Both Figures 5 and 6 indicate that a discretization of two kilometers produces the smallest error. Most dramatic was the increase in error associated with element dimensions greater than 2.5 kilometers. Over the whole island, the numerical flow model underestimated the true water table elevations by an average of 0.03 meters or 0.01 percent when a two kilometer discretization was used. An element dimension of three kilometers produced average errors of 0.6 meters or 12 percent. The percent error in predicted head increased for discretizations less than two kilometers (see Figure 6). When a discretization of one kilometer was used the number of equations required to predict hydraulic head at each element increased. With more equations in the groundwater flow model the number of numerical calculations to reach solution convergence increased. The additional numerical calculations produced a growth in roundoff errors. The accumulation of roundoffs could be curtailed by solving the system of model equations with a more efficient alogrithm or by relaxing the error criterion used to test solution convergence. Hence, it should be noted that the errors presented in Figures 5 through 8 reflect model errors generated over different discretization scales when using the Gauss-Seidel solution technique and the previously described convergence criterion.
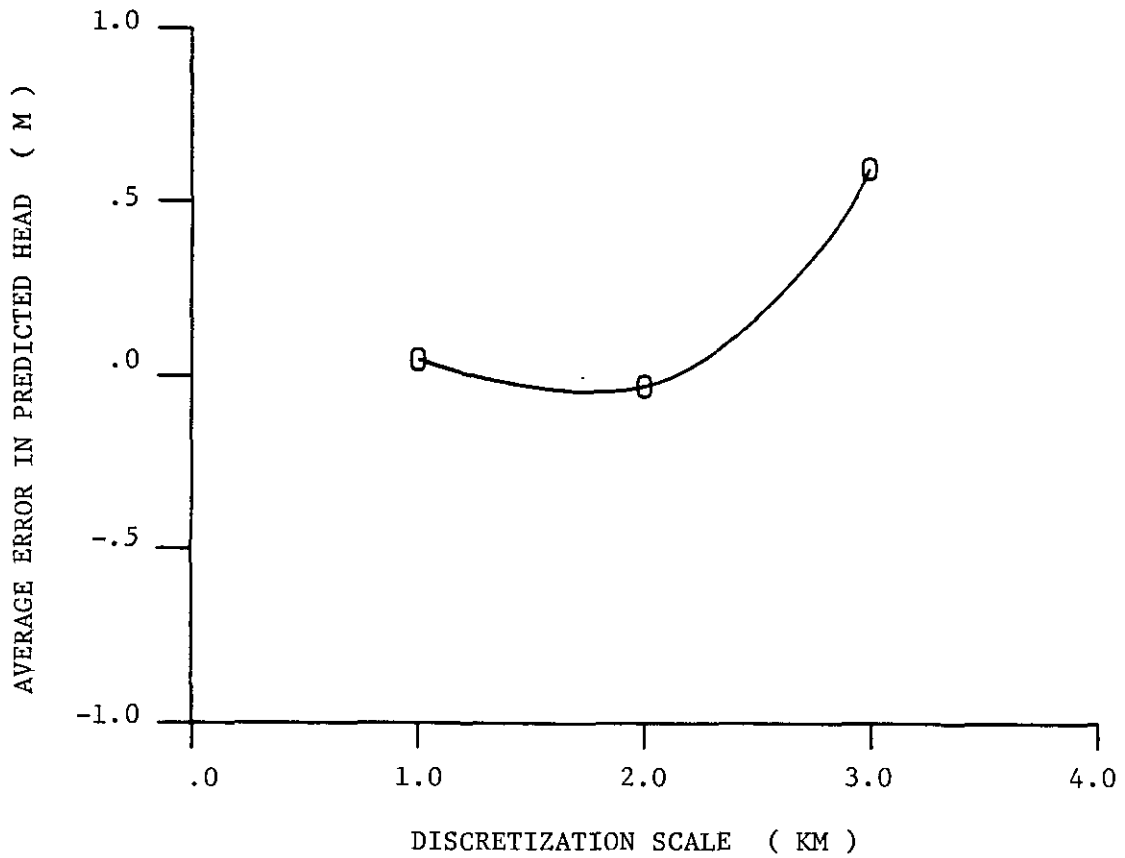
Figure 5.    Average error in predicted head (meters above mean sea
level) as a function of the discretization scale (for the
numerical groundwater flow model of the hypothetical island
aquifer).
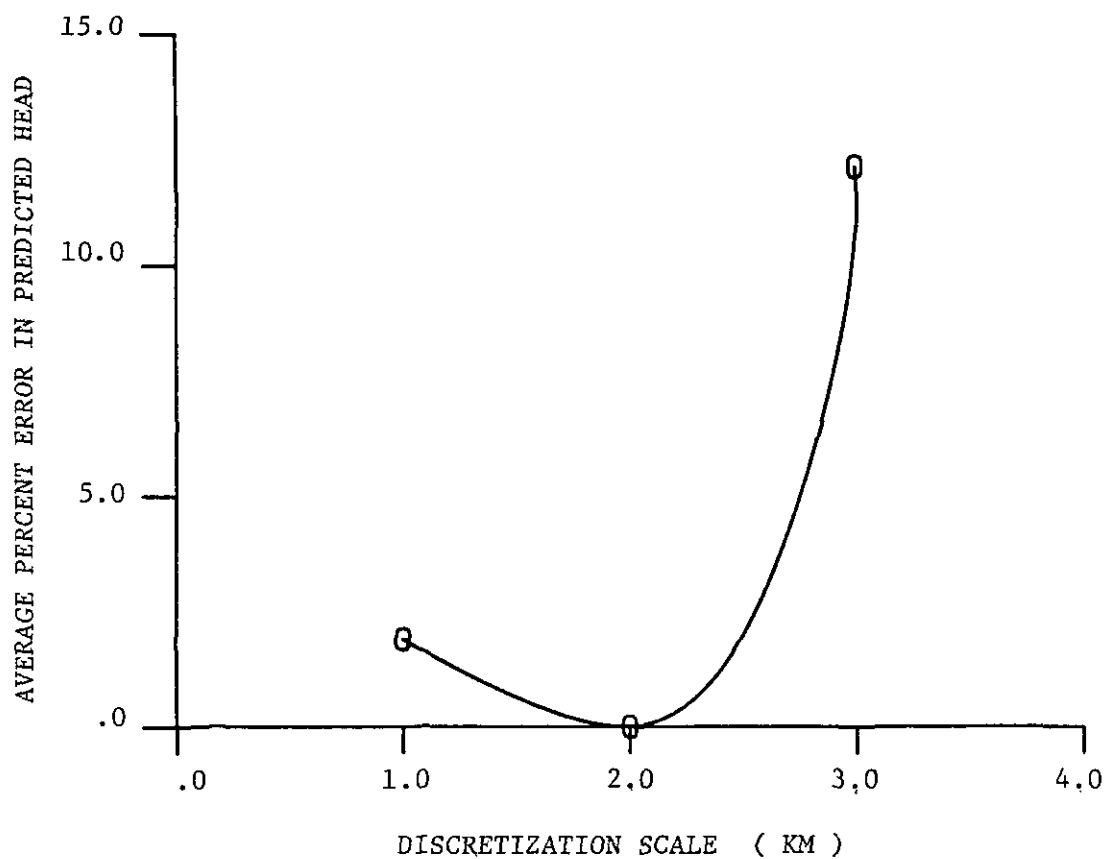
Figure 6.    Average percent error in predicted head (above mean sea
             level) as a function of the discretization scale (for the
             numerical groundwater flow model of the hypothetical island
             aquifer).
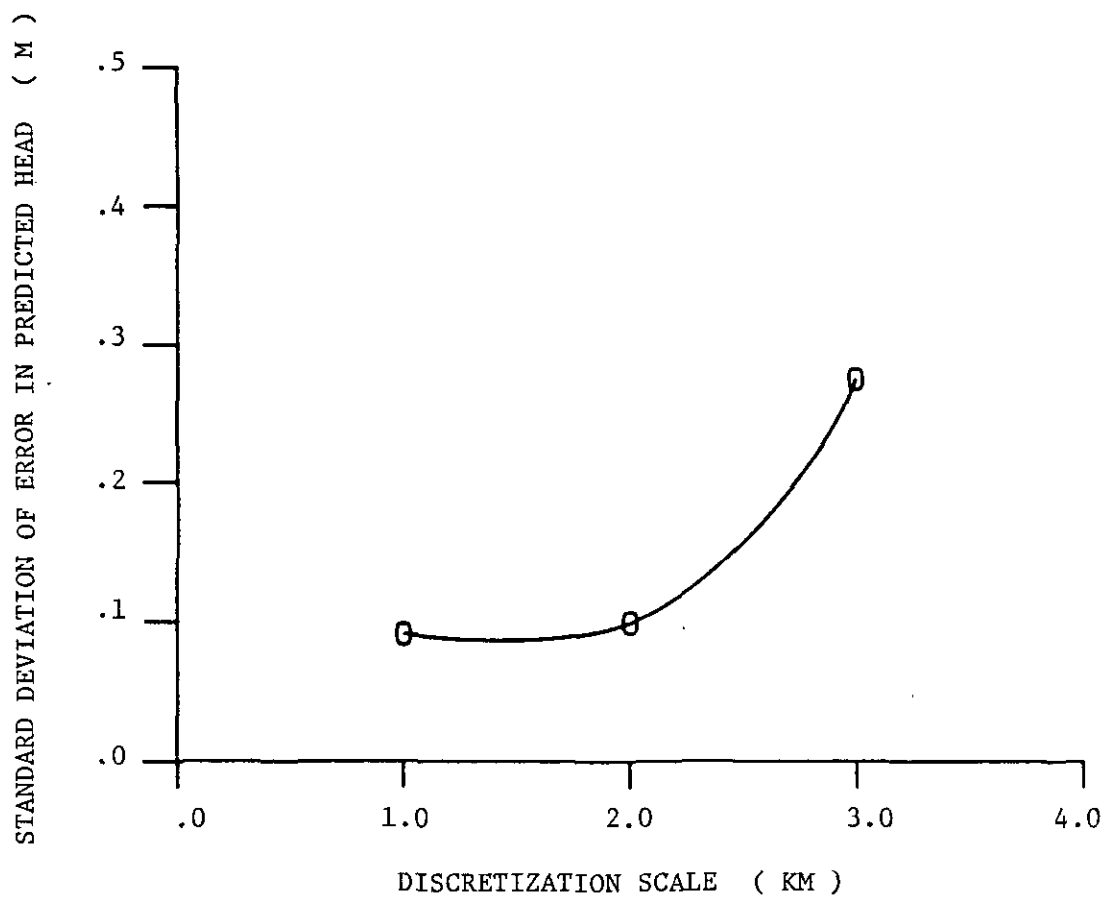
Figure 7. Standard deviation of error in predicted head (meters above mean sea level) as a function of the discretization scale (for the numerical groundwater flow model of the hypothetical island aquifer).
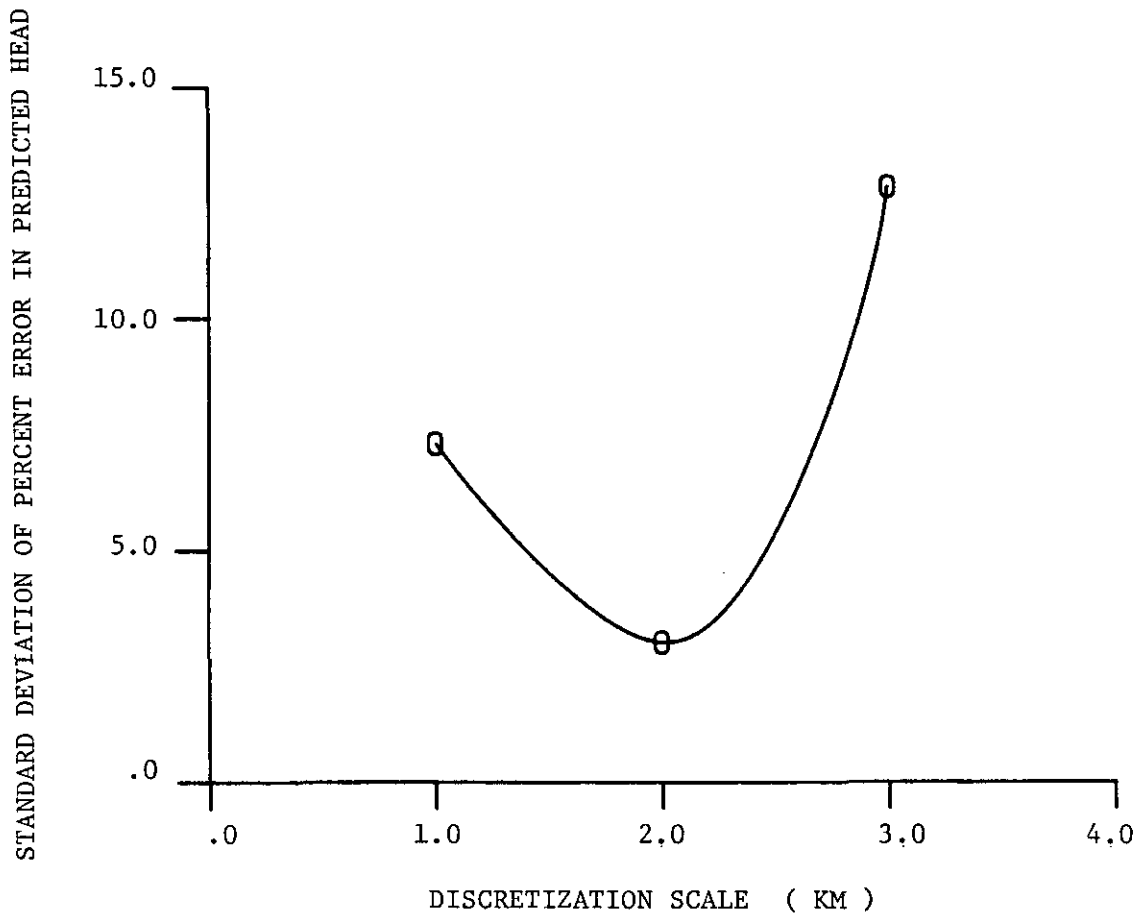
Figure 8.    Standard deviation of percent error in predicted head
             (above mean sea level) as a function of the discretization
             scale (for the numerical groundwater flow model of the
             hypothetical island aquifer).

The standard deviation of the errors and the standard deviation of relative errors in estimated water table elevations were also plotted against the discretization used. As illustrated in Figures 7 and 8 there was an increase in the deviations of model errors for numerical discretizations greater than two kilometers. At the two kilometer grid scale the standard deviation of model errors was 0.1 meter or 3 percent while at the three kilometer discretization the standard deviation of errors increased to 0.3 meters or 12.8 percent.

A mass balance check was performed to investigate the presence of model anomalies which introduce or remove water within the bounded hypothetical island. The percent error in the mass balance was small at all scales of discretization. A total gain of 0.008 percent was calculated for the model simulations at 2 kilometer element dimensions. This increase was not regarded as significant.

The two kilometer discretization appeared to be the optimal choice in regard to minimizing model error. The validation process showed that the numerical groundwater flow model could match analytical results within an error standard deviation of $\pm$ 5 percent while producing small mass balance errors. It was concluded therefore that the numerical model was successful at describing groundwater flow in the test island. For a field study similar to the hypothetical problem presented in this chapter, one would expect, at best, similar numerical accuracies.

## 4.3   Development of the Numerical Inverse Model

Application of a steady-state distributed parameter numerical groundwater model requires estimates of aquifer transmissivities at each discrete element.  For study sites where data on aquifer characteristics are sparse, a simple model is presented to estimate spatial aquifer transmissivities from water table data.  The inverse model is a two stage procedure.  In the first stage the average water table elevations from each element are used in a properly posed inverse boundary value problem (Nelson, 1961) to obtain initial estimates of the elemental transmissivities.  The first approximation of the transmissivity field serves as a starting point from which a numerical groundwater flow model can be used to perform simulations that approximate observed water table elevations.  During stage two the initial set of transmissivities are modified as necessary until the observed water table is predicted within acceptable errors.

### 4.3.1   Formulation of Stage I

Formulation of the first stage is predicated on the work of Nelson (1961), where the equation for steady flow through isotropic media (22) is expanded to obtain

$$\frac{\partial h \partial T}{\partial x \partial y} + \frac{\partial h \partial T}{\partial y \partial y} + \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) \cdot T = P - Z - S - U - Q \qquad (30)$$

The classical inverse problem treats T (transmissivity) as the dependent variable in a hyperbolic partial differential equation (30) where the coefficients $\partial h/\partial x$, $\partial h/\partial y$, $\partial^2 h/\partial x^2$, and $\partial^2 h/\partial y^2$ are known.  The inverse

problem is a Cauchy problem, which is properly posed when a single transmissivity estimate (Cauchy data) is specified at some point along each characteristic (flow line) of the region being studied (Emsellem and Marsily, 1971 and Neuman, 1973). For the continuous inverse boundary value problem, insufficient Cauchy data can render nonunique solutions. Neuman (1973) observed the phenomenon of cross-characteristic influence of Cauchy data with discrete inverse models (i.e., finite difference and finite element). This phenomenon allowed the unique identification of parameters for discrete formulations even when Cauchy data was insufficient for a continuous problem.

The finite difference approximation for equation (30) is the following explicit formulation, presented by Nelson (1961).

$$(\frac{\partial h}{\partial x})_{i,j} \cdot (\frac{T_{i,j} - T_{1,j-1}}{\Delta x}) + (\frac{\partial h}{\partial y})_{i,j} \cdot (\frac{T_{i,j} - T_{i-1,j}}{\Delta y})$$

$$+ (\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2})_{i,j} \, T_{i,j} = P_{i,j} - Z_{i,j} - S_{i,j} - U_{i,j} - Q_{i,j} \quad (31)$$

In the work here the coefficients are calculated as

$$(\frac{\partial h}{\partial x})_{i,j} = \frac{h_{i,j} - h_{i,j-1}}{\Delta x} \quad (32)$$

$$(\frac{\partial h}{\partial y})_{i,j} = \frac{h_{i,j} - h_{i-1,j}}{\Delta y} \quad (33)$$

$$(\frac{\partial^2 h}{\partial x^2})_{i,j} = (\frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{(\Delta x)^2}) \quad (34)$$

$$(\frac{\partial^2 h}{\partial y^2})_{i,j} = (\frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{(\Delta y)^2}) \quad (35)$$

A properly posed inverse boundary value problem requires that the transmissivities be specified along a line that intersects all flow lines, or at some point along each characteristic (Emsellem and Marsily, 1971). Assuming the water table elevation is known at each element of a discretized aquifer, then the nodal transmissivities are determined in an explicit fashion through the sequential application of equation (31) in a direction from a boundary where the transmissivities are specified.

## 4.3.2 Formulation of Stage II

Stage II begins with the execution of a validated groundwater flow model using the transmissivity estimates from the first stage. Results from the simulation model are used to compute model errors in simulated water table elevations and hydraulic gradients. The errors are used to alter values of elemental transmissivities in a manner which will reduce the error between simulated and observed water table elevations and hydraulic gradients. For elements where the hydraulic head is simulated, the transmissivities are adjusted using the errors between simulated and observed water table elevations. For elements where the hydraulic head is specified, the errors used in transmissivity adjustments are the errors between the simulated and observed hydraulic gradients. Iterations of the simulation/adjustment procedure continue until an allowable number of iterations is reached or until results from groundwater flow simulations meets prescribed performance criteria.

4.3.2.1  Transmissivity Adjustments at Elements Where the Hydraulic Head

   Is Simulated

For discrete numerical elements, where the hydraulic head is

simulated, the error $(E_{i,j})$ in the simulated water table elevation at

node i,j is calculated as

$$E_{i,j} = h_{i,j} - H_{i,j} \qquad (36)$$

where $h_{i,j}$ = simulated head at element i,j

   $H_{i,j}$ = observed head at i,j

From Coats, Dempsey and Henderson (1970) the error $E_{i,j}$ at node i,j is a

function of the set of elemental transmissivities used within the

discretized aquifer hence

$$E_{i,j} = \theta_{i,j} \left( T_{1,1}, \ T_{1,2} \cdots T_{n,m} \right) \qquad (37)$$

Slater and Durrer (1971) suggest that $\theta_{i,j}$ may be linear for small

changes in the $T_{i,j}$ for all i,j.  Without knowing the function $\theta_{i,j}$ it

was assumed that the error $E_{i,j}$ at an element i,j is most sensitive to

transmissivity changes in node i,j and less sensitive to changes in

transmissivities in cells located further away from the target node.  If

equation (37) can be reduced to

$$E_{i,j} = \psi_{i,j} \left( T_{i,j} \right) \qquad (38)$$

then local transmissivities can be modified over small magnitudes as a

means of reducing the error in local hydraulic head simulations.  For

elements where the water table is simulated (not specified),

transmissivities are adjusted between simulations by an amount proportional to the absolute error in simulated head.

The direction of adjustments are determined by the product of the sign of error in simulated head ($E_{i,j}$) and the sign of the elemental mass balance condition. A negative mass balance condition exists if total elemental groundwater withdrawal exceeds nodal recharge. A positive mass balance condition exists if recharge dominates pumpage.

Under a negative mass balance condition the local transmissivity is increased if the simulated head is less than the observed head. Increasing the transmissivity permits more water to flow into the region. If the predicted head is greater that observed, the transmissivity is decreased to reduce the rate of ground water flow into the element.

Positive mass balance conditions call for a reduction of the elemental transmissivity if the calculated water table elevation is lower than the observed. The reduction of transmissivity restricts the ease of groundwater flow out of the element; consequently, the simulated water level rises. If the predicted hydraulic head exceeds observed elemental head, the local transmissivity is increased to allow greater flow from the node so that the local hydraulic head will drop. Table 1 summarizes the decision rules to determine the direction of transmissivity adjustments from predicted hydraulic head errors.

Table 1.  The Direction (Positive or Negative) of Adjusting
Transmissivities in Elements where the Hydraulic
Head is Simulated.

| Status of Elemental Mass Balance | Status of Prediction | |
| --- | --- | --- |
| | Over Estimated (Positive Error) | Under Estimated (Negative Error) |
| Net Recharge (positive) | + | − |
| Net Discharge (negative) | − | + |

### 4.3.2.2 Transmissivity Adjustments at Elements Where the Hydraulic Head Is Specified

At locations where elemental water table elevations are specified (i.e, along a coastline) errors in transmissivity estimates can induce errors in the hydraulic gradients for flow entering or leaving nodes. If the hydraulic head is specified at element i,j, then the error $(G_{x_{i,j}}$ and $G_{y_{i,j}})$ in the hydraulic gradient in the X and Y coordinate direction at node i,j can be expressed as a function of the transmissivity of the element.

$$G_{x_{i,j}} = \phi_{i,j}\left(T_{i,j}\right) \text{ and } G_{y_{i,j}} = \Omega_{i,j}\left(T_{i,j}\right) \qquad (39)$$

Errors in the simulated hydraulic gradients can be reduced with proper adjustment of the transmissivities. For groundwater flowing in the direction of increasing coordinate values i and j, the errors in simulated hydraulic gradients are calculated as

$$G_{x_{i,j}} = \left(\frac{h_{i,j-1} - H^S_{i,j}}{\Delta x}\right) - \left(\frac{H_{i,j-1} - H^S_{i,j}}{\Delta x}\right) \qquad (40)$$

$$\underset{\text{in the x direction}}{\text{Simulated gradient}} \qquad \underset{\text{in the x direction}}{\text{Observed gradient}}$$

$$G_{y_{i,j}} = \left(\frac{h_{i-1,j} - H^S_{i,j}}{\Delta y}\right) - \left(\frac{H_{i-1,j} - H^S_{i,j}}{\Delta y}\right) \qquad (41)$$

$$\underset{\text{in the y direction}}{\text{Simulated gradient}} \qquad \underset{\text{in the y direction}}{\text{Observed gradient}}$$

where $G_{x_{i,j}}$ = the error in simulated hydraulic gradient in the x

direction;

$G_{y_{i,j}}$ = the error in simulated hydraulic gradient in the y

direction;

$h_{i,j}$ = simulated head at element i,j;

$H_{i,j}$ = observed head at element i,j;

$H^s_{i,j}$ = specified head at element i,j.

Appropriate equations similar to (40) and (41) can be constructed for other directions of groundwater flow.

Adjustments in elemental transmissivities are made in an amount proportional to the average error in predicted hydraulic gradients. The direction of modification is determined by the sign of the average error. If the simulated hydraulic gradient is too steep the error will be positive and transmissivity will be increased. Increasing the transmissivity will allow the same flow to occur under a lower gradient. If the calculated water table slope is too shallow then the error will be negative. Decreasing the transmissivity will increase the predicted hydraulic gradient as is necessary to drive the flow through a less permeable aquifer.

### 4.3.3  Programming the Inverse Model

The numerical Inverse Model is coded in Fortran. The model requires data on elemental hydraulic heads, recharge rates, pumpage

rates, element areas, horizontal element dimensions, and boundary conditions (where transmissivity is specified and the hydraulic head is specified).

## 4.4 Validation of the Inverse Model

The numerical inverse model was validated against transmissivity and hydraulic head data obtained from an analytical model. The effects of numerical discretization scale on accuracy of model predictions were investigated.

## 4.4.1 Hypothetical Aquifer and the Analytical Model

The hypothetical island aquifer presented in section (4.2.1) and the associated analytical groundwater flow model (4.2.2) were used to generate discrete transmissivity estimates in the validation test problem. A product of the aquifer hydraulic conductivity (60 m/day) and the local saturated thickness (equal to the hydraulic head) was used to approximate the true elemental transmissivity.

A comparison was performed between analytical transmissivities and predictions obtained from the numerical inverse model. The transmissivities predicted by the inverse model were also used in the validated groundwater flow model to obtain simulated water table elevations. The predicted groundwater levels were compared with analytical heads in the hope of revealing the error in simulated heads resulting from the errors in transmissivities acquired through inverse modeling.

4.4.2  Application of the Numerical Inverse Model over the Hypothetical

Aquifer

The numerical inverse model was applied over discretized versions of the hypothetical island aquifer. Transmissivities were predicted for each numerical element using data on the water table elevation at every element. The hydraulic head data was obtained from a numerical simulation of groundwater flow on the island using the true transmissivities.

Numerical estimates of groundwater levels were used over analytical heads for two reasons. First, numerical results from the flow model using the true transmissivities represent the best achievable approximation of the observed water table. Secondly, stage two of the inverse model employs the validated groundwater flow model to modify transmissivities from the first stage. This limits the inverse model in identifying a set of discrete aquifer transmissivities that yield predicted water table elevations, that are as good but no better than direct numerical simulations using the true aquifer transmissivities.

Stage one of the inverse model was completed after initial estimates of elemental transmissivities were found over the whole island aquifer. During stage one, the island was sectioned along groundwater divides into quarters. The sections of the island are smaller but identical inverse problems.

Figure 9 shows the boundary conditions used with the first quarter of the overall inverse problem at the two kilometer numerical discretization scale. The boundary conditions used in each quarter were
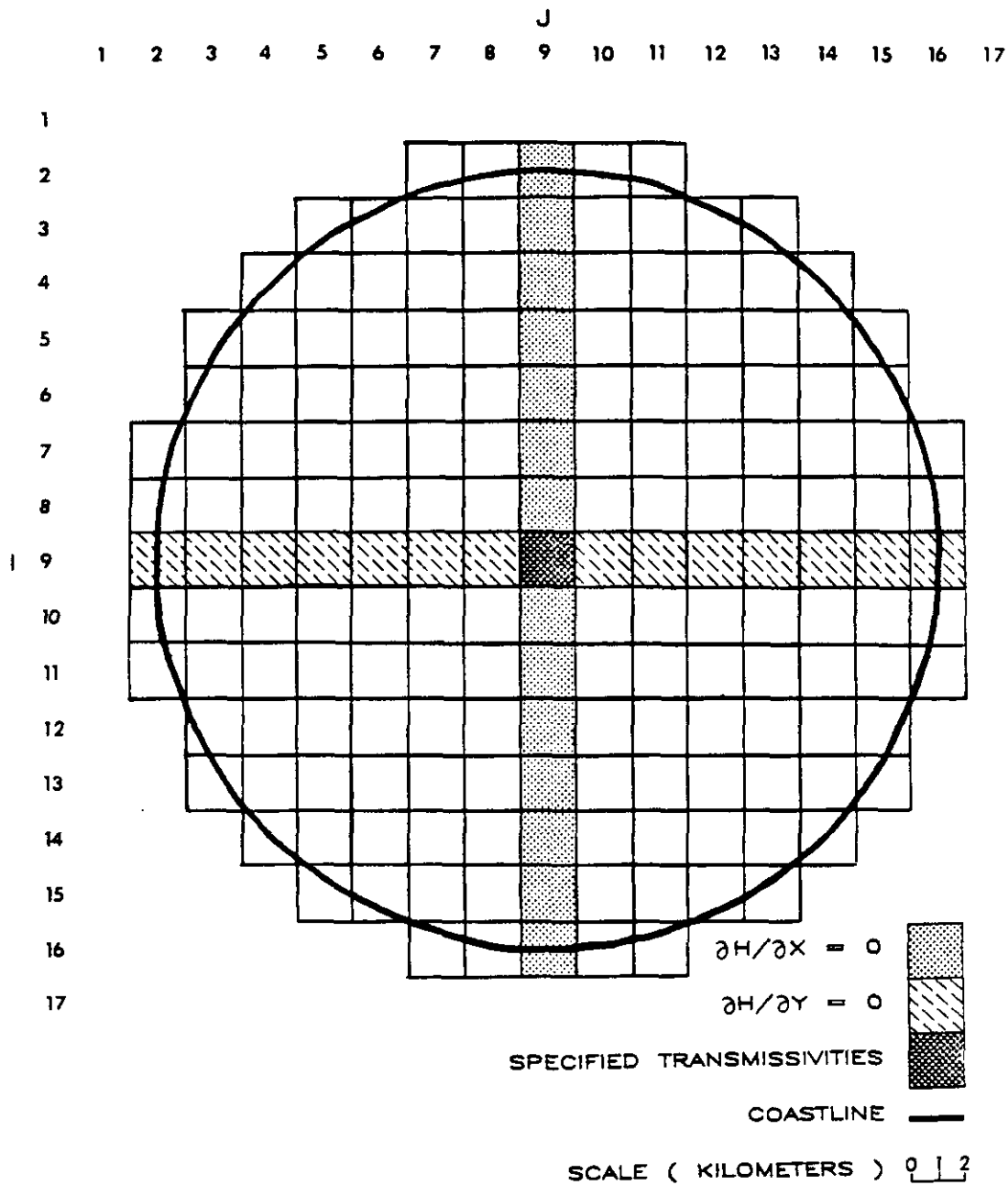
Figure 9.    Two-dimensional and numerical discretization of the hypothetical island aquifer with illustrated boundary conditions for the inverse model.

the same. Because all flow lines emanate from the center of the island
the inverse problem would be properly posed if the transmissivity at the
origin were specified. For elements along the groundwater divide, which
runs parallel to the y axis, it was assumed that groundwater flow in the
x direction was negligible hence

$$\frac{\partial h}{\partial x} = 0 \tag{42}$$

In those elements along the groundwater divide, which runs parallel to
the x axis, it was assumed that groundwater flow in the y direction
could be ignored hence

$$\frac{\partial h}{\partial y} = 0 \tag{43}$$

Transmissivities for each quarter were calculated through the sequential
application of equation (31) in a direction radiating from the center of
the island where the transmissivity was specified.

Stage two of the inverse model began after initial estimates of
transmissivities for the whole island were compiled from stage one.
During stage two 100 simulation/adjustment iterations were performed.
The magnitude of any single adjustment was 50 percent of the error in
the simulated water table elevation or 50 percent of the error in the
predicted hydraulic gradient.

### 4.4.3  Validation Results

The accuracy of the inverse model was tested over three scales of
numerical discretization. Comparisons were made between island
transmissivities estimated by the inverse model and transmissivities

given by the analytical model. The relative error in transmissivity estimates was expressed as the ratio of the error in predicted transmissivity over the analytical transmissivity.

$$\text{Relative Error} = \frac{\text{Inverse model transmissivity - analytical transmissivity}}{\text{analytical transmissivity}}$$

(44)

Figure 10 shows that the average percent error in transmissivities predicted by the inverse model ranged between .7 to 1.6. While the average percent error increased with discretization scale the standard deviation of the error remained relatively constant ranging between 5.2-5.3 percent. As indicated by the small positive errors, the inverse model tended to overestimate transmissivities in the island problem.

Numerical groundwater flow simulations were performed using the true transmissivities and estimated transmissivities from the inverse model. Table 2 shows the flow model prediction errors that result from each set of aquifer transmissivities. At every discretization scale, the errors in predicted heads were essentially the same between the simulations using the true transmissivities or the transmissivities generated by the inverse model.

Two conclusions can be drawn from the validation of the inverse model. First, the model was successful in estimating aquifer transmissivities for the hypothetical problem within a 5.3 percent standard deviation of errors. Secondly the errors in estimated transmissivities did not produce a significant reduction in the numerical accuracy of predicted water table elevations.
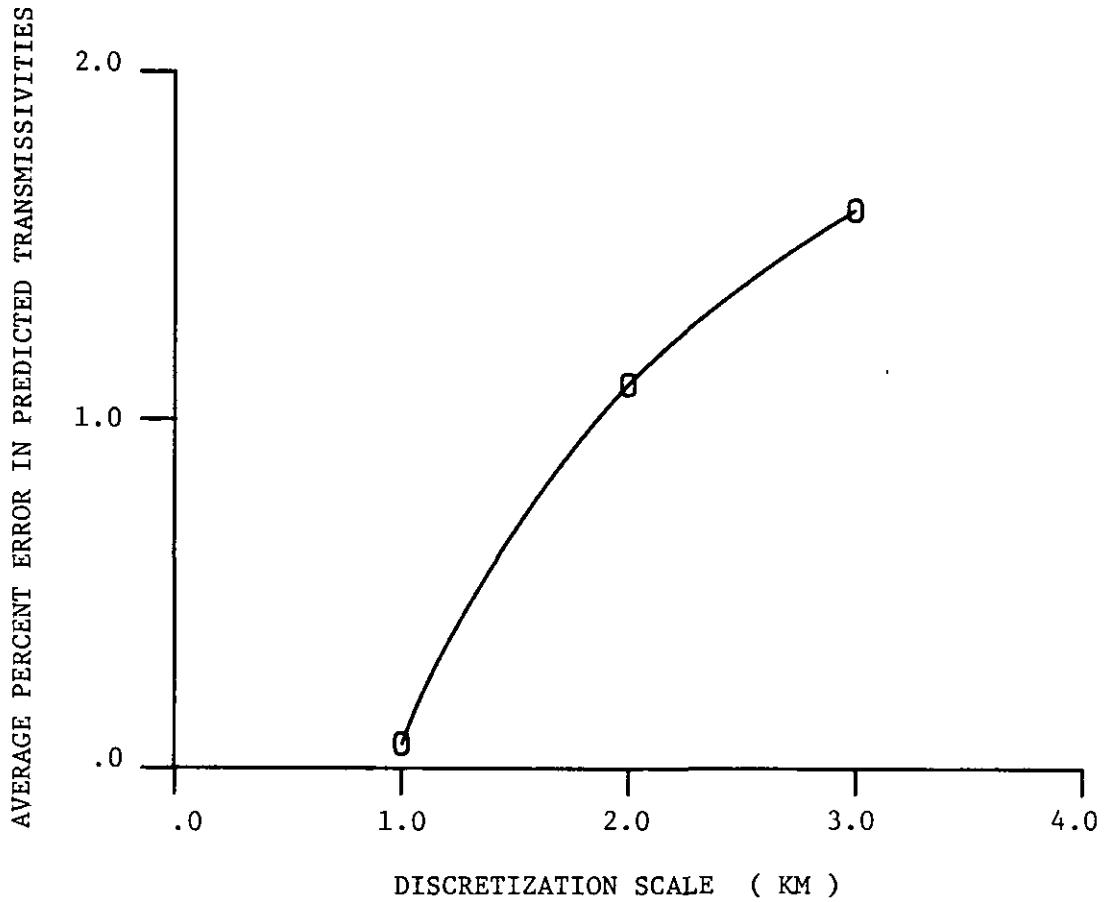
Figure 10.   Average percent error in predicted transmissivities as a
             function of the discretization scale (for the numerical
             inverse model of the hypothetical island aquifer).

Table 2.   Numerical Flow Model Prediction Errors Using True
and Estimated Transmissivities.

| | Discretization Scale [km] | Errors when using True Transmissivity [m] | Errors when using Estimated Transmissivity [m] |
|---|---|---|---|
| Average head error | 1 | 0.045 | 0.037 |
| Standard deviation of head error | 1 | 0.091 | 0.093 |
| Average head error | 2 | -0.032 | -0.410 |
| Standard deviation of head error | 2 | 0.099 | 0.100 |
| Average head error | 3 | 0.591 | 0.582 |
| Standard deviation of head error | 3 | 0.274 | 0.278 |

CHAPTER 5


NONPOINT SOURCE GROUNDWATER POLLUTION MODEL


The creation of continuity constraints for groundwater quality

management models is predicated on the validated discrete approximations

of the steady-state groundwater contaminant transport equation.  This

chapter presents the governing equation for steady-state, two-

dimensional horizontal, advective transport of a conservative

contaminant through an unconfined aquifer.  A finite difference

approximation of the governing contaminant equation is used to construct

numerical equations for a discretized hypothetical nonpoint source

groundwater pollution transport problem.  The resultant set of algebraic

equations are embedded as continuity constraints in a simple linear

program.  The solution to the linear program is the numerical estimate

of the concentration a dissolved constituent in each element of the

discretized aquifer.  The linear program solution is compared with

contaminant levels obtained from an analytical model of the hypothetical

groundwater problem.


5.1  Development of the Numerical Contaminant Transport Model for

     Nonpoint Source Groundwater Pollution

     The governing equation for steady-state, two-dimensional

horizontal, advective transport of a conservative solute approximates

the response of an aquifer receiving distributed contaminant loadings.

It is assumed that the nonpoint source pollution is delivered from surface directly to the zone of saturation, and that the contaminant is completely mixed vertically. The governing equation for steady-state, two-dimensional horizontal, advective transport of a conservative contaminant is

$$V_x \cdot B \cdot \frac{\partial c}{\partial x} + V_y \cdot B \cdot \frac{\partial c}{\partial y} + (Z + S + Q + U) \cdot C$$

$$= + C_w \cdot W + C_z \cdot Z + C_s \cdot S + C_u \cdot U + C_q \cdot Q$$

(44)

where $C$ = the concentration of a conservative contaminant, $(M/L^3)$;

$V_x, V_y$ = horizontal Darcian fluid velocities in the x and y directions respectively, $(L/t)$;

$B$ = saturated thickness of the aquifer, $(L)$;

$W, Z, S, U$ = volumetric flux of recharge per unit area of aquifer from four sources of groundwater contamination, $(L/t)$;

$C_w, C_z, C_s, C_u$ = the contaminant concentrations in the four fluxes of groundwater recharge W, Z, S, and U, $(M/L^3)$;

$Q$ = natural recharge, $(L/t)$;

$C_q$ = concentration of contaminant in the natural recharge, $(M/L^3)$.

In equation (44) the horizontal Darcian fluid velocities and the thickness of the saturated zone are obtained from field measurements or

simulations with a calibrated groundwater flow model. The contaminant concentrations and injection flows from all sources of recharge are assumed to be known. Though pumpage (P) does not appear in equation (44) its influence is expressed through the accurate estimation of the Darcian velocity coefficients from numerical groundwater flow simulations.

### 5.1.1 Formulation of the Finite Difference Equations for Contaminant Transport

The linear algebraic equations of the numerical contaminant model are developed from finite difference approximation of the governing equation for solute transport (44). A second order central difference scheme is used to construct the following discrete solute-transport equation

$$+ V_x \cdot B \Big|_{i,j} \cdot \left( \frac{C_{i,j+1} - C_{i,j-1}}{2\Delta x} \right)$$

$$+ V_y \cdot B \Big|_{i,j} \cdot \left( \frac{C_{i+1,j} - C_{i-1,j}}{2\Delta y} \right) + \left( Z_{i,j} + S_{i,j} + Q_{i,j} + U_{i,j} \right) \cdot C_{i,j}$$

$$= + \left( C_w \cdot W \right)_{i,j} + \left( C_z \cdot Z \right)_{i,j} + \left( C_s \cdot S \right)_{i,j} + \left( C_u \cdot U \right)_{i,j} + C_q \cdot Q_{i,j} \quad (45)$$

where i,j          = the respective y and x coordinates of an element in a grid superimposed over an aquifer being modeled;

$C_{i,j}$          = a variable for contaminant concentration in element (i,j), $(M/L^3)$;

$V_x B \Big|_{i,j}$ = horizontal mass flux of flow through element (i,j) in the x direction, ($L^2$/t);

$V_y B \Big|_{i,j}$ = horizontal mass flux of flow through element (i,j) in the y direction, ($L^2$/t);

$W_{i,j}, Z_{i,j}, S_{i,j}$, = total contaminant source flows in element i,j

and $U_{i,j}$ (L/t);

$(C_w \cdot W)_{i,j}, (C_z \cdot Z)_{i,j}$ = contaminant mass flux from four sources in

$(C_s \cdot S)_{i,j}$, and element i,j, ($M/L^2 \cdot t$)

$(C_u \cdot U)_{i,j}$

$Q_{i,j}$ = natural recharge flows in element (i,j), (L/t);

$C_q$ = contaminant concentration in natural recharge flows, ($M/L^3$);

$\Delta x$ and $\Delta y$ = the distance between the center of elements or the Length scale of discretization, (L).

The horizontal fluxes of flow, $V_x B \Big|_{i,j}$ and $V_y B \Big|_{i,j}$, are approximated as described by Trescott, Pinder, and Larson (1976) using the harmonic mean to estimate the transmissivity between nodes.

$$V_x \cdot B \Big|_{i,j} = T_{i,j+.5} \cdot \left(\frac{h_{i,j+1} - h_{i,j}}{2\Delta x}\right) + T_{i,j-.5} \cdot \left(\frac{h_{i,j} - h_{i,j-1}}{2\Delta x}\right)$$

$$(46)$$

$$V_y \cdot B \Big|_{i,j} = T_{i+.5,j} \cdot \left(\frac{h_{i+1,j} - h_{i,j}}{2\Delta y}\right) + T_{i-.5,j} \cdot \left(\frac{h_{i,j} - h_{i-1,j}}{2\Delta y}\right)$$

$$(47)$$

Equations (46) and (47) make use of hydraulic head data at each element which was obtained from previous simulations with a calibrated groundwater flow model.

For the simplest contaminant model the only variables appearing in equation (45) are those representing the dissolved contaminant concentrations; the $C_{i,j}$'s. To construct a numerical model for a study-site discretized into n·m elements, equation (45) is applied at each node to construct n·m finite difference equations for solute transport. Each discrete equation is constructed from elemental data on hydraulic head, aquifer transmissivity, node dimensions, node area, contaminant source flows, and contaminant source concentrations. The system of equations which comprise the contaminant transport model for nonpoint source groundwater pollution can be expressed as

$$G_{1,1}(C_{1,1}, C_{1,2}, \ldots C_{1,m}, C_{2,1}, \ldots C_{n,m}) = + (C_w \cdot W)_{1,1} + (C_s \cdot S)_{1,1}$$
$$+ (C_z \cdot Z)_{1,1} + (C_u U)_{n,m} + (C_q \cdot Q)_{1,1}$$

$$G_{1,2}(C_{1,1}, C_{1,2}, \ldots C_{1,m}, C_{2,1}, \ldots C_{n,m}) = + (C_w \cdot W)_{1,2} + (C_s \cdot S)_{1,2}$$
$$+ (C_z \cdot Z)_{1,2} + (C_u U)_{1,2} + (C_q \cdot Q)_{1,2}$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$G_{n,m}(C_{1,1}, C_{1,2}, \ldots C_{1,m}, C_{2,1}, \ldots C_{n,m}) = + (C_w \cdot W)_{n,m} + (C_s \cdot S)_{n,m} \quad (48)$$
$$+ (C_z \cdot Z)_{n,m} + (C_u U)_{n,m} + (C_q \cdot Q)_{n,m}$$

where $G_{1,2}$ = a linear algebraic function corresponding to terms on

the left side of the equal sign in equation (45) at

element $(i = 1, j = 2)$.

In this paper, the system of equations from the numerical

contaminant transport model is expressed as vectors

$$[G]\{C\} = + [C_w]\{W\} + [C_s]\{S\} + [C_z]\{Z\} + [C_w]\{U\} + [C_q]\{Q\} \quad (49)$$

where $\{C\}$ = $(n \cdot m)$ x 1 vector of variables for contaminant

concentrations at every element;

$[G]$ = $(n \cdot m)$ x $(n \cdot m)$ vector of coefficients

generated from the algebraic manipulation of

the finite difference approximation of the

governing flow equation;

$[I]$ = $(n \cdot m)$ x $(n \cdot m)$ identity matrix;

$[C_w], [C_z], [C_s],$   =  dissolved contaminant concentrations in four

and $[C_u]$                (n·m) x (n·m) diagonal matrices of elemental

                              contamination concentration in every source

                              flows of each element;

$\{U\}, \{W\}, \{S\},$   =  (n·m) x 1 vector of known recharge fluxes

                              from four sources in every element

and $\{Z\}$

$C_q$           =  Background contaminant concentrations in

                              natural recharge flows;

$\{Q\}$         =  (n·m) x 1 vector of known natural recharge

                              fluxes in every element.

## 5.1.2  Boundary Conditions

Obtaining a solution to equations describing contaminant transport requires specification of boundary conditions.  Two types of boundary conditions employed in this research are

-Zero contaminant flux boundaries, as would exist across a
groundwater divide, are treated by specifying zero Darcian
velocities across the divide, for elements located on the divide,

-Specified contaminant concentrations, as would exist wherever the
concentration of contaminant is known and assumed constant.

### 5.1.3 Solving the Numerical Contaminant Transport Model

The numerical contaminant transport model is a set of algebraic equations which are comprised of variables $C_{i,j}$ (for all i and j) representing contaminant concentrations at each node. The set of algebraic equations can be solved with the same algorithm employed to solve linear programs (Gorelick, 1981). The process is one of treating the $C_{i,j}$'s as decision variables, while using the set of algebraic equations from the numerical model as continuity constraints in a simple linear program. The objective function in the linear program is

$$\text{Maximize} \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} \tag{50}$$

The constraint set contains continuity and nonnegativity constraints.

$$[G]\{C\} = + [C_w]\{W\} + [C_s]\{S\} + [C_z]\{Z\} + [C_u]\{U\} + [C_q]\{Q\}$$

$$C_{i,j} \geq 0 \quad \forall \text{ i and j}$$

The total number of variables in this problem is same as the total number of constraints, which equals the number of discrete elements in the groundwater study area. In the continuity constraints, the terms on the right-hand side of the equal sign are known constants.

A solution to the linear program is a set of values for the $C_{i,j}$'s which maximize the value of the objective function and satisfies the continuity and nonnegativity constraints. The linear program is solved using one among several available algorithms which identify the optimum feasible solution through an iterative procedure.

### 5.1.4  Programming the Contaminant Transport Model

A Fortran program was written to construct the linear program. The program contains the groundwater flow model and an algorithm for constructing the objective function and the continuity constraints. The constructed linear program is passed to a computer file where it can be solved through existing optimization packages.

The data requirements for executing the program are 1) the number of elements in the discretized study area, 2) the area and dimensions of the elements, 3) the elemental recharge rates of all sources, 4) the elemental pumpage rates, 5) the piecewise aquifer transmissivities (or the aquifer hydraulic conductivity with the depth to aquifer floor), 6) the hydraulic and the contaminant boundary conditions, and 7) the contaminant concentrations in all recharge flows.

### 5.2  Validation of the Numerical Contaminant Transport Model for Nonpoint Source Groundwater Pollution

A numerical contaminant transport model was constructed for a nonpoint source groundwater pollution scenario which could be modeled analytically. The accuracy and precision of the linear programming solution to the numerical solute transport model were evaluated against analytical results at three levels of discretization. Linear programming simulations were also performed using aquifer transmissivities generated over the flow field by the inverse model. The simulations were executed to assess potential errors in contaminant predictions affected by errors in the estimated transmissivity field.

As a final check on the numerical model, mass balance calculations were completed for each simulation.

### 5.2.1  The Hypothetical Problem

An analytical model was developed to validate the numerical contaminant model for the subsurface transport of a conservative pollutant. The contaminant chosen was nitrate. Freeze and Cherry (1979) and Zaporozec (1983) have described nitrate as a non-adsorbing contaminant which does not undergo degradation in groundwaters which are oxygenated and have low dissolved organic content: this is the condition of groundwaters in Cape Cod where the management models will be ultimately applied.

The analytical model used to validate the numerical contaminant model employed the same hypothetical groundwater-flow field as used in the validation of the subsurface flow model. As before, aquifer permeability was uniform and isotropic. Natural recharge was constant and uniform over the radial distance from $r_w$ to L. From the circular region located in the center of the island, the aquifer received constant recharge from a fully penetrating well.

Figure 11 shows a plan view and profile of the island aquifer. Notice there are two circular regions. Within the small circular region of radius $r_w$, nitrates were delivered to the subsurface environment through recharge flow, (I). The nitrate concentration ($C_I$) in this recharge was 500 µg/l. Beyond the inner circular region were two
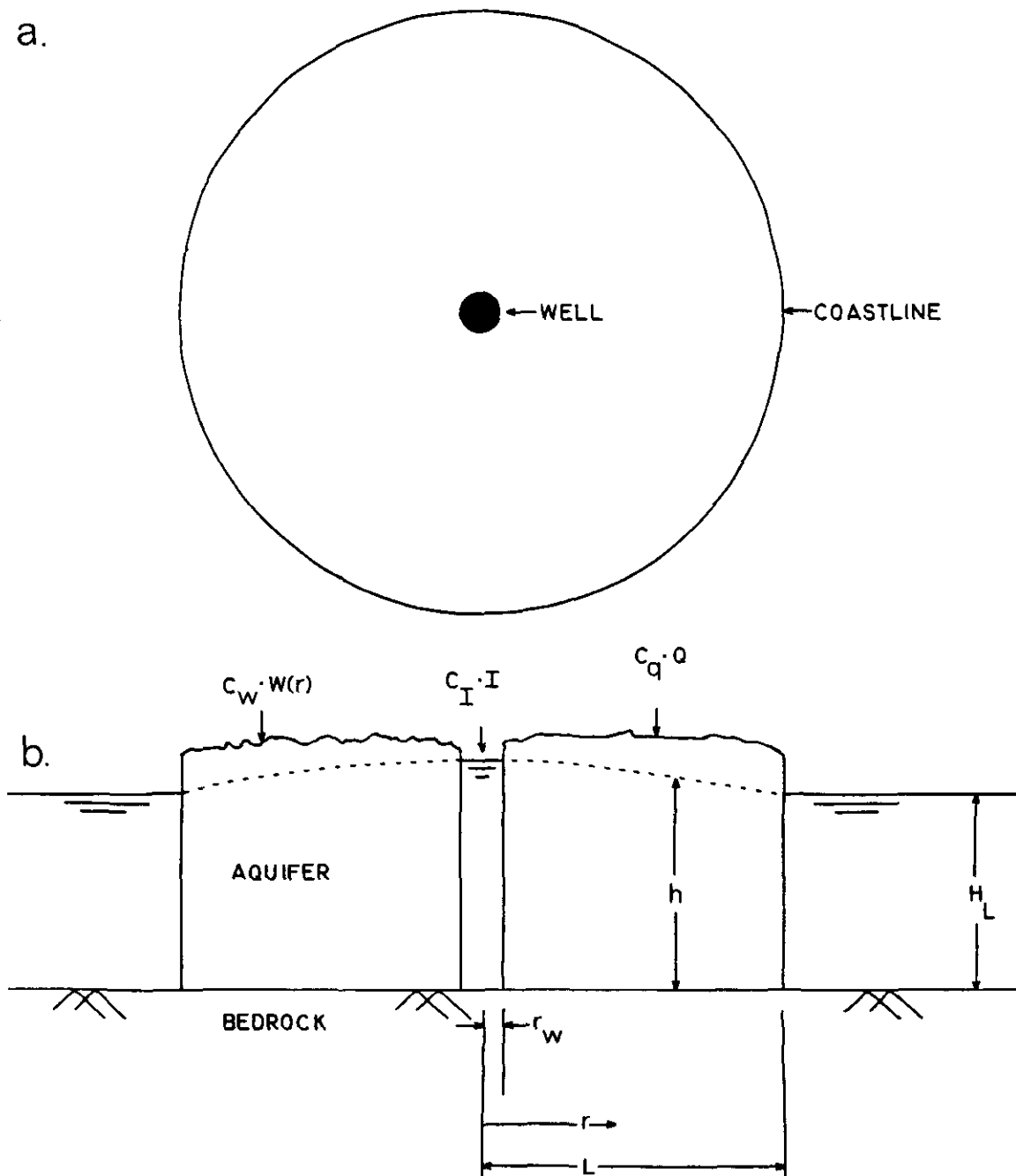
Figure 11. Plan view (a) and profile view (b) of the hypothetical two-dimensional, horizontal, and unconfined island aquifer receiving sustained nonpoint source pollution.

sources of groundwater nitrate contamination. The nitrate pollution was delivered from natural recharge (the assumed nitrate concentration in the recharge was 500 µg/l) and from the subsurface discharges of septic systems connected to houses on private water supply wells. The nitrate concentration $(C_w)$ in the waste effluent was assumed 35,000 µg/l as N; this concentration was typical of effluents from septic systems that reduce total nitrogen load by 50 percent (U.S. Environmental Protection Agency, 1980). Subsurface disposal flows from septic systems increased from zero at radial length $r_w$ to a maximum surface loading rate of $(W_L)$ near the coast. The function

$$\left(1 - \ell^{-sk \cdot \left(r - r_w\right)}\right) \tag{52}$$

at some $r \geq r_w$, expressed the extent to which septic system recharge had approached the maximum feasible surface loading rate. At any radial length greater than or equal to $r_w$ the function describing nitrate loading was

$$NO_3 \text{ loading } (r) = C_w \cdot W_L \cdot \left(1 - \ell^{-sk \cdot \left(r - r_w\right)}\right) \tag{51}$$

where $W_L$ = maximum recharge rate from septic systems in a mature residential development, (L/t);

$C_w$ = nitrate concentration (µg/l as N) in septic system effluents, $(M/L^3)$;

$r$ = radial distance from the center of the island aquifer, (L);

$r_w$ = radius of the circular region at the center of the island, (L);

sk = a constant, (1/L).

From Figure 11 the symbols are defined

h = elevation of the water table above bedrock, (L);

$H_L$ = elevation of water table at the coast, (L);

I = recharge in the circle defined by $r_w$, ($L^3/t$);

L = the radial distance from the center of the island to the coast, (L);

C = nitrate concentration in the aquifer at any radial distance between $r_w$ and L,($M/L^3$);

Q = surface recharge rate, (L/t);

$C_I$ = nitrate concentration in the recharge I, ($M/L^3$);

$C_q$ = nitrate concentration in the natural recharge at $r \geq r_w$, ($M/L^3$);

$C_w$ = nitrate concentration in septic system effluent ($M/L^3$).


## 5.2.2  The Analytical Model

The equation used to describe steady-state, horizontal, radial, advective transport of a conservative contaminant over the radial distance from $r_w$ to L was

$$v \cdot h \cdot \frac{\partial c}{\partial r} + Q \cdot C = C_q \cdot Q + C_w \cdot W_L \cdot \left(1 - \ell^{-sk\left(r - r_w\right)}\right) \qquad (53)$$

s.t. the boundary condition:  $C = C_I$  at  $r = r_w$.

For  $r \geq r_w$, the groundwater flow,  $v \cdot h$, was defined analytically from equation (27) as

$$vh = \frac{r \cdot Q}{2} - \frac{r_w^2 \cdot Q}{2 \cdot r} + \frac{I}{2 \pi r} \qquad (54)$$

The solution to equation (52) was

$$C = \left[ C_I \cdot I + \left( \pi \cdot Q \cdot C_q + \pi \cdot W_L \cdot C_w \right) \cdot \left( r^2 - r_w^2 \right) \right. \qquad (55)$$

$$+ \left( - 2 \cdot \pi \cdot W_L \cdot C_w \right) \cdot \left( \frac{sk \cdot r + 1}{sk \cdot sk} \right) + \left( 2 \cdot \pi \cdot W_L \cdot C_w \ell^{sk \cdot \left( r_w - r \right)} \right) \cdot$$

$$\left. \left( \frac{sk \cdot r + 1}{sk \cdot sk} \right) \right] / D_c$$

where  $D_c = r^2 \cdot \pi \cdot Q - r_w^2 \cdot \pi \cdot Q + I$.  The hypothetical groundwater pollution problem used the same groundwater flow conditions as those applied in the test problem for the validation of the numerical flow and inverse models. Beyond the hydraulic conditions specified in section 4.2.2, other conditions specific to the contaminant transport problem were

$C_w$ = 35,000 µg/l nitrate as N;

$C_q$ = 500 µg/l nitrate as N;

$C_I$ = 500 µg/l nitrate as N;

$W_L$ = 0.000375 m/d;

$sk$ = 0.00025 1/m.

5.2.3  Validation Results

A total of six numerical models were constructed. The six models include three at different discretizations (1, 2, and 3 kilometers) using the true transmissivities and three others constructed at the same discretization scales using transmissivities predicted by the inverse model.

Results from numerical groundwater flow models which employed both the true and the inverse modeled transmissivities were incorporated into the numerical contaminant transport models. The predicted heads at each element and the elemental transmissivities were used with equations (46) and (47) to define coefficients for horizontal flow which appear in the finite difference approximation of steady-state, two-dimensional, horizontal, advective contaminant transport (equation 45).

Figure 12 illustrates the two kilometer discretized version of the horizontal island aquifer. The two dimensional hyperbolic contaminant transport equation problem is properly posed when a single concentration estimate is specified at some point along each characteristic of the region being studied. In our hypothetical problem it was sufficient to specify the concentration where all characteristics originate; hence, the concentration in the discrete model was specified at an element corresponding to the center of the island (element $i,j = 9,9$ in figure 12). Discrete approximations of contaminant concentrations were related to subsurface disposal of pollutants through the continuity equations. Equation 45 was used as the continuity equation in interior nodes (non coastal nodes). For coastal nodes, equation (45) was modified with
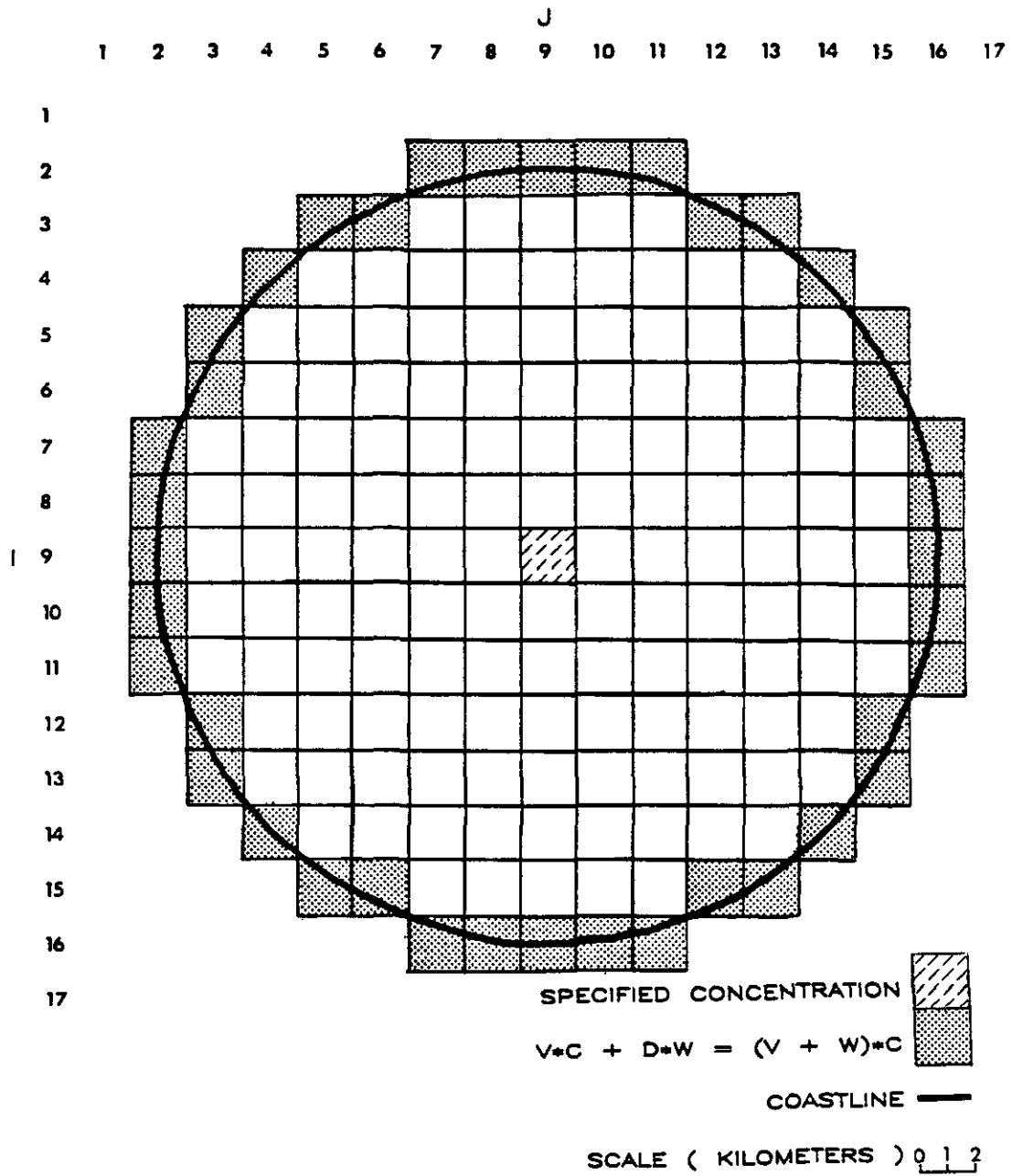
Figure 12.    Two-dimensional and horizontal numerical discretization of
the hypothetical island aquifer with illustrated boundary
conditions for the contaminant transport model.

appropriate second order backward and forward difference molecules, which allowed approximation of the hydraulic and contaminant gradients at the coast, with nodes inside the coastal boundary. Figure 12 illustrates how the coastal nodes were treated. The mass of contaminant entering the boundary from interior nodes $(V_i C_i)$ plus the mass of contaminant entering the aquifer from surface pollution occurring on the boundary $(DW_b)$ is equated to the product of total flow leaving the boundary $(V_i + W_b)$ and the concentrations of contaminant at the boundary $(C_b)$.

Subsurface nitrate loading from septic systems was calculated as

$$\left(C_w \cdot W\right)_{i,j} = \left(C_w \cdot W_L \cdot \left(1 - \ell^{-sk\left(r - r_w\right)}\right)\right) \tag{56}$$

where $W_{i,j}$ = the subsurface recharge flows from septic system effluent at node i,j, (L/t);

$r$ = radial distance as determined from the center of element i,j, (L).

In elements near $r = r_w$ the nitrate load was calculated in an amount reflecting the percent of elemental area covered by both the inner region $(r \leq r_w)$ and the outer region $(r > r_w)$. Nitrate fluxes for coastal elements were reduced to an amount reflecting the percent of elemental area representing aquifer.

The system of equations of the different numerical contaminant models were solved using linear programming. Analytical and numerical estimates of nitrate concentrations are plotted as contours in Figure

13; the numerical results are from the model using the true hydraulic conductivities and a two kilometer discretization. The plots illustrate the excellent agreement between the numerical solution to the whole island problem and the analytical solution. Most of the prediction error is located near the center of the island. The model over estimates nitrate concentrations near $r_w$ because it overestimates the magnitude of fluid flow in the region. Comparisons were made between numerical and analytical contaminant levels at each element. Numerical accuracy and precision were investigated over each specified discretization scale. The numerical error in predicted nitrate concentrations was calculated as

$$\text{Error} = \frac{\text{Numerical } NO_3}{\text{Concentration}} - \frac{\text{Analytical } NO_3}{\text{Concentration}} \qquad (57)$$

The relative error in numerical nitrate estimates was expressed as

$$\frac{\text{Relative}}{\text{Error}} = \frac{\text{Error}}{\frac{\text{Analytical } NO_3}{\text{Concentration}}} \qquad (58)$$

Data on the average numerical error ($\mu g/l\ NO_3$) and the average relative error (%) are plotted in Figures 14 and 15 respectively. Both Figures depicted an increase in model error as numerical discretization increases beyond one kilometer. The numerical nitrate predictions tended to be greater than the analytical estimates; consequently the numerical model provides conservative simulations of the true extent and impact of a groundwater pollution scenario. When a two kilometer element dimension was used, the average prediction error was less than 0.1 mg/l as $NO_3$-N or 2.2 percent. The model nitrate concentrations
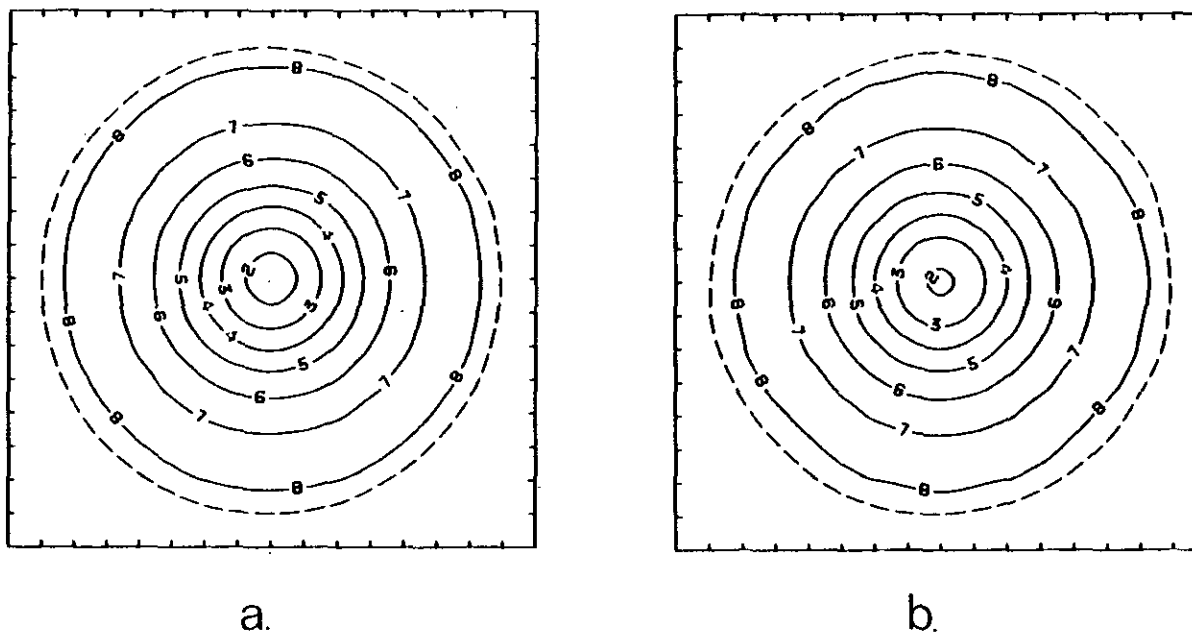
Figure 13. Contour plots of predicted nitrate nitrogen concentrations
(mg/l) from an analytical model (a) and from a numerical
model using a two kilometer discretization scale (b).
Dashed lines represent the coastline, while solid lines
correspond to the iso-concentration contours for nitrate.
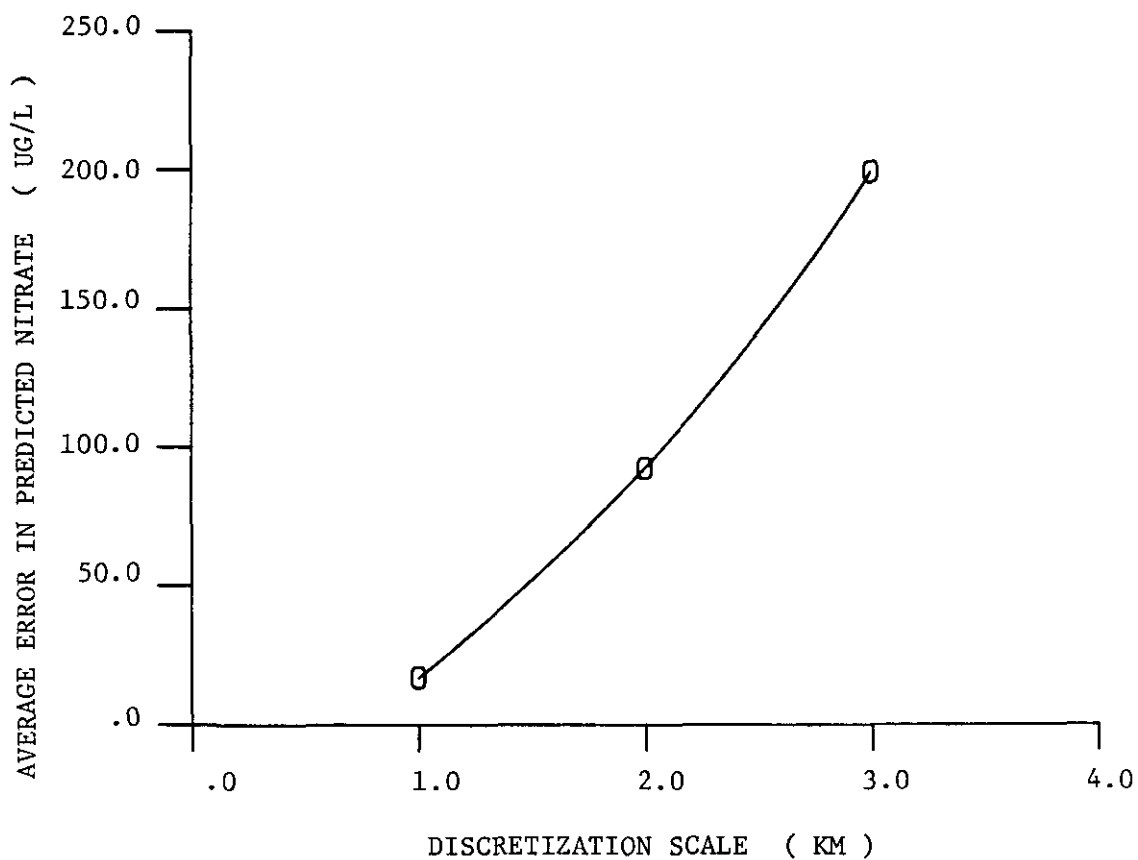
Figure 14. Average error in predicted nitrate concentrations as a function of the discretization scale (from the numerical contaminant transport model for the hypothetical island aquifer).
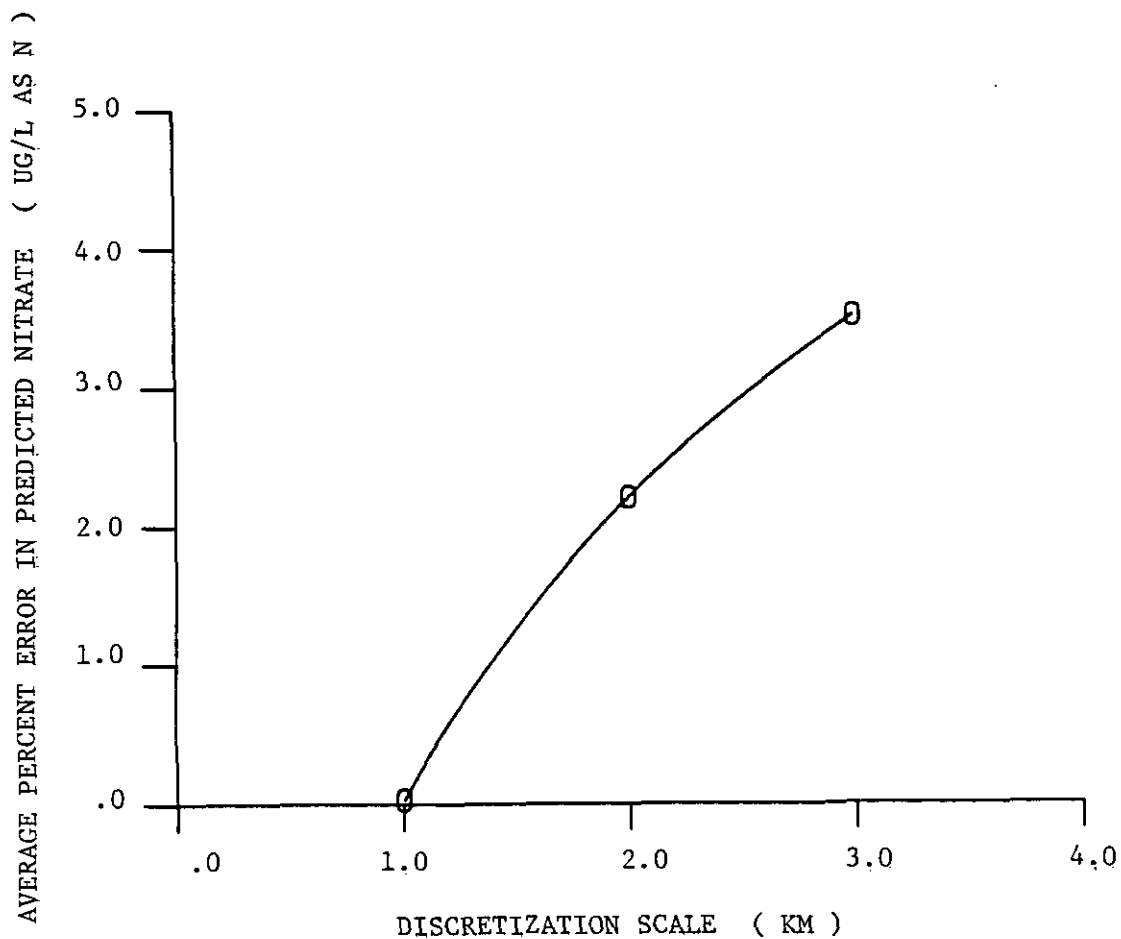
Figure 15.    Average percent error in predicted nitrate concentrations as a function of the discretization scale (from the numerical contaminant transport model for the hypothetical island aquifer).
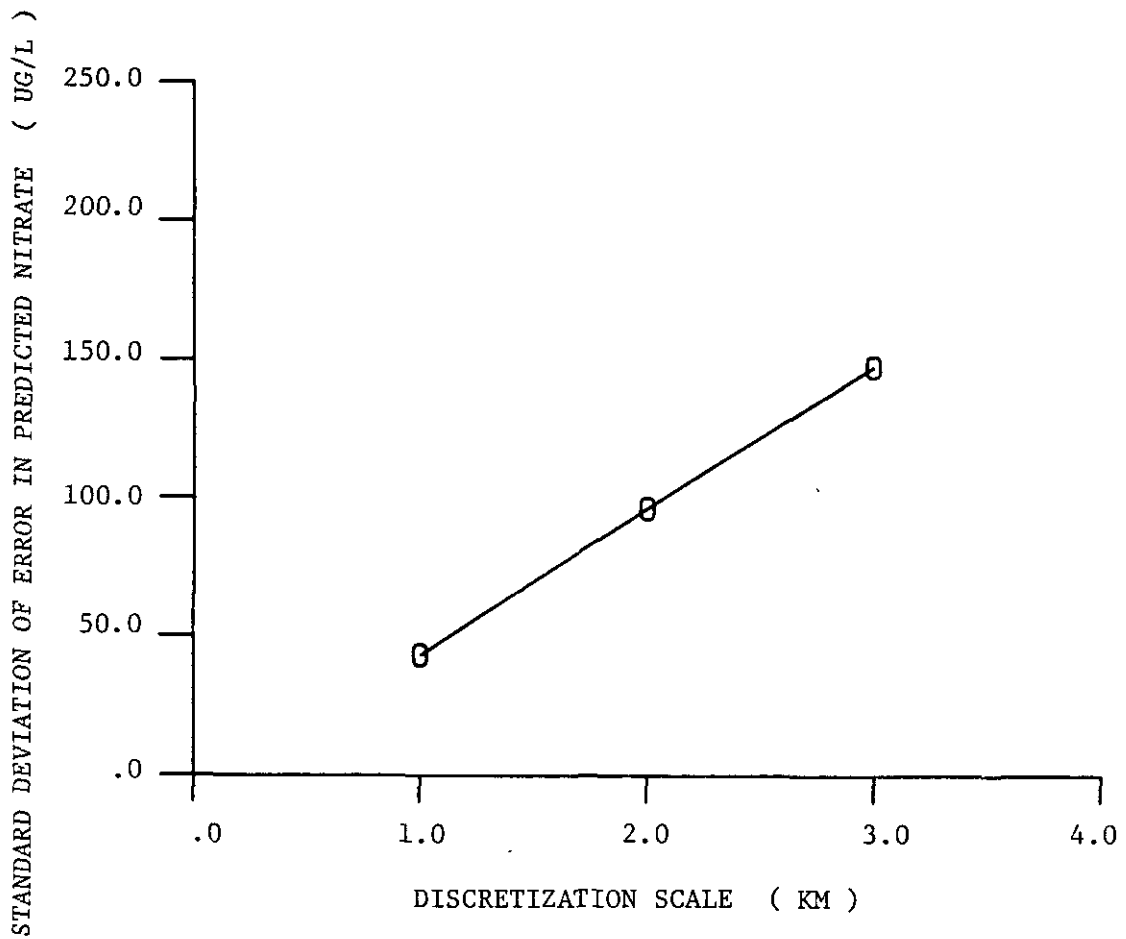
Figure 16.  Standard deviation of error in predicted nitrate
            concentrations as a function of the discretization scale
            (from the numerical contaminant transport model for the
            hypothetical island aquifer).
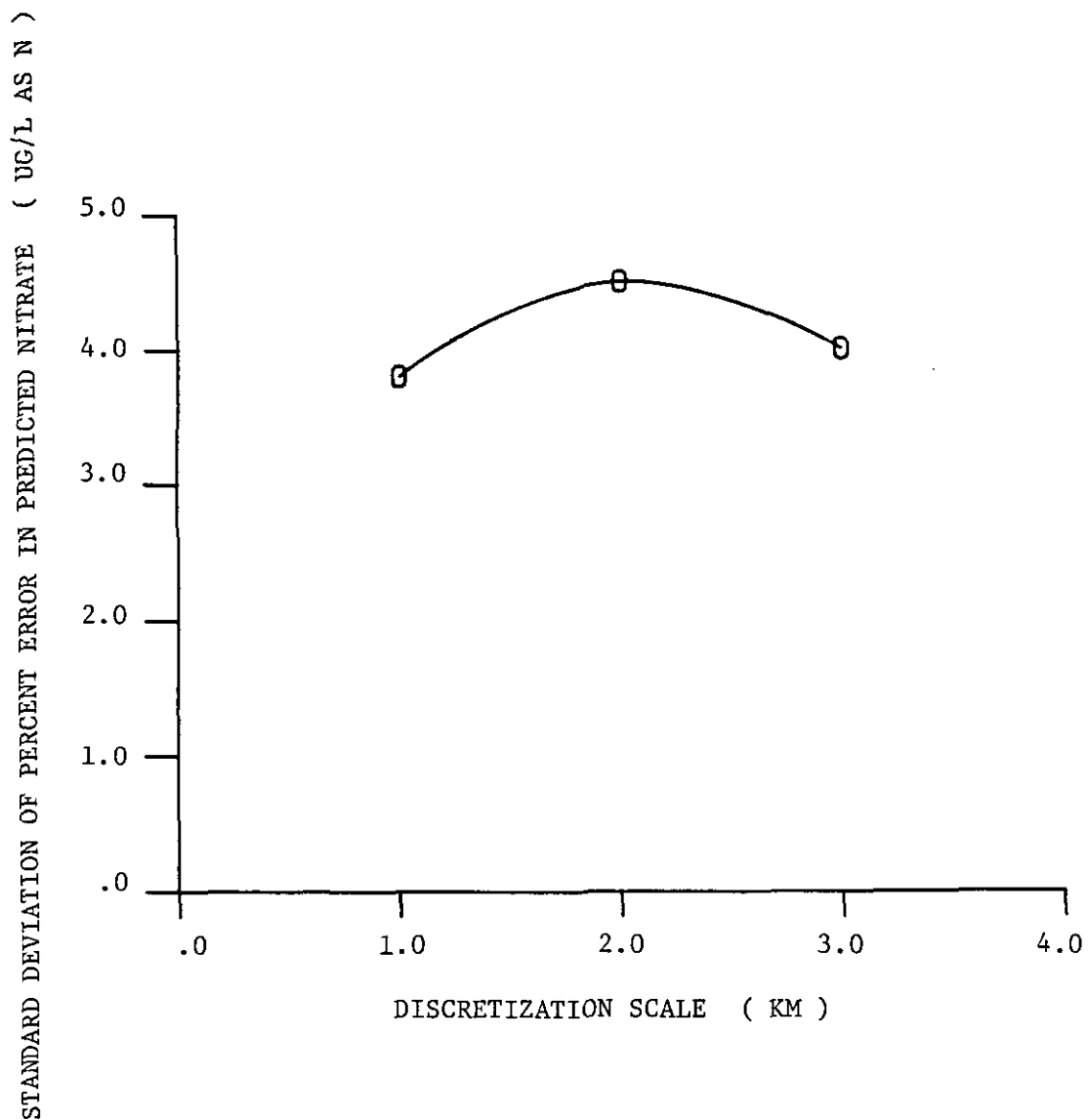
Figure 17. Standard deviation of percent error in predicted nitrate concentrations as a function of the discretization scale (from the numerical contaminant transport model for the hypothetical island aquifer).

Table 3. Difference Between Numerical Contaminant Model Predictions Using True Transmissivities and Transmissivities Estimated with the Inverse Model.

| Discretization Scale [km] | Average Difference [µg/l] | Standard Deviation of Average Difference [µg/l] | Average Percent Difference | Standard Deviation of Average Percent Difference |
|---|---|---|---|---|
| 1 | -0.500 | 3.000 | -0.001 | 0.040 |
| 2 | 0.600 | 4.000 | 0.010 | 0.050 |
| 3 | 4.200 | 4.000 | 0.060 | 0.050 |

represented average regional predictions over aquifer elements which were each four square kilometers in area. A two percent average error in simulated nitrate concentrations was considered more than acceptable for regional contaminant predictions.

The standard deviation of errors in predicted nitrates ($\mu$g/l as $NO_3$-N) and the relative standard deviation of errors are plotted as functions of discretization scale in Figures 16 and 17. The standard deviations of model errors increased in a linear fashion as the element dimension increased from one kilometer to three. At the two kilometer element scale the standard deviation of errors in predicted nitrate levels was less than 0.1 mg/l. Figure 17 showed that the greatest standard deviation of percent error (4.5) occurred not with the largest elements (three kilometers) but with the two kilometer discretization. The effect of discretization scale on the standard deviation of percent error could not be explained. Difference between the largest and smallest relative standard deviations was less than 0.7 percent.

Numerical simulations were also performed using transmissivities estimated by the inverse model. These simulations were designed to test the influence of errors in the transmissivity field on numerical contaminant estimates. Shown below Table 3 presents the average and the average relative difference between simulation results obtained with the true aquifer hydraulic conductivities and the inverse modeled transmissivities. The simulation results were the same regardless of the source of transmissivity estimates.

The last evaluation of the contaminant transport model was a mass balance analysis. The percent error in the nitrate balance was small at all scales of discretization; see Figure 18. A total loss of 0.2 percent of the contaminant applied was noted at the two kilometer scale. As the scale of discretization increased the mass balance condition changed from one of losing nitrate to gaining dissolved contaminant in an amount of 0.4 percent.

The validation of the numerical contaminant transport model clarifies the level of expected model accuracy at three discretization scales for a groundwater pollution scenario comparable to the hypothetical problem. For a numerical discretization of two kilometers it was shown that model errors were 2.2 $\pm$ 4.5 percent. This level of accuracy was considered acceptable given that model results correspond to regional nitrate predictions and not point estimates. The numerical contaminant estimates produced from simulations using transmissivities generated from the inverse model were excellent. Errors present in the elemental transmissivities obtained with the inverse model did not significantly affect predicted contaminant distributions.
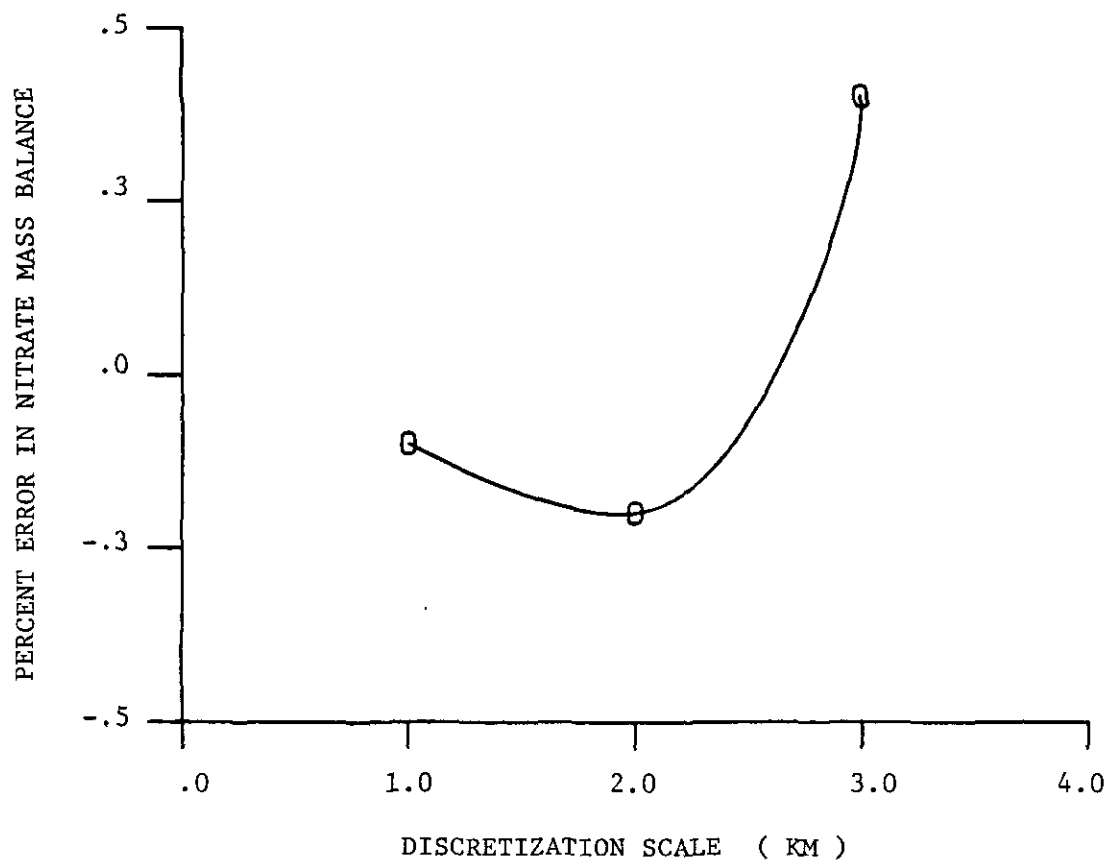
Figure 18.   Percent error in nitrogen mass balance as a function of the discretization scale (from the numerical contaminant transport model for the hypothetical island aquifer as a function of the discretization scale.

CHAPTER 6

CALIBRATION OF A GROUNDWATER FLOW MODEL FOR

WESTERN CAPE COD

The application of a nonpoint source groundwater pollution
management model to a study site must follow the successful mathematical
description of the groundwater flow field. This chapter is devoted to
the modeling of groundwater flows in the aquifer underlying Western Cape
Cod. The intense pressures for residential development portend future
nonpoint source groundwater pollution problems for this area of
Massachusetts; consequently, this area was chosen to serve as an
application region for the management models.

The chapter begins with a brief description of the primary
groundwater pollution problems on Cape Cod, followed by a discussion of
the hydrology and geology of the study area. The next section of the
chapter presents the preliminary assumptions that underlie efforts to
model the Cape Cod Aquifer with the validated subsurface flow model.
Calibration of the numerical flow model is accomplished with the inverse
model. Final simulation results are presented as water table maps and
numerical error scatter plots.

6.1  Nonpoint Source Groundwater Pollution on Cape Cod

The aquifer under Barnstable County, Massachusetts (Figure 19) has
over a hundred public wells and has been designated a Sole Source
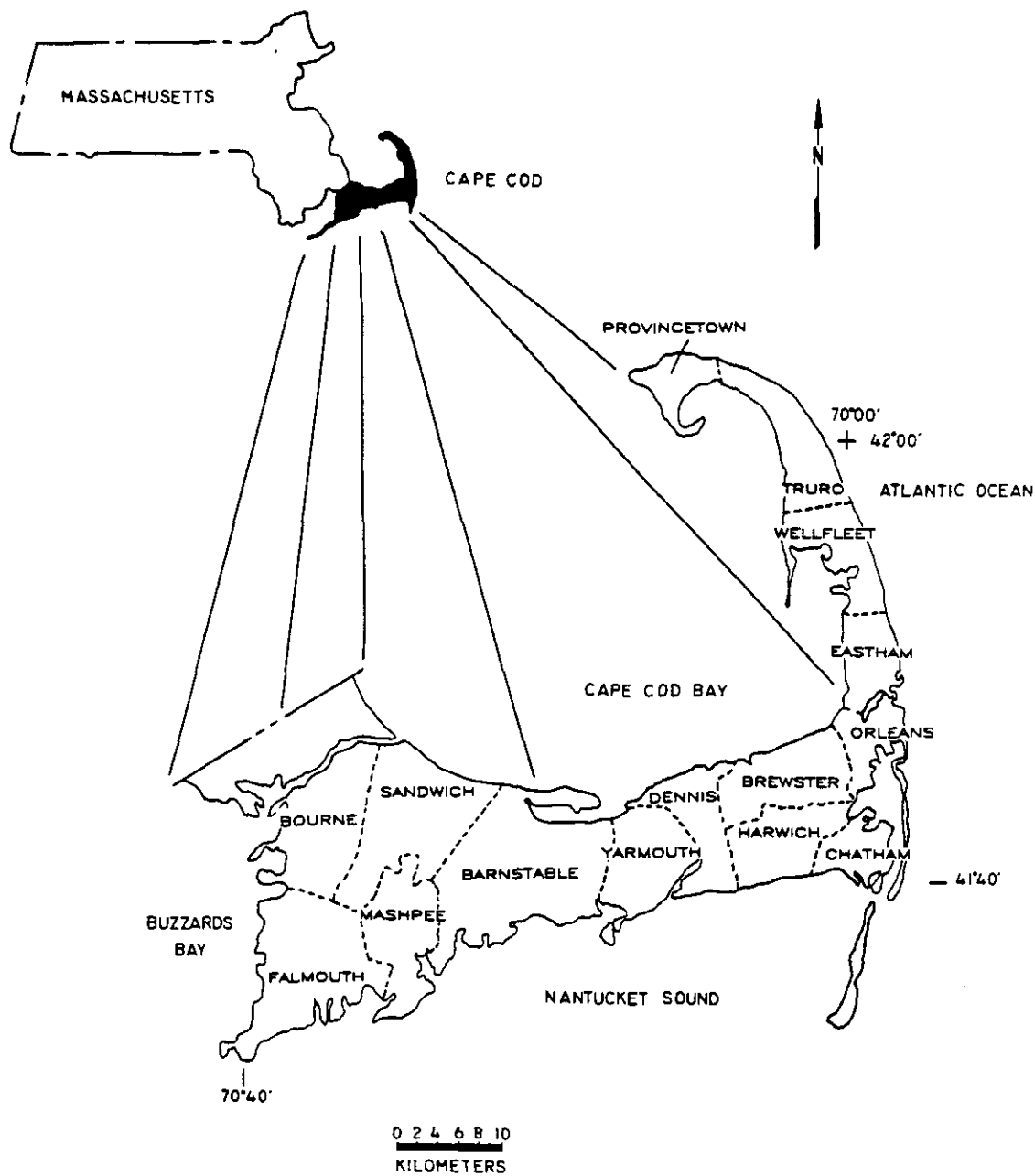
Figure 19.    Map of Barnstable County, Massachusetts with associated towns.

Aquifer by the U.S. Environmental Protection Agency. The aquifer is threatened by nonpoint source pollution from road runoff, lawn fertilizers, septage lagoons and pits, and underground fuel and chemical storage tanks. On-site domestic waste disposal systems (septic systems and cesspools), however are among the largest and most ubiquitous source of distributed groundwater contamination on Cape Cod; 88 percent of the year-round residents use septic tanks or cesspools (Quadri, 1984). Another major source of groundwater nitrate contamination is the use of lawn fertilizers.

Research has linked the expanded use of on-site domestic waste disposal systems and the application of lawn fertilizers to increases in groundwater nitrate levels on Cape Cod and elsewhere (Yates, 1985, Flipse et al., 1984, Quadri, 1984, Porter, 1980, Katz, Linder, and Ragone, 1980). Nitrate pollution of groundwaters is recognized as a significant problem because nitrate is a persistent contaminant in the subsurface environment which poses potential health hazards (Zaporozec, 1983) and contributes to the eutrophication of coastal waters. Elevated nitrate concentrations in groundwaters may also indicate the presence of other contaminants (e.g., bacteria and viruses), which like nitrate could be public health hazards that are expensive to remove from water supplies (Perkins, 1984).

Sewer systems are often installed to mitigate water quality problems derived from septic system pollution. The 208 Water Quality Plan for Cape Cod (Cape Cod Planning and Economic Development Commission, 1987) favored the continued use of septic systems because

they are an inexpensive method of treating domestic wastes and because the artificial recharge helps to maintain water levels within the aquifer.

Future population growth is expected to expand the use of septic systems and lawn cultivation into sensitive regions where major public wells are now situated; consequently, the nitrate pollution on Cape Cod could develop into a regional groundwater quality problem. To protect groundwaters, a cooperative effort will be needed by all communities of Barnstable County to determine where and to what extent residential development should be permitted.

## 6.2  The Hydrology and Geology

The western section of Barnstable County, Massachusetts, was chosen as the application region for the nonpoint source groundwater pollution management models (see Figure 20). Cape Cod is primarily composed of unconsolidated deposits of clay, silt, sand, gravel, and boulders laid down by glacial and fluvial action during the pleistocene age. The unconsolidated material extends to depths of 25-152 meters below mean sea level with general increases in depth occurring from Bourne eastward (Burns, et al., 1975). Underlying the unconsolidated deposits is a granitic basement rock.

The major geological features of the area are the Buzzards Bay Moraine, the Sandwich Moraine, and the Mashpee Outwash Plain. Figure 21 illustrates the location of each formation. The moraines are elevated formations which are composed of unconsolidated deposits that range in
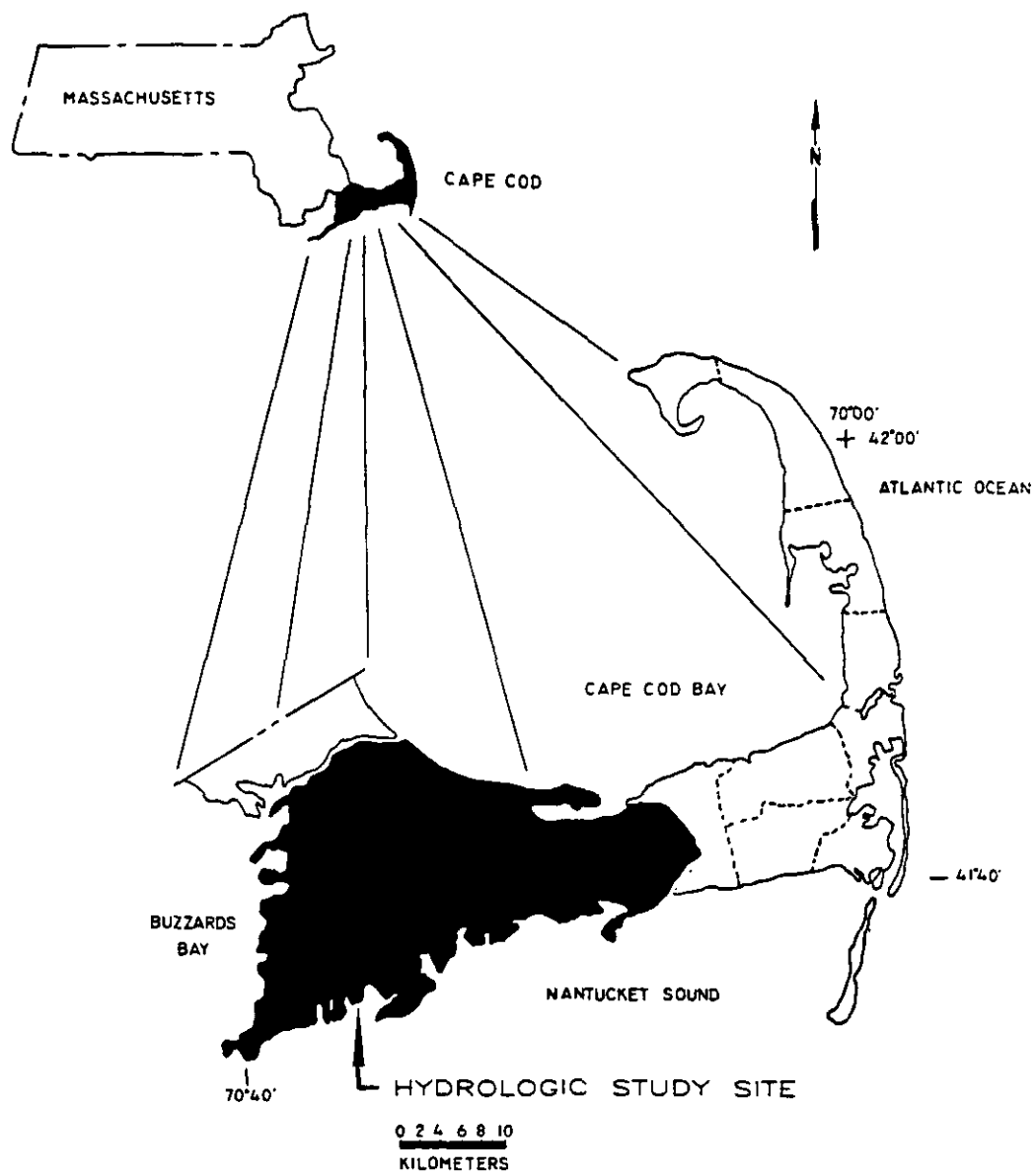
Figure 20.   Map of Western Cape Cod which serves as the hydrologic
             study site for the application of the numerical groundwater
             flow model.
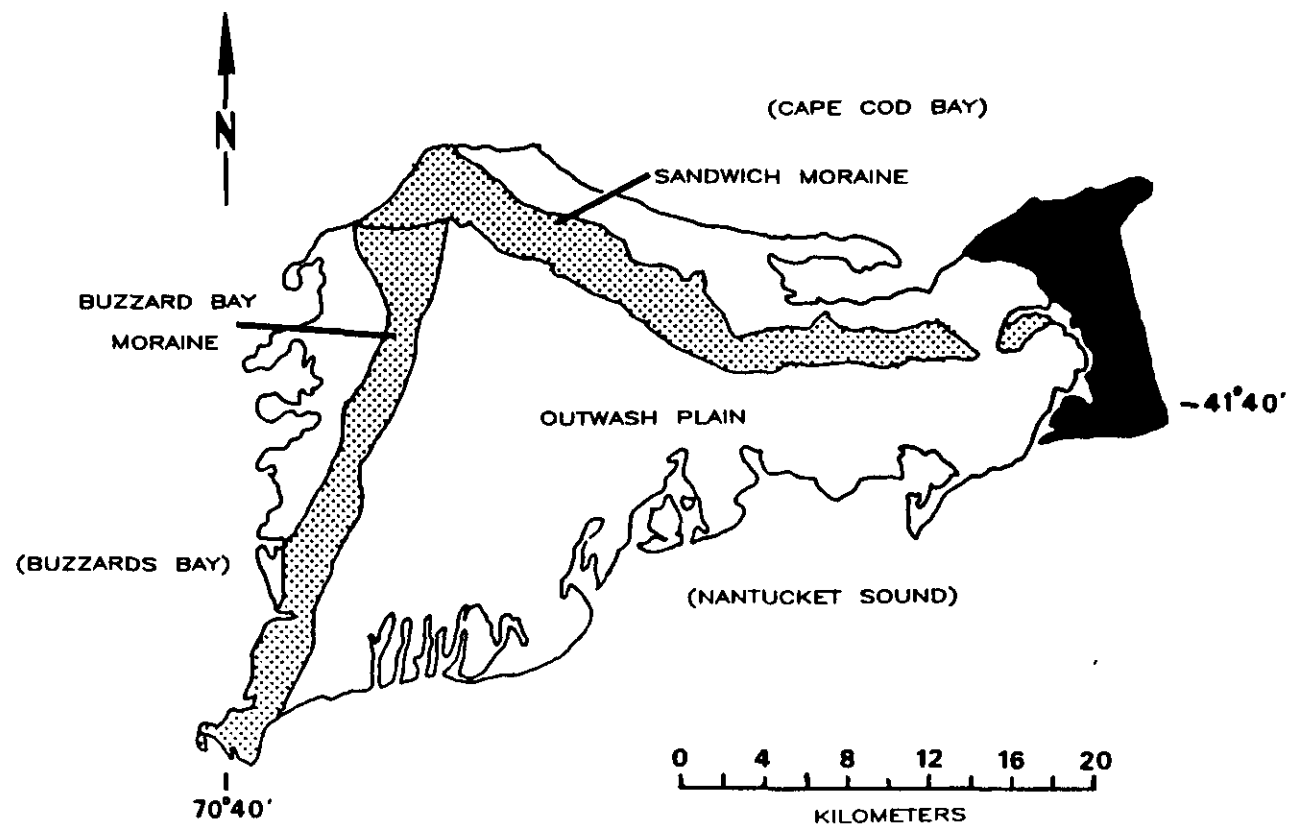
Figure 21.  Geologic map of Western Cape Cod.  Adapted from Guswa and LeBlanc (1985).

grade from clay to larger boulders. The water transmitting properties of the moraines are generally considered poor, because the lithology (and probably the permeability) of the moraine aquifer tends to vary greatly over short distances (Guswa and LeBlanc, 1985). The outwash plain is highly permeable and the depth to the water table is more shallow here than in the moraines. The deposits in the outwash plain are generally sand and gravel with lenses of clays and silt. The grades of the deposits decrease in a direction south and east of the moraines.

The Sole Source Aquifer for the residents of Western Cape Cod is an unconfined lense of fresh water floating on top of salt water (see Figure 22). Surface waters in this region are primarily water table ponds and a few minor streams which receive flow principally through groundwater seepage (with small contributions from surface runoff). Annual variation of pond surface elevation is greatest within the interior regions. Marshes and streams near the coast have relatively constant surface elevations and operate as significant points of groundwater discharge (Guswa and LeBlanc, 1985).

Annual recharge (from precipitation only) on the western cape aquifer is 30-56 centimeters (Guswa and LeBlanc, 1985, LeBlanc, 1984, Burns, et al., 1975, and Strahler, 1972). Figure 22 shows the cross-sectional flow paths for water entering the aquifer from surface precipitation. Near the morainal deposits (central rib) water travels in horizontal and vertical directions. As some of the water flows vertically, it is deflected in an horizontal direction due to the

N    MORAINE    WATER TABLE    OUTWASH    S

m.s.l.

-30-
-60-
-90-

GLACIAL DEPOSITS    AQUIFER

a.

BASEMENT ROCK AQUIFUGE

SALT GROUND WATER

FRESH GROUND WATER

GLACIAL DEPOSITS

0
-300
-600

b.

FRESH-SALT INTERFACE    BASEMENT ROCK

TRUE-SCALE CROSS

SECTION
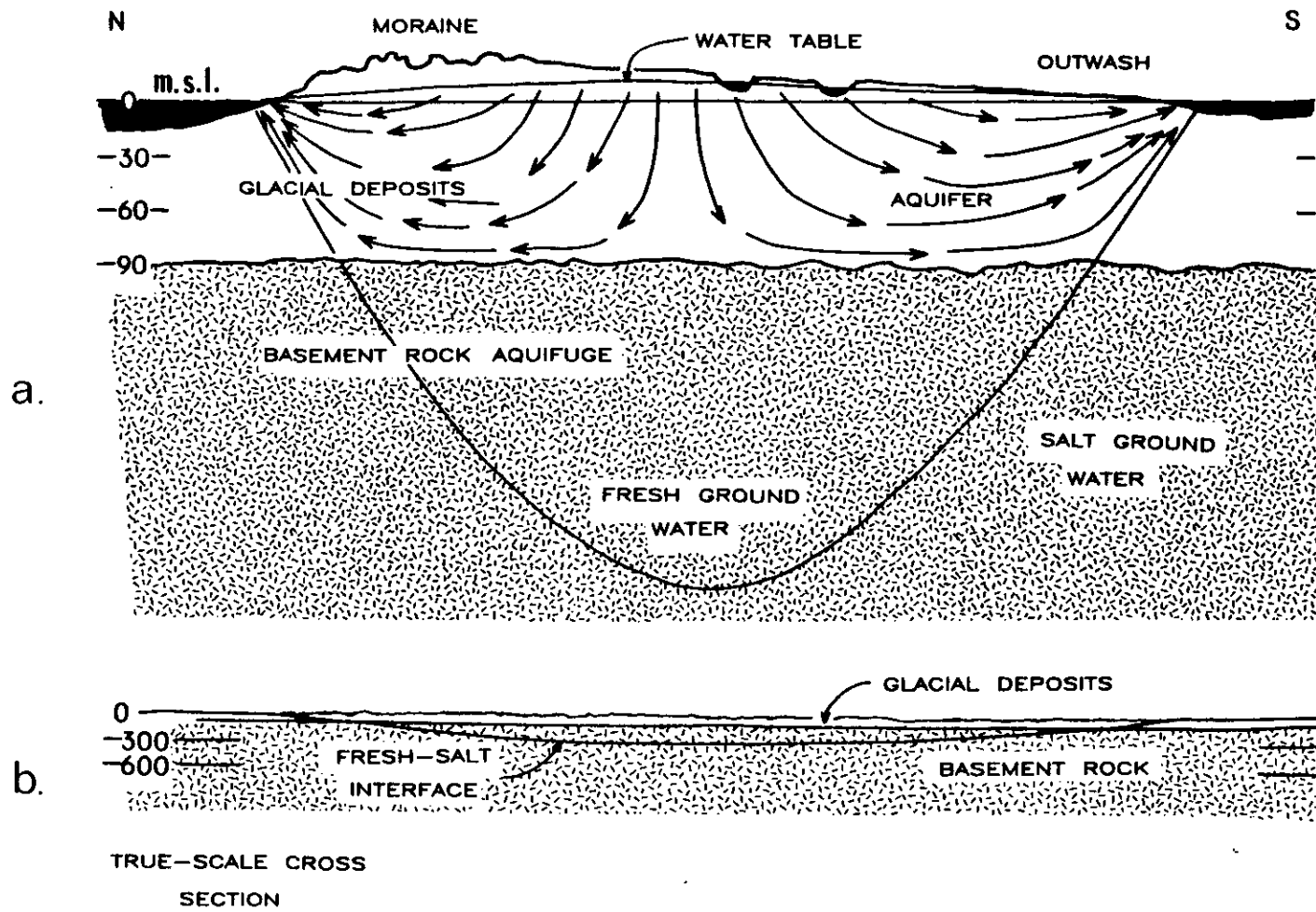
Figure 22.   Profile view of the unconfined aquifer of Western Cape Cod
showing the idealized groundwater flow field (a) and the
aquifer drawn to scale (b). Adapted from Strahler (1972).

decreasing vertical permeability of outwash deposits with depth. Figure 22 does not show the presence of fine sand, silt or clay layers; however, LeBlanc (1984) reported the existence of such layers in Falmouth, and Strahler (1972) identified a layer of silt/clay 150 feet thick from data collected from a well drilled in Harwich. Both surmised that these formations have much lower permeabilities than overlying outwash deposits and both located these layers approximately 30 meters below Mean Sea Level (MSL). Stahler (1972) suggested that the principal groundwater movement could be limited to the highly permeable unsolidated deposits overlying this layer. LeBlanc (1984) investigated a contaminant plume in the outwash plain of Falmouth, Massachusetts and observed that a layer of fine sands and tills underlying the coarse sand and gravel deposits precluded vertical movement of the plume. Thus, the highly permeable layer of sand/gravel outwash which overlies the fine sand and till deposits appears to be the major conduit of groundwaters and groundwater contaminants.

Figure 23 shows a gently sloping water table in the outwash plain which suggests that groundwater moves easily and horizontally from the central rib through the sand and gravel deposits of the glacial outwash plain toward the coast (LeBlanc, 1984 and Bear, 1979). Near the coast the flows are deflected upward by the denser sea water at the freshwater/saltwater interface (see Figure 22). Because groundwater recharge is seasonal, annual fluctuations in the water table have been observed; the greatest range of fluctuations occur inland (2-3 feet)
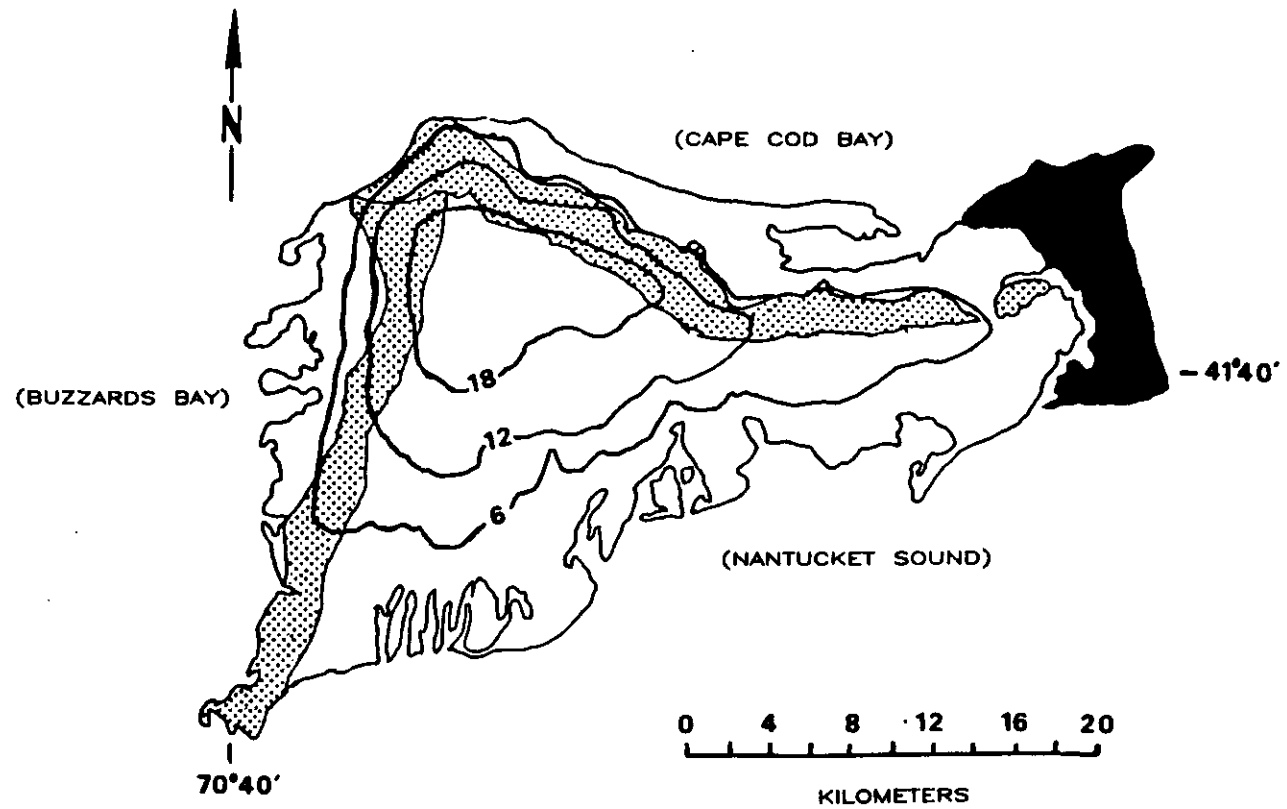
Figure 23.  Geologic Map of Western Cape Cod with the steady-state
water table elevation contours overlaid.  Adapted from
Guswa and LeBlanc (1985).

while only a few inches of fluctuation occur near the coast (Strahler, 1972).

## 6.3 Modeling Unconfined Groundwater Flow

The validated steady-state, two-dimensional horizontal groundwater flow model was calibrated for Western Cape Cod. Several assumptions were made regarding the characteristics and boundaries of the aquifer, the nature of groundwater flow in the region, the sources of significant recharge, and finally the location of discharge boundaries. Data to estimate hydrologic stresses were collected by Guswa and LeBlanc (1985) from lengthy records of climatological data (precipitation, temperature, etc.) and observation well data. The shortest records used were for groundwater pumpage and artificial recharge; estimates of these activities amounted to less than five percent of the total recharge budget. Calibration of the flow model was efficiently accomplished with inverse modeling. Data on aquifer stresses and elemental water table elevations were used in an inverse model to identify discrete aquifer transmissivities which would reproduce observed hydraulic heads during numerical flow simulation.

### 6.3.1 Model Assumptions

The following assumptions were made regarding the characteristics of the aquifer:

1) Previous work from Guswa and LeBlanc (1985), Burns, et al., (1975), and Sterling (1963) had shown that the aquifer on

Western Cape Cod is composed of layered unconsolidated nonhomogeneous deposits of boulders, gravel, sand, silt, and clay which suggests that the aquifer is nonhomogeneous. The horizontal deposition of sediments have created an aquifer which is anisotropic with respect to fluid flow in the vertical direction, but relatively isotropic with respect to fluid flow in the horizontal direction;

2) To use the numerical groundwater flow model, the aquifer is discretized into elements where the physical properties of the aquifer are assumed piecewise constant within each element. Large discretizations are acceptable, if the phenomenon of interest is greater than the scale of aquifer inhomogeneities (Bear 1979);

3) The difference in density between freshwater and saltwater prevents significant mixing along the freshwater/saltwater interface; therefore, the elevation of that interface wherever it intersected the highly permeable outwash deposits was treated as an impermeable bottom boundary;

4) Both LeBlanc (1984) and Strahler (1972) identified the basement rock as an aquifuge; however, LeBlanc also observed that the less permeable layers of fine sands, silts, and clays behaved as impermeable boundaries to significant fluid flow and contaminant transport when compared to the overlying highly permeable sand and gravel outwash deposits. It was assumed therefore, that the top elevation of layers of fine sand, silt,

or clay would be used as the elevation of the impermeable

boundary except where the highly permeable glacial outwash

deposits extend all the way to bedrock.

Several assumptions were made regarding the nature of subsurface

flow on Cape Cod:

1) Piezometric readings by LeBlanc (1984), Guswa and LeBlanc

(1985), and Burns, et al., (1975) support assumptions that, on

a regional scale, groundwater flows are principally horizontal.

It was assumed that regional groundwater flow in Western Cape

Cod aquifer is unconfined and horizontal; vertical flows are

minor over most of the aquifer except near the coast, but these

flows are not considered;

2) Surface water elevations for ponds in the interior of the study

area were assumed to reflect local water table elevations;

3) Seasonal changes in the regional piezometric surface are a

reflection of seasonal variations in the groundwater recharge

buffered by a huge reservoir of fresh water stored in the

Western Cape Cod aquifer. Consumptive losses are estimated to

be less than one percent of the total annual natural recharge.

It was assumed that averaged water table elevations reflect the

approximate steady-state conditions;

4) Since the density of septic tank effluent is close to that of

distilled water, it is assumed that groundwater flow is density

independent;

5) Because the groundwater model considers only horizontal flows, the dimensions of the discrete elements must be sufficiently large that horizontal flows dominate. Bear (1979) noted that vertical flows resulting from local hydraulic perturbations (e.g., pumping) become minor over horizontal distances of 1.5-2.0 times the thickness of the saturated zone (the distance between the water table boundary and impermeable bottom). On Western Cape Cod the horizontal dimensions of each element were 2000 meters which is over seven times the maximum thickness of the aquifer.

Regarding hydrologic stresses in the study area, there were a few other assumptions:

1) Based upon the work of Guswa and LeBlanc (1985), Burns, et al., (1975), and Strahler (1972) it was assumed that the principal sources of groundwater recharge were precipitation and return water from septic systems;

2) Throughout any discrete element, it was assumed that the aquifer receives evenly delivered stresses from constant recharge originating from precipitation, septic tank effluents, or other identified sources and constant discharge from pumping or other forms of natural and artificial groundwater withdrawal;

3) Groundwater discharge along the shore (i.e., through coastal marshes) and through the seabed is controlled by the elevation of the coastal waters. It is assumed that the hydraulic head

along the coast is a constant taken to equal mean sea level above some stated datum.

## 6.3.2 Numerical Discretization

A uniform numerical discretization was applied over the aquifer (see Figure 24). The two kilometer discretization was used because excellent results were obtained at this scale during the validation of both the flow and the contaminant transport models. The area of each element was assumed to equal four square kilometers unless it was determined that the effective recharge area was less (e.g., for elements along the coastline). Where elements had less than four square kilometers of area the recharge was reduced proportionately.

## 6.3.3 Boundary Conditions

Two primary boundary conditions were incorporated into the mathematical modeling of subsurface flow on Western Cape Cod. The first condition specified that the aquifer floor was impermeable and that no flow was lost to leakage. This assumption is reasonable since a granitic basement rock and/or clay underlie the permeable sandy aquifer (Strahler, 1972).

The second condition concerns the proper approximation of the boundary where groundwater discharge occurs along the shore (i.e., through coastal marshes) and through the sea bed. It was assumed that the elevation of the coastal water elevation determined water table elevations and groundwater discharge along the coast. Groundwater

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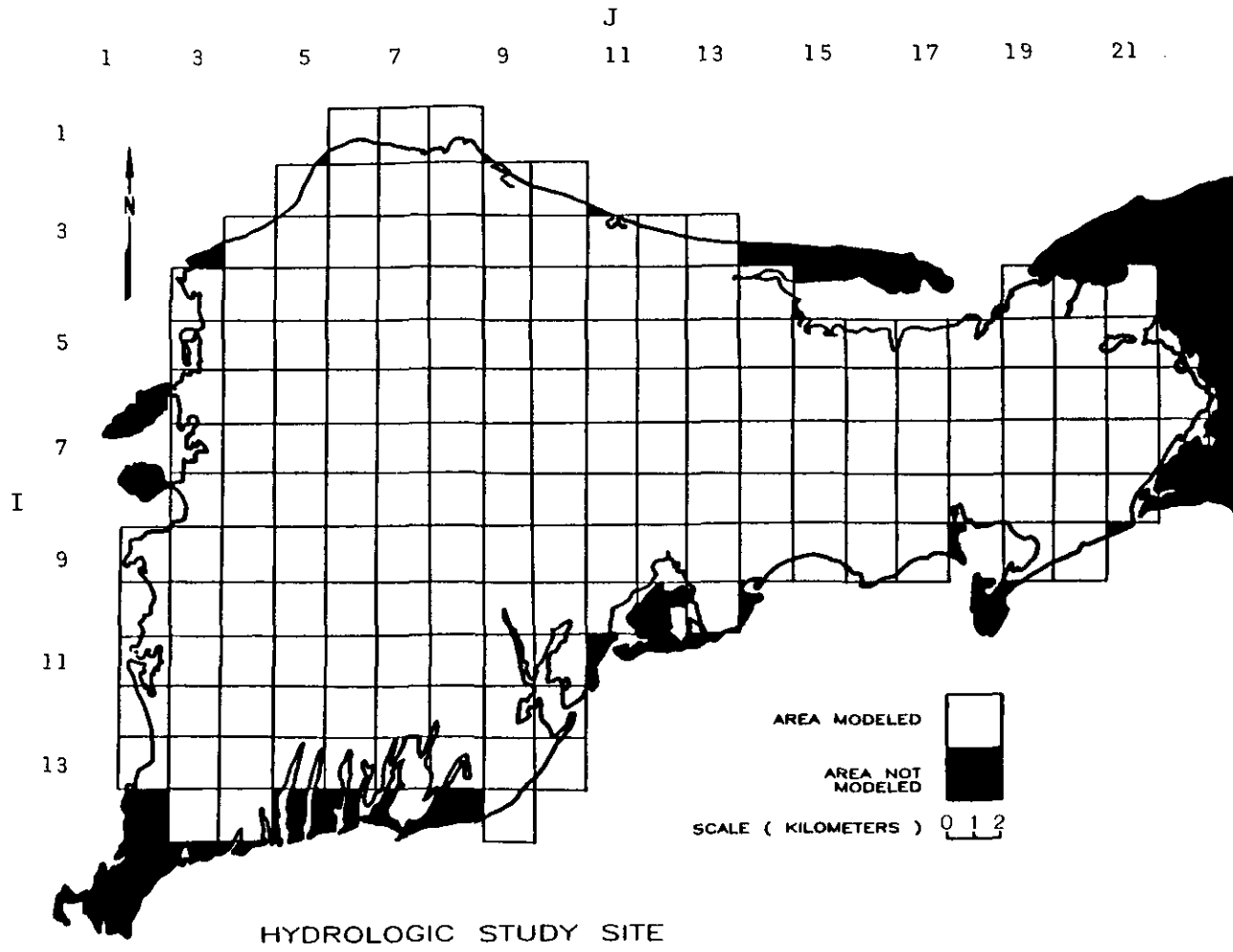
Figure 24.  Map of two-dimensional, and horizontal discretized Western
Cape Cod showing regions included in the groundwater flow
model.  Elements are addressed by the i,j coordinate
system.

elevations along the coast are somewhat constant with elevations
fluctuating from 0 to 0.6 meters (Frimpter and Fisher 1983);
consequently, subsurface flow at the coast may be approximated with a
specified head boundary condition.  For all elements along the coast,
the hydraulic heads were specified at mean sea level; see Figure 25.

## 6.3.4  Data Requirements and Acquisition

The numerical flow model requires data on spatial changes in
groundwater recharge rates, pumpage rates, and aquifer transmissivity.
Data was collected from the U.S. Geological Survey on hydrologic
stresses that would yield elemental recharge rates and pumpage rates.
It was not possible to calculate discrete transmissivities from aquifer
thickness and elemental permeability information because existing
data was inadequate to define regional permeabilities at each node.
Therefore the discrete transmissivities were estimated with the inverse
model.  The inverse model required hydraulic head estimates at each node
and the estimated transmissivity at some point along each flow line in
addition to the hydraulic stresses data.  The specifics of the data
and the methods of acquisition are presented below.

## 6.3.4.1  Natural and Artificial Recharge

Aquifer recharge on Cape Cod is derived from natural precipitation
and artificial recharge from septic system effluents and wastewater
treatment plants.  Guswa and LeBlanc (1985) calculated natural recharge
as the difference between average annual precipitation and average
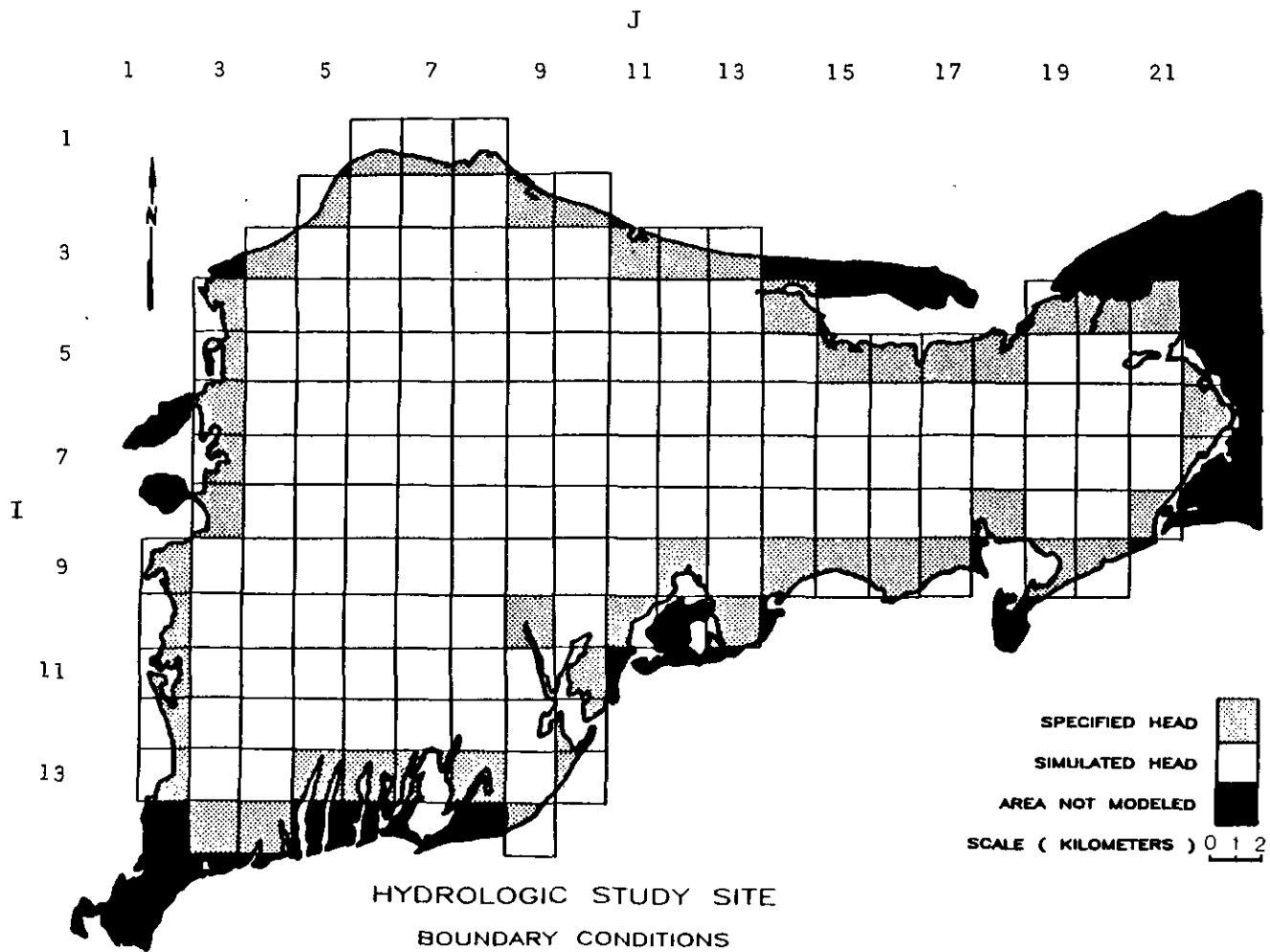
Figure 25.  Map of discretized Western Cape Cod showing hydrologic
            boundary elements of specified hydraulic head and elements
            where the hydraulic head is predicted.

annual evapotranspiration. Precipitation was calculated from rainfall data for 1947-1976. The annual evapotranspiration rate was calculated by the Thornthwaite method (Strahler, 1972). Data from 1947-1976 on mean monthly temperature was used in conjunction with the geographic latitude of the area to obtain estimates of the annual rate of evapotranspiration. The estimated natural recharge rate increased from 48 cm/year in Yarmouth (in the east) to 56 cm/year in Falmouth (in the west).

Over all of Cape Cod, 88 percent of the population uses septic systems as a means of disposing domestic wastes. From the 1980 census, 90 percent of the population uses municipal water; consequently, the water exported from municipal wells appears as artificial recharge from septic systems. The total artificial recharge from each water district was calculated from the average daily volume of water pumped by the water district. Table 4 shows the average daily pumpage from each district during 1975-76. Uniform artificial recharge rates from each water district were determined from the ratio of a district's average daily pumpage and the area served. Artificial recharge rates for each element were adjusted proportionally to the fraction of element area served by each municipal water supply. Nodes which received artificial recharge are identified in Figure 26.

Aquifer recharge from the two sewage treatment plants (the Otis and Barnstable plants) were equated to the flows reported by Guswa and LeBlanc (1985) in their three-dimensional model for Western Cape Cod.

Table 4. Artificial Recharge Sources Over Western Cape Cod in 1975/76.

| Water Supply Source | Average Daily Pumpage [m³/day] | Area Served [km²] | Percent Flow Applied |
|---|---|---|---|
| Barnstable Fire District | 1079 | 20 | 100 |
| Barnstable Sewage Treatment Plant | 2650 | 1 | 100 |
| Barnstable Water Company | 5856 | 24 | 100 |
| Bourne Water District | 2306 | 24 | 100 |
| Centerville-Osterville and Cotuit Fire Districts | 6149 | 32 | 100 |
| Falmouth Water District | 10138 | 34 | 100 |
| Highwood Water Company | 484 | 3 | 100 |
| Otis Air Force Base | 2298 | 1 | 100 |
| Sandwich/South Sagmore Water District | 1490 | 21 | 100 |
| Yarmouth Water District | 9270 | 28 | 100 |
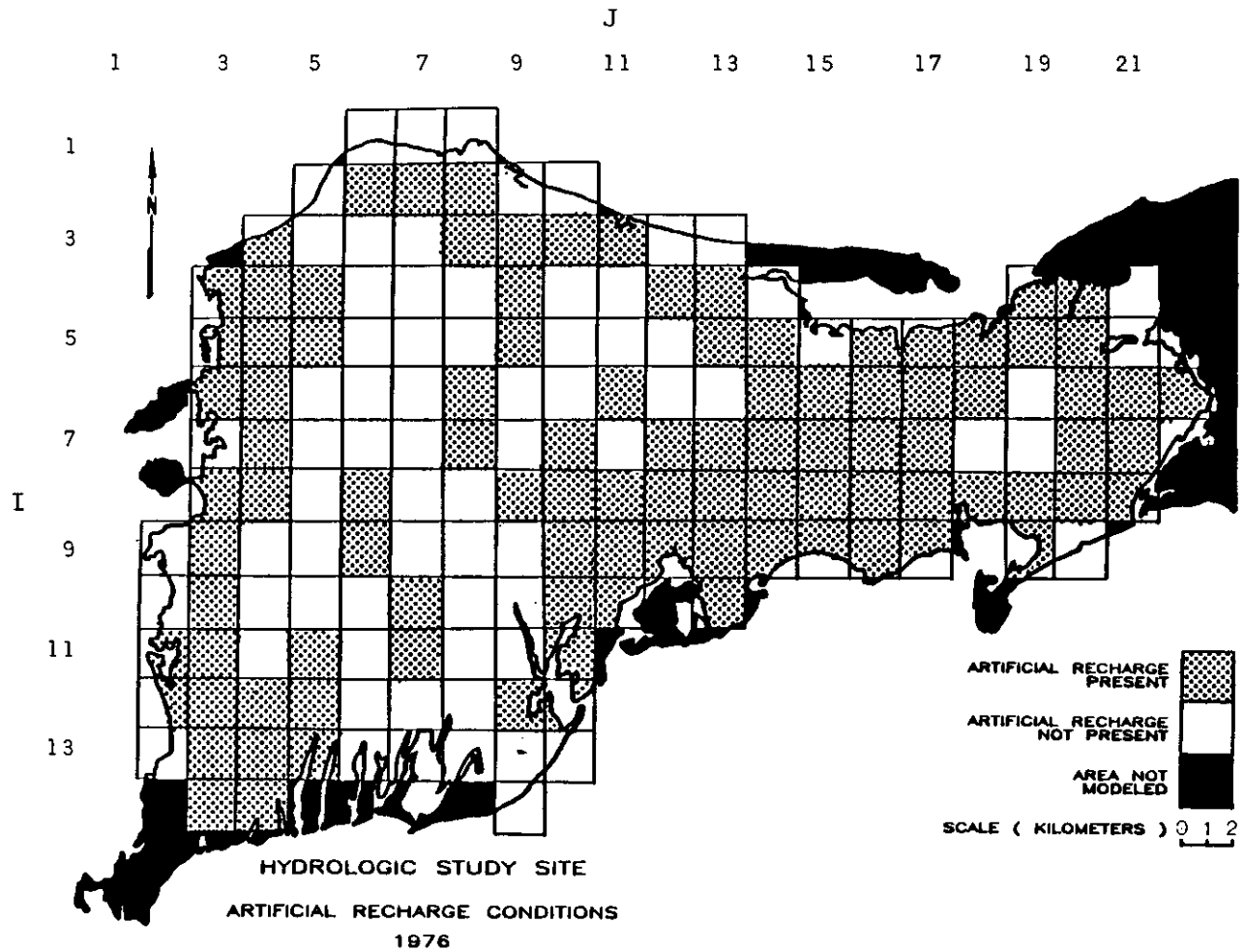
Figure 26. Map of discretized Western Cape Cod showing which elements
received artificial recharge as of 1976
residential/commercial development patterns.

6.3.4.2  Groundwater Pumpage

Municipal water demands comprise the greater part of all groundwater withdrawal on Cape Cod.  Pumpage occurs throughout the Western Cape Cod Area.  Figure 27 shows which elements had active wells during 1975-76.  Table 5 lists each active well and the average daily pumpage over 1975-76.

6.3.4.3  Water Table Elevation Data

The inverse model was applied over Western Cape Cod as a means of estimating elemental aquifer transmissivities.  The data requirements of the inverse model include hydraulic head estimates at each element in addition to the recharge and pumpage data presented above.  The hydraulic head at each element was estimated from a combination of observation wells, pond levels, and interpolations from water table maps.  Observed average water levels from 1950-82 were obtained from LeBlanc and Guswa (1977) and Letty (1984).  Additional pond levels were obtained from U.S. Geological Survey topographic maps for Cotuit, MA (1974), Dennis, MA (1974), Falmouth, MA (1979), Hyannis, MA (1979), Onset, MA (1967), Pocasset, MA (1979), Sagmore, MA (1979), Sandwich, MA, (1972), and Woods Hole, MA (1967).  In a few areas of Cape Cod, well water and surface water levels were not available; for these areas the hydraulic head was equated to the average graphic interpolation from three water table maps (Guswa and LeBlanc, 1985, Redfield, in Strahler, 1972, and Cape Cod Planning and Economic Development Commission, 1982).
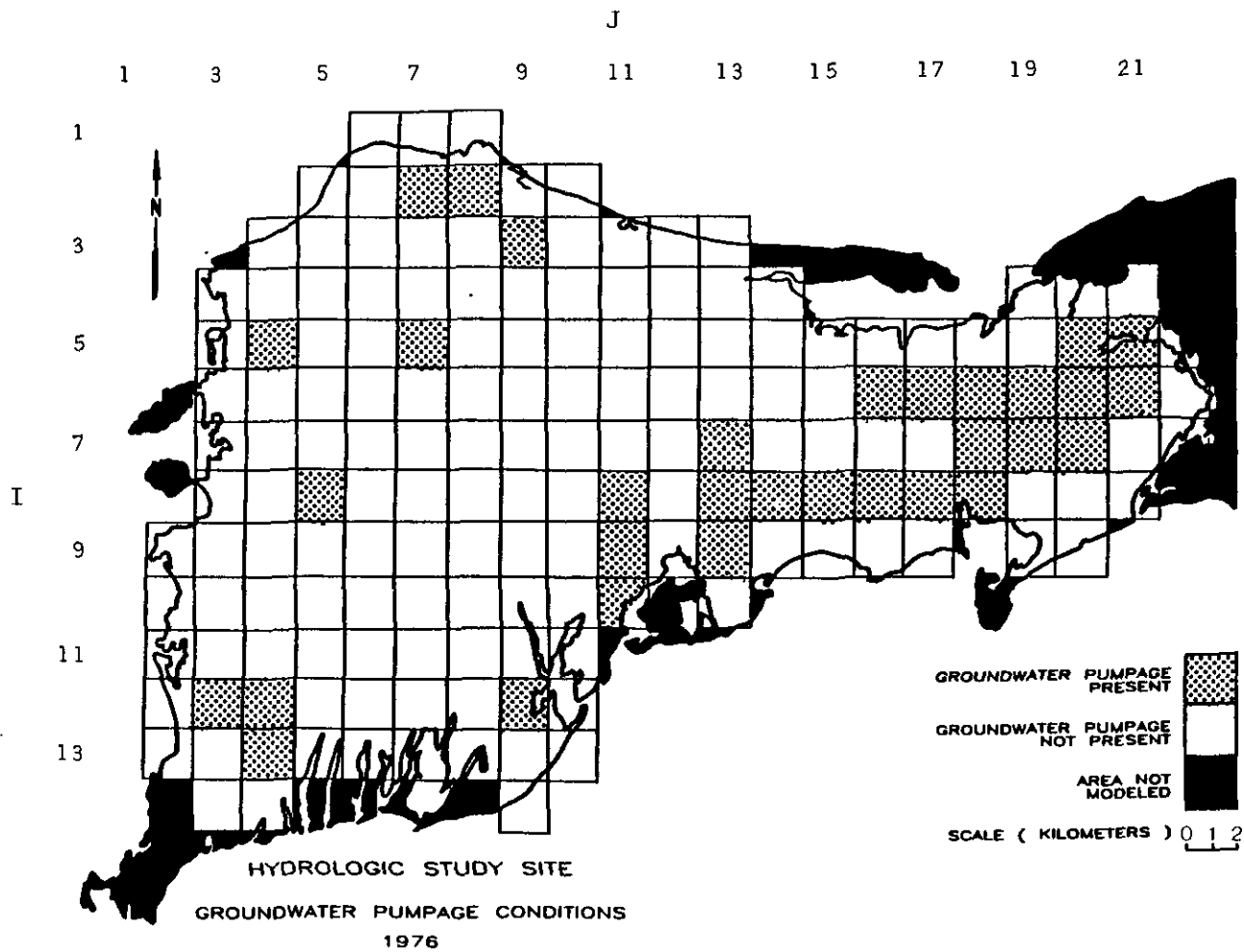
Figure 27.  Map of discretized Western Cape Cod showing which elements contained pumpage as of 1976 for residential/commercial needs.

Table 5.   Summary of Active Wells Over Western Cape Cod in 1975/76.

| U. S. Geological Survey Well | Location Node (i,j) | Discharge 3 [m /d] |
|---|---|---|
| A1W59 | 8,11<br>9,11 | 9<br>9 |
| A1W159, A1W107<br>A1W249, A1W158, A1W160 | 9,13 | 2863 |
| A1W224 | 10,11 | 113 |
| A1W228 | 6,16 | 421 |
| A1W226<br>A2W227<br>A1W368 | 8,15<br>8,16 | 291<br>291 |
| A1W229, A1W384 | 8,16 | 2877 |
| A1W251 | 9,11 | 274 |
| A1W259, A1W373 | 8,13<br>8,14 | 614<br>614 |
| A1W369 | 9,11 | 276 |
| A1W370 | 6,17 | 658 |
| A1W371, A1W372 | 7,13 | 798 |
| A1W376 | 8,16 | 1108 |
| A1W377, A1W385, A1W386 | 7,17<br>7,18<br>8,17<br>8,18 | 779<br>779<br>779<br>779 |
| A1W402 | 6,17<br>7,17 | 88<br>88 |
| A1W403, A1W383, A1W387 | 7,17 | 1329 |
| BHW 22,232 | 2,7 | 303 |
| BHW 23 | 8,5 | 1241 |

Table 5.  Summary of Active Wells Over Western Cape Cod in 1975/76,
          Continued.

| U. S. Geological<br>Survey Well | Location<br>Node<br>(i,j) | Discharge<br>$[m^3/d]$ |
|---|---|---|
| BHW233, BHW1-3, BHW136<br>BHW199 | 5,4 | 1600 |
| Long Pond | 12,3<br>12,4<br>13,4 | 3385<br>3385<br>3385 |
| MIW32, MIW32 | 12,9 | 484 |
| SDW 27,37 | 3,9 | 1037 |
| SDW 155 | 7,7 | 1059 |
| SDW 249,250 | 2,8 | 152 |
| YAW42, YAW43 | 6,18<br>6,19 | 629<br>629 |
| YAW53, YAW144, YAW146 | 6,21 | 1754 |
| YAW54 | 6,21 | 1116 |
| YAW58 | 7,19 | 759 |
| YAW61, YAW63 | 6,20 | 1192 |
| YAW64, YAW65 | 7,20 | 1600 |
| YAW103 | 5,20 | 1091 |
| YAW126, YAW127 | 5,21<br>6,21 | 89<br>89 |
| YAW128 | 7,19 | 306 |

Figure 28 distinguishes nodes where hydraulic heads were estimated from observation data or from interpolation.

### 6.3.4.4  Aquifer Transmissivities

Inadequacies in available field data precluded direct estimation of transmissivities for the Cape Cod aquifer.  Available information on aquifer properties represents data compiled from well logs and pump tests which was collected over aquifer intervals that average less than ten percent of the total aquifer thickness.  Few wells actually log the aquifer down to bedrock.  Most pumping wells penetrate no more than 15 meters into the saturated zone (Horsley, 1983).  For virtually all of Western Cape Cod little is known about the lower 75 to 90 percent of the aquifer.

To obtain transmissivity estimates for each element of the discretized aquifer, therefore, the inverse model was employed. Application of the inverse model to western Cape Cod was accomplished using the same approach successfully applied with the hypothetical problem in Chapter 4.  During the first stage the whole study area was sectioned along the major groundwater divides into quarters (See Figure 29).  Transmissivities for each quarter were calculated through the sequential application of equation (31) in a direction radiating from the element $i = 6$, $j = 7$ where the transmissivity was specified.  The boundary conditions used in each quarter were the same.  Those conditions were

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ILLUSTRATION OF THE LOCATION
OF OBSERVED, ASSUMED, AND
INTERPOLATED WATER TABLE ELEVATIONS
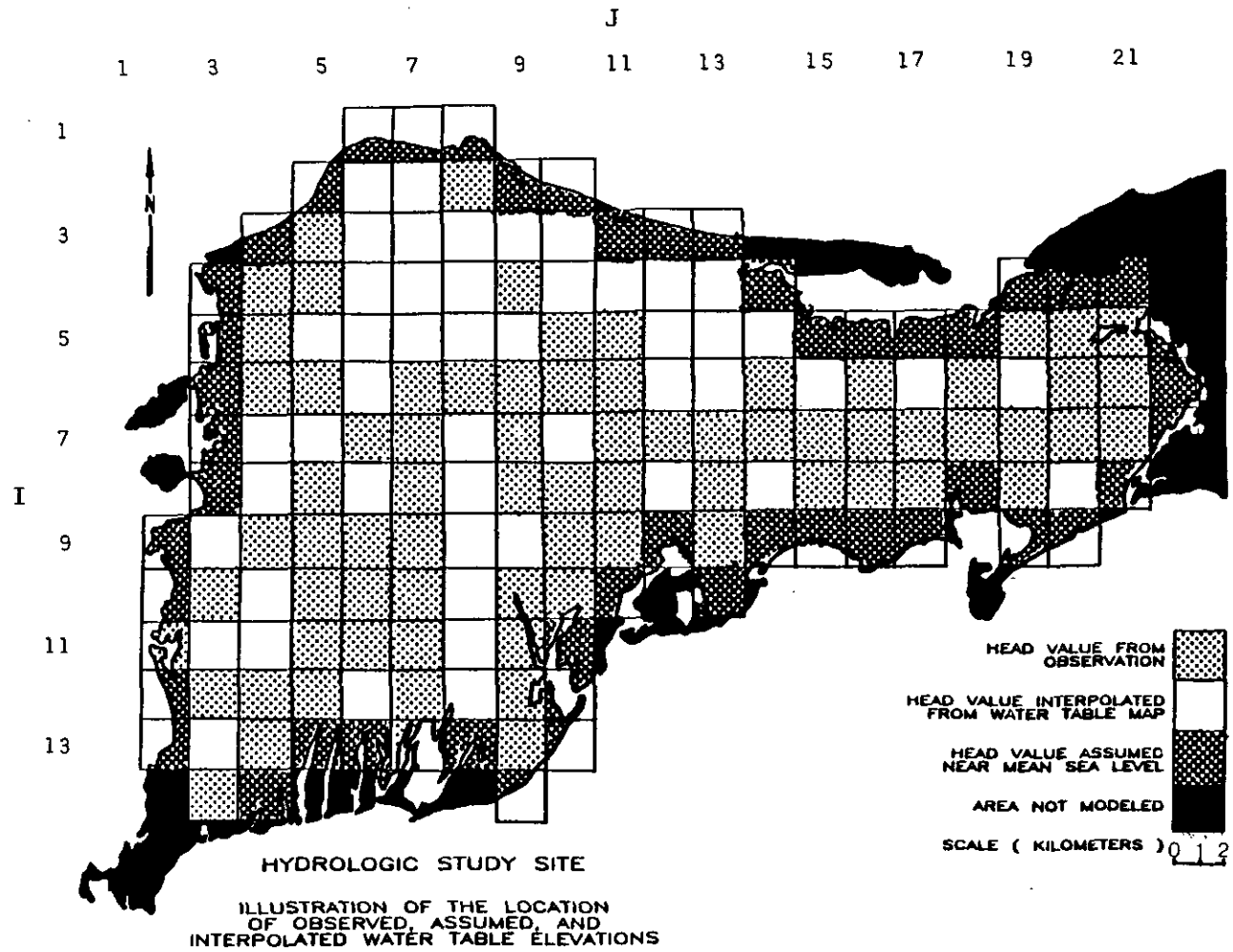
Figure 28.   Map of discretized Western Cape Cod showing the location of
observed, assumed, and interpolated water table elevations.
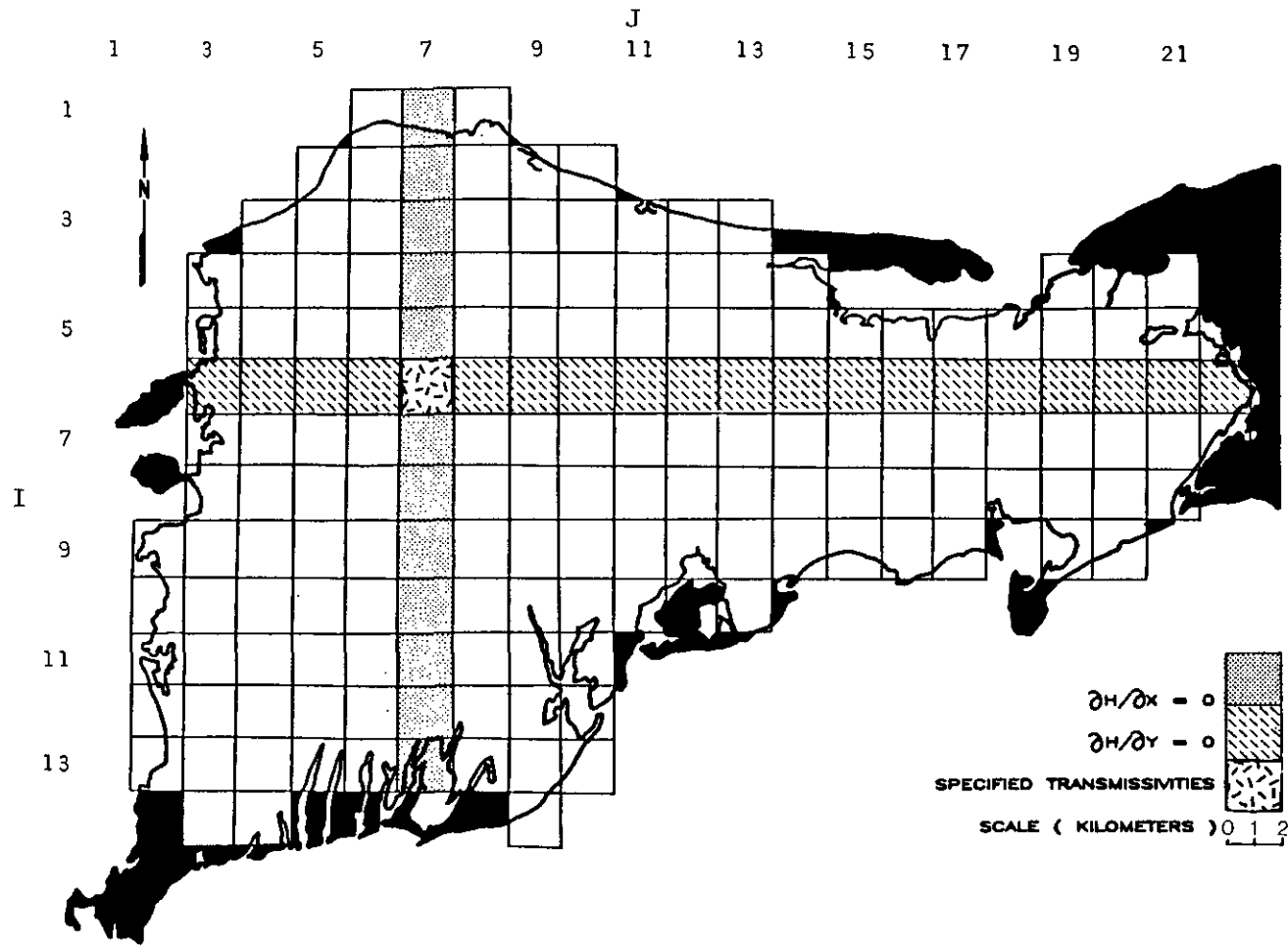
Figure 29. Map of discretized Western Cape Cod showing the boundary conditions for the inverse model for the hydrologic study area.

1)  Specified Transmissivity at a node representing the origin of all flow lines

$$T_{6,7} = 2472 \ m^2/day \tag{59}$$

2)  No flow across the boundary for elements along the groundwater divide which parallels the y axs

$$\frac{\partial h}{\partial x} = 0 \tag{60}$$

3)  No flow across the boundary for elements along the groundwater divide which parallels the x axis

$$\frac{\partial h}{\partial y} = 0 \tag{61}$$

The transmissivity at node i=6, j=7 was calculated from the product of an assumed permeability of 30.9 m/day and estimated saturated thickness of 80 meters from Guswa and LeBlanc (1985). The assumed permeability was predicated on estimates used by Burns, et al., (1975) to model two-dimensional horizontal groundwater flow in the Truro area.

During the second stage of inverse modeling, 100 simulation/adjustment iterations were performed. The magnitude of any adjustment was 50 percent of the error in predicted water table elevations or hydraulic gradients.

The hydraulic conductivity at each node was calculated by dividing the elemental transmissivity by the estimated discrete aquifer thickness. Hydraulic conductivities for interior nodes (the hydraulic conductivity of coastal nodes was not calculated because the saturated thickness of the aquifer was not known) ranged from 5 to 116 m/d. This range of hydraulic conductivities was much greater than the range (3.4

to 36 m/d) reported by Burns, et al., (1975), but much smaller than the range of 0.3-183 m/day used by Guswa and LeBlanc (1985) to model three-dimensional flows on Western Cape Cod. Hydraulic conductivity values obtained from the inverse model represent regional estimates which may not agree with any point value; this is because aquifer lithology is known to vary over length scales shorter than the discretization scale.

Spatial changes in hydraulic conductivity can induce severe changes in the hydraulic gradient. Associated with spatial changes in the hydraulic conductivity are regional changes in the geology. Two prominant physical features of Western Cape Cod are the Buzzards's Bay and Sandwich Moraines. The hydraulic conductivity of the moraines is not well known. Both high yielding wells and dry holes have been reported there. Some wells on the moraine penetrated silt and clay layers several hundred feet thick (Guswa and LeBlanc, 1985). Figure 30 shows major regions over Western Cape where the inverse model predicted hydraulic conductivities less than 20 and 25 m/day. These regions correspond well with the locations of the moraines depicted in Figure 21. If the inverse model can be used to identify spatial changes in hydraulic conductivity then it may be able to identify the subsurface boundaries of geologic formations having different permeabilities.

## 6.3.5 Simulation Results

The discrete transmissivity estimates were used with the pumpage data, recharge data, and boundary conditions to perform the groundwater flow simulations of the Western Cape Cod aquifer. Total recharge was
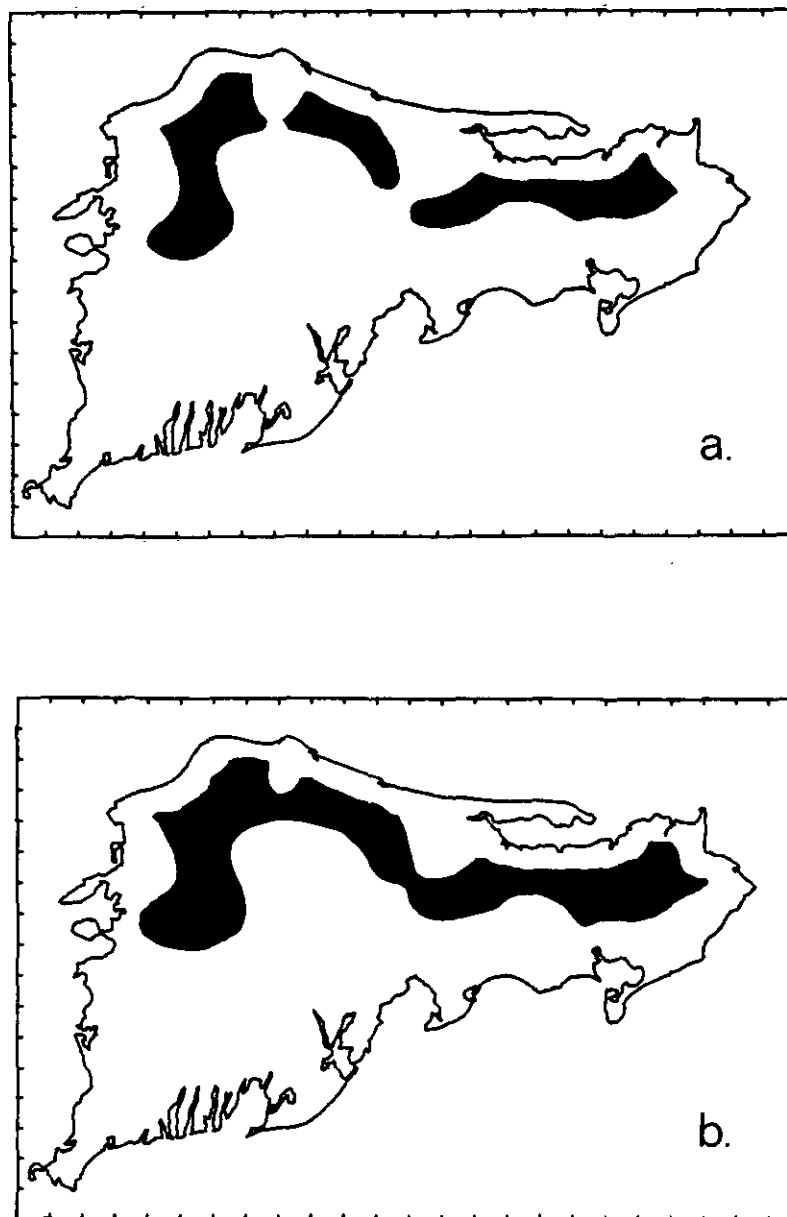
Figure 30.    Results of the inverse model for Western Cape Cod showing
regions of aquifer hydraulic conductivities less-than-or-
equal-to 20 m/d in (a) and regions of aquifer hydraulic
conductivities less-than-or-equal-to 25 m/d in (b).  Note
that regions correspond to the location of the moraines.

771,572 m$^3$/day. Total daily pumpage was 41,596 m$^3$/day which is five percent of the total recharge. Mass balance errors were very small, averaging in length 0.002 percent, indicating a very slight gain in subsurface flows.

Predicted heads were plotted as contours in Figure 31. The contour map depicts slight hydraulic gradients to the south and east sections of the aquifer and steeper gradients along the north shore. The steeper water table reflects the energy required to force flow through the less permeable moraine deposits.

The errors in predicted heads are summarized in Table 6. The average percent error in calculated head was 0.08 percent with a standard deviation of five percent. Actual error was always less than one meter and on the average was 0.05 meters. Model prediction errors (Figure 32) were generally positive (over estimates) regardless of where the node was located (small values of observed water table elevations indicate regions near the coast). Because nodes near the coast have water table elevations near mean sea level (0) the model was less accurate for these nodes in terms of percent error (see Figure 33).

Calibration of the groundwater flow model was considered successful from the perspective of satisfying three goals. First, a set of transmissivities were identified for the region which yield hydraulic conductivities that were not only within the range of previous observations, but were also distributed spatially in a pattern commensurate with known geologic formations. Secondly, the standard deviations of the errors of the hydraulic predictions were within five
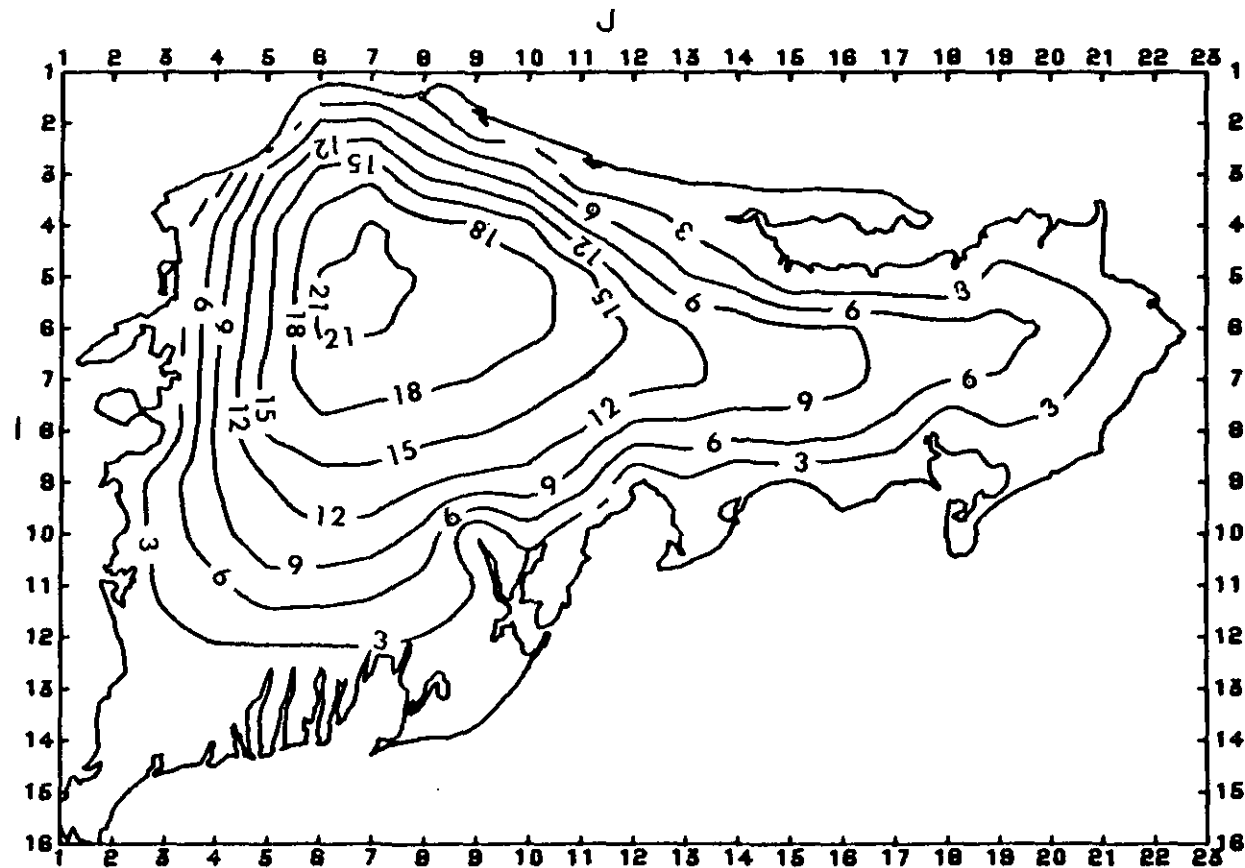
Figure 31.    Contour plots of predicted steady-state water table
              elevations (meter above mean sea level) from the
              groundwater flow model for Western Cape Cod.

Table 6.  Summary of Flow Model Predictions Errors Following
Calibration to Western Cape Cod.

| | |
|---|---|
| Average Head<br>Error; [m] | 0.051 |
| Standard Deviation<br>of Average Head<br>Error; [m] | 0.224 |
| Average Percent<br>Error in Predicted<br>Head | 0.080 |
| Standard Deviation<br>of Average Percent<br>Error in Predicted<br>Head | 5.040 |
| Percent Mass Balance Error | 0.002 |
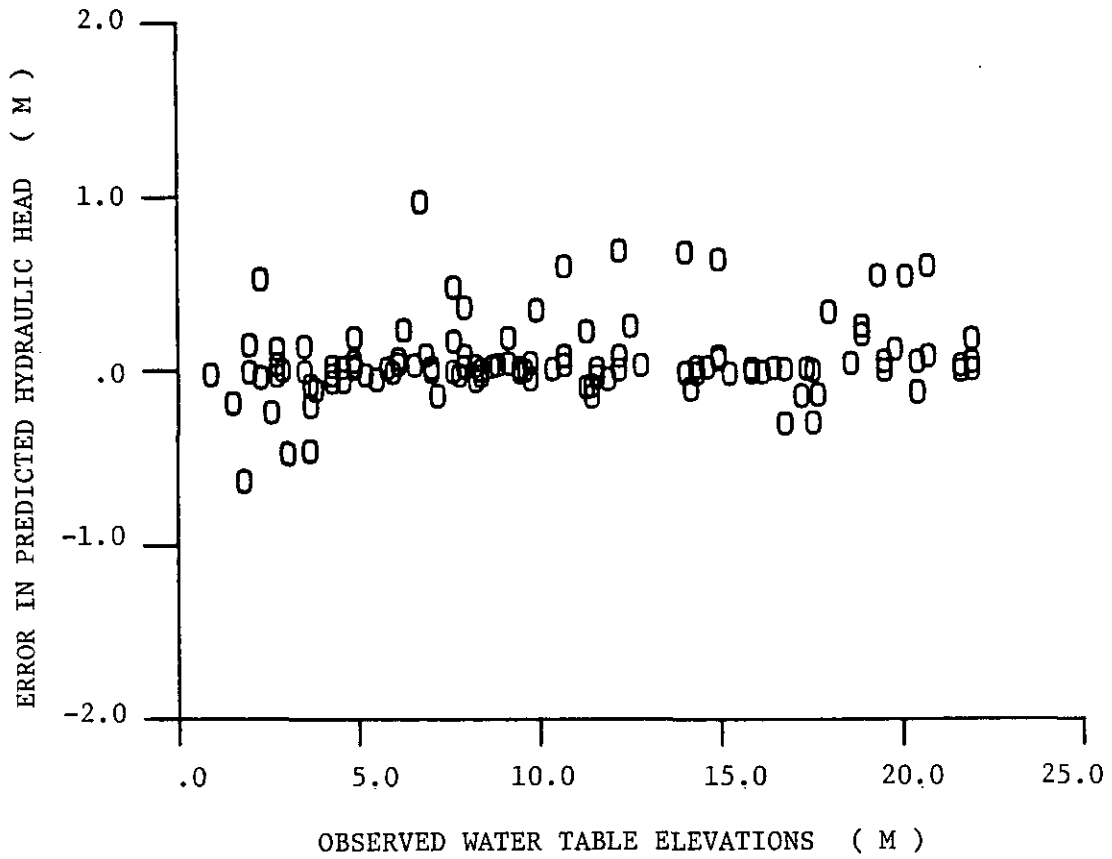
Figure 32. Scatter plots of error in predicted hydraulic head from the numerical groundwater flow model of Western Cape Cod as a function of observed water table elevations (meters above mean sea level).
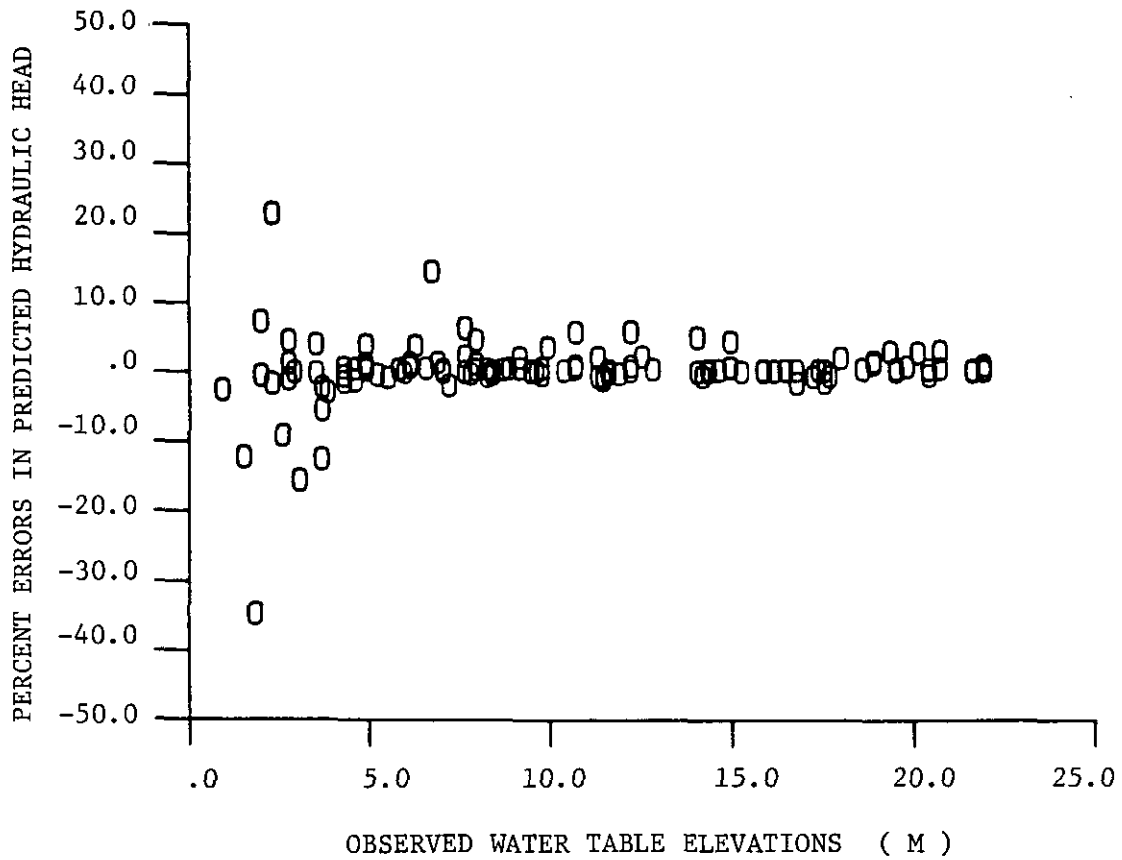
Figure 33.   Scatter plot of percent errors in predicted hydraulic head
(above mean sea level) from the numerical groundwater flow
model of Western Cape Cod as a function of observed water
table elevations (meters above mean sea level).

percent.  Finally, the low mass balance errors corroborated the excellent numerical predictions of the piezometric surface.

CHAPTER 7


MODELING TO ELUCIDATE CRITICAL RECHARGE AREAS


Two non-point source groundwater pollution management models are formulated and applied in this chapter. The first model identifies areas within the regional groundwater flow system which are most critical to the preservation of area-wide groundwater quality. The second model can be implemented to delimit critical recharge zones surrounding municipal water supplies.

Both models are applied to a section of Western Cape Cod. Long term regional groundwater nitrate distributions from the 1980 development pattern in Falmouth and Bourne are projected. The projections are based on nitrate contamination from septic systems, lawn fertilizers, leaky sewers, background loads, and subsurface recharge from secondary municipal sewage. Model assumptions which facilitate direct incorporation of the validated solute transport model as continuity constraints of the management models are enumerated.

Both management models identify the water quality tradeoffs associated with alternative subsurface disposal patterns, regardless of whether water quality impacts are regional in nature or simply at a water supply well. Modeling results include contour plots of the steady-state regional nitrate distributions from the 1980 land use patterns for Bourne and Falmouth, Massachusetts. In addition, maps showing regional and local water quality impact isopleths are presented.

## 7.1 Formulation of Management Model I

The purpose of the first management model is to identify areas within a regional groundwater flow system which are most critical to the preservation of area-wide groundwater quality. The model is formulated to reveal the steady state distribution of a dissolved contaminant produced from nonpoint source pollution. In addition values of dual variables can be obtained to delineate recharge areas significant to protecting long term regional water quality. The first management model is a simple linear program. The objective function is formulated as a direct summation of all the elemental contaminant variables in the management area divided by the number of discrete nodes under management. The optimum value of the objective function represents the maximum average element concentration under specified subsurface disposal activities expressed through the continuity constraints. The objective function is written

$$\text{Maximize} \quad \frac{1}{n \cdot m} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} \tag{62}$$

where the product of n and m is the total number of elements within the boundaries of the management area.

The continuity constraints appear similar to equation (49); however, all subsurface disposal activities are specified as known terms on the right side of inequalities. Recall, that the continuity constraints operate as an expressed approximation of the relationship between subsurface disposal activities and the resultant groundwater contamination. Two inequality continuity constraints are constructed

for each node in lieu of a single equality constraint. Presenting the equality constraint as combinations of inequality constraints ensures that pollutant concentrations reflect specified contaminant loadings and creates dual variables for use in sensitivity analysis. The continuity constraint set is expressed as

$$[G]\{C\} \geq [C_w]\{W\} + [C_s]\{S\} + [C_z]\{Z\}$$
$$+ [C_u]\{U\} + C_q \cdot [I]\{Q\} \qquad (63)$$

$$[G]\{C\} \leq [C_w]\{W\} + [C_s]\{S\} + [C_z]\{Z\}$$
$$+ [C_u]\{U\} + C_q \cdot [I]\{Q\} \qquad (64)$$

where all the vectors and coefficients are defined as with equation (49).

The complete formulation of model I follows.

Objective function:

$$\text{Maximize} \quad \frac{1}{n \cdot m} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij}$$

s.t.

Continuity Constraints:

$$[G]\{C\} \geq [C_w]\{W\} + [C_s]\{S\} + [C_z]\{Z\}$$
$$+ [C_u]\{U\} + C_q \cdot [I]\{Q\}$$

$$[G]\{C\} \leq [C_w]\{W\} + [C_s]\{S\} + [C_z]\{Z\}$$
$$+ [C_u]\{U\} + C_q \cdot [I]\{Q\}$$

Nonnegativity constraints:

$$C_{i,j} \geq 0 \quad \forall \ i \text{ and } j$$

Model I yields the steady state contaminant distribution under a defined flow field and a specified distribution of contaminant loadings. Plotting the dual variables associated with the less-than-or-equal constraints depicts discrete estimates of the regional water quality tradeoffs from small changes in subsurface disposal fluxes. Iso-water-quality-impact contours are interpolated between plotted values of the dual variables. These contours will delineate areas within a regional aquifer which are most critical to the preservation of area-wide groundwater quality.

## 7.2 Formulation of Management Model II

The purpose of management Model II is to delimit critical recharge zones surrounding municipal water supplies. The formulation of Model II is a simplified version of model I where the scope of the objective function has been reduced to include fewer elements. The new objective function contains one or more contaminant concentration decision variables ($C_{i,j}$'s) that coincide with elements containing water supply wells or significant surface or groundwater resources. The value of the objective function represents the average steady state concentration of contaminant in groundwaters underlying discrete elements containing the water supply wells. The formulation of the objective function is

$$\text{Maximize } \frac{1}{\beta} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} \cdot \gamma_{i,j} \qquad (65)$$

where $\gamma_{i,j}$ = the value of one if the contaminant concentration in

element i,j is the target of interest (i.e., an element

containing municipal well);

$\beta$ = $\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} \cdot \gamma_{i,j}$, which equals all the number of elements

containing target water supply wells.

As in Model I this model produces the steady-state contaminant

distribution resulting from the specified aquifer disposal rates

expressed as constant terms on the right-hand-sides of the continuity

constraints. The optimum values of the dual variables can be plotted in

a pattern around the node (or nodes) of interest. Iso-water-quality-

impact contours can be interpolated between discrete values of the dual

variables. These contours yield estimates of the unit changes in

contaminant concentration at the target node (or nodes) induced by

changes in subsurface disposal activity in all elements within the

managed area. The contour pattern delimits critical recharge zones

surrounding the target node (or nodes).


7.3 Application of Models I and II to Bourne and Falmouth,

Massachusetts

Models one and two were applied to a small section (268 square

kilometers) of Western Cape Cod (see Figure 34). The region lies west

of the groundwater divide extending north to south along the eastern

borders of Bourne and Falmouth, Massachusetts. Figure 35 depicts

the locations of significant municipal water wells, sewage treatment
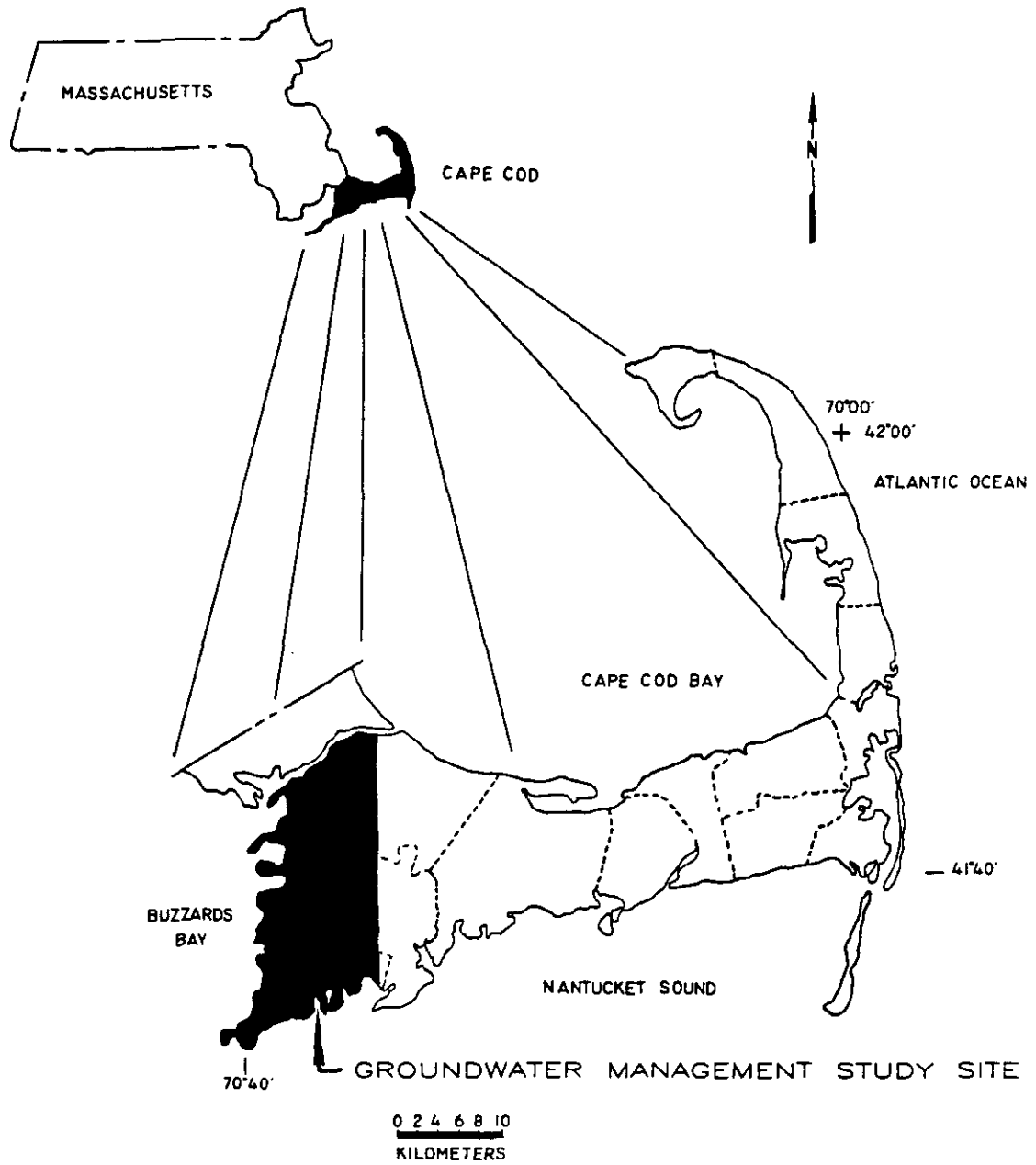
Figure 34. Map depicting the study site for the application of the nonpoint source groundwater pollution management models.

GROUNDWATER MANAGEMENT STUDY SITE
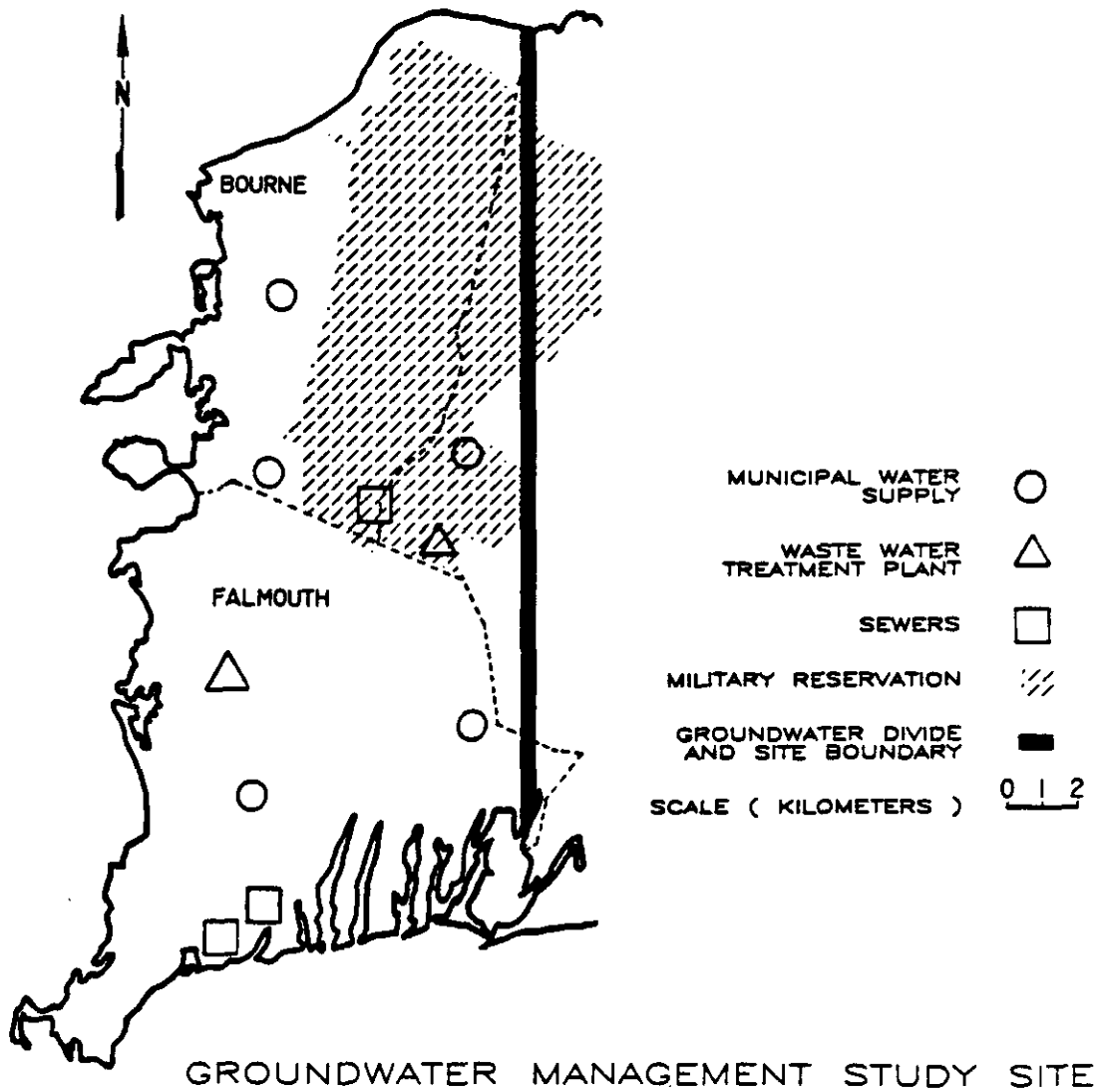
Figure 35.   Map illustrating details of the study-site for the
application of the nonpoint source groundwater pollution
management models.

plants, sewers, town borders, the military reservation, and the groundwater divide.

The study area was discretized into two kilometer elements (see Figure 36). This discretization dimension was chosen because the steady-state discrete contaminant transport equations were successfully validated at this scale in Chapter 5. The discrete formulation of the validated numerical contaminant transport model served as continuity constraints for each management model. The solution of both linear programs revealed the long term regional groundwater quality impacts from land use activities producing dispersed nitrate pollution from septic systems, lawn fertilizers, secondary sewage recharge, leaky sewers, and background loading. The simulations were performed with estimates of pertinent nitrate source concentrations and flux rates from the year 1980.

### 7.3.1  Fundamental Modeling Assumptions

Assumptions were made regarding 1) the boundaries of the Bourne-Falmouth study-site, 2) the characteristics of the aquifer and groundwater flow, 3) the nature of diffuse sources of subsurface nitrate pollution, and 4) the fundamental mechanisms affecting subsurface contaminant fate and transport. The specific assumptions made regarding the boundaries of the study area were:

1) Boundary conditions along the groundwater divide: contaminant transport across the groundwater divide was assumed negligible.
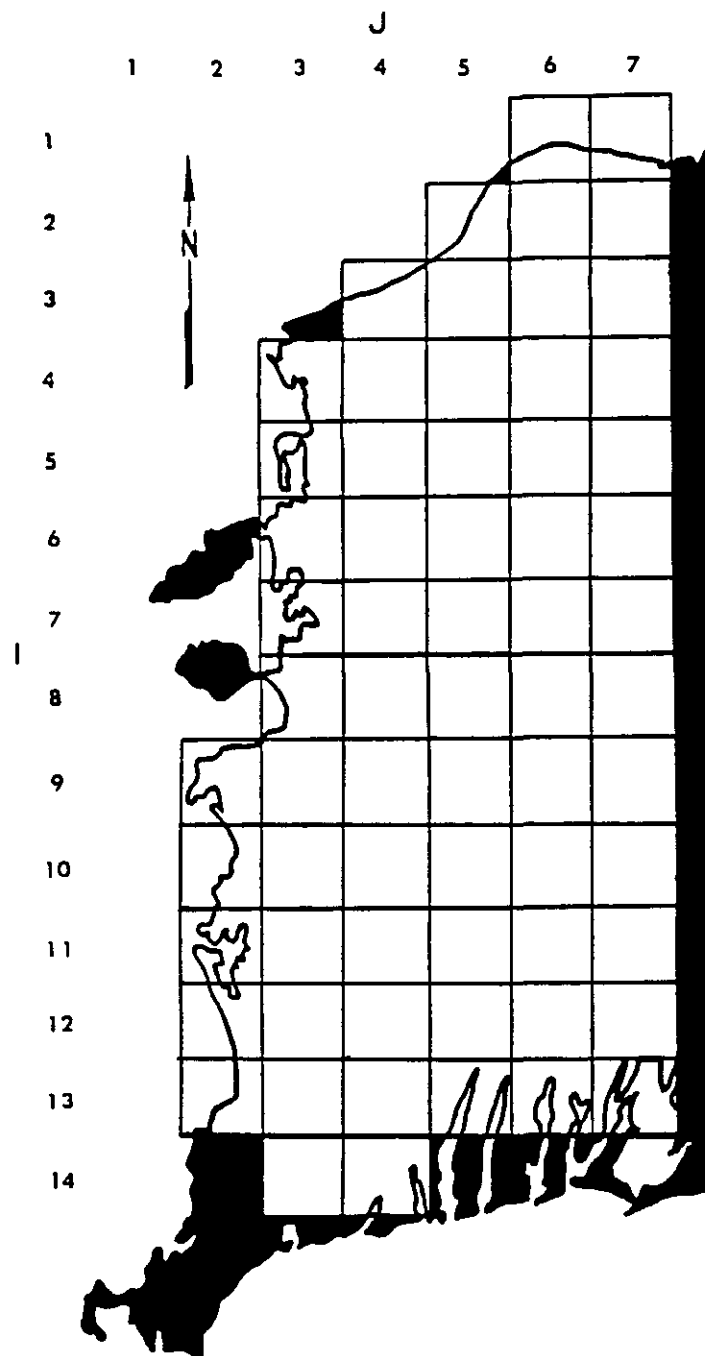
Figure 36.   Map of discretized study site for the application of the
             nonpoint source groundwater pollution management models.

This boundary was treated by specifying zero Darcian velocities across the divide; see Figure 37;

2) Boundary conditions at the coast: most contaminants in the aquifer discharge to the ocean along the coast. It was assumed that contaminants entering boundary nodes from upgradient flows and boundary recharge must equal the product of groundwater discharge and contaminant levels at the boundary. Figure 37 illustrates how the coastal nodes were treated. The mass of contaminant entering the boundary from interior nodes ($VC_{ii}$) plus the mass of contaminant entering the aquifer from surface pollution occurring on the boundary ($DW_b$) is equated to the product of total flow leaving the boundary ($V_i + W_b$) and the concentration of contaminant at the boundary ($C_b$);

3) Boundary conditions at the groundwater mound: the numerical problem is not properly posed unless the contaminant concentration is specified at least somewhere along each characteristic. Because element $i,j = 6,7$ corresponds to the peak of the groundwater mound all characteristics originate from this node; hence, for convenience the contaminant concentrations in the element was specified;

4) No loss of nitrate occurred across the impermeable aquifer floor or the water table.

The assumptions made regarding aquifer characteristics and flow were:

GROUNDWATER MANAGEMENT STUDY SITE
BOUNDARY CONDITIONS
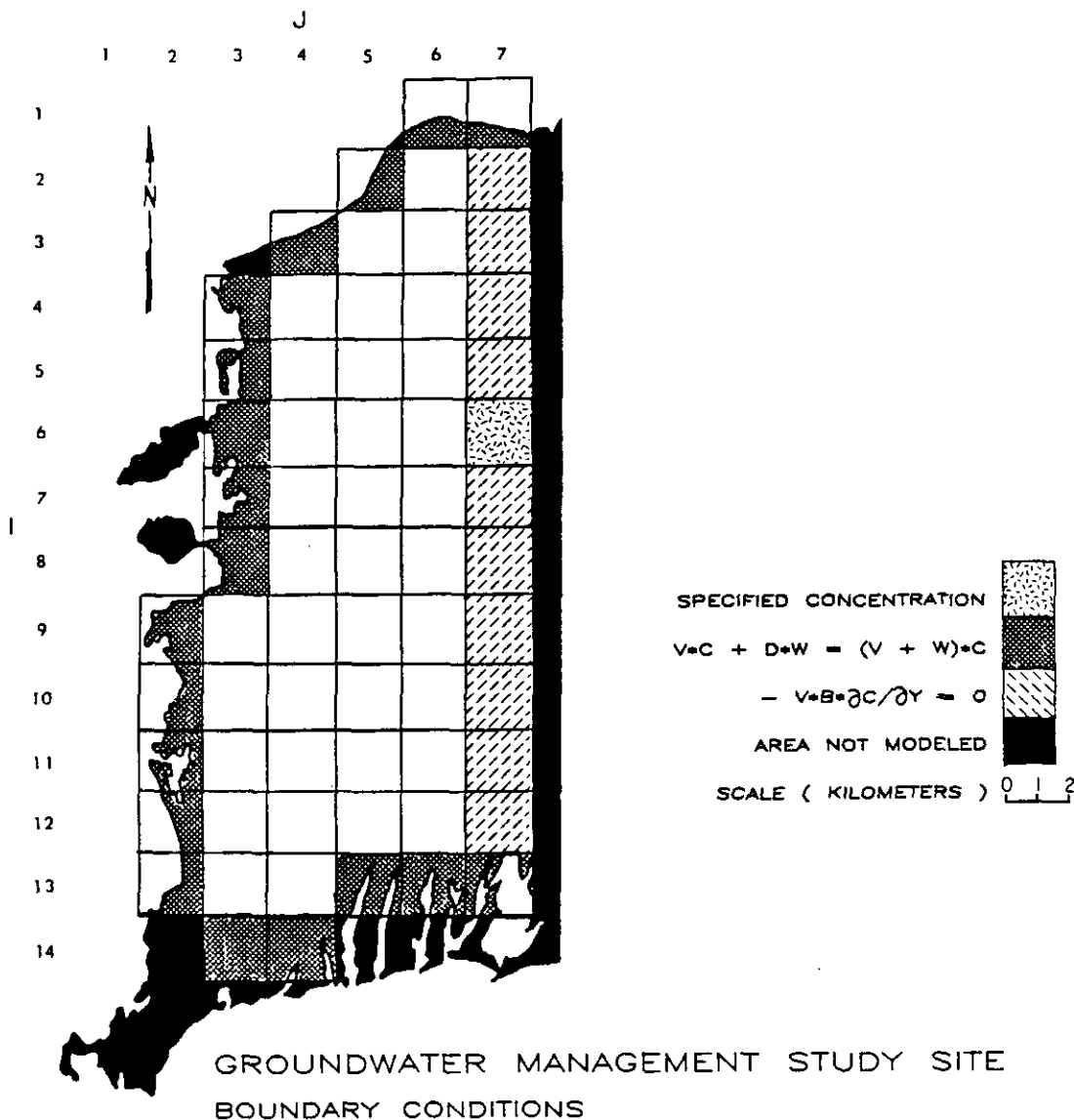
Figure 37.    Map illustrating the boundary conditions used to construct
the continuity constraints in the groundwater quality
management models.

1) Western Cape Cod aquifer is composed of unconsolidated nonhomogeneous deposits of boulders, gravel, sand, silt, and clay. It was assumed that the aquifer was nonhomogeneous, but isotropic with respect to horizontal dissolved contaminant transport;

2) Regional groundwater flow in Western Cape Cod aquifer has been unconfined and predominantly horizontal; therefore, regional advective contaminant transport was assumed unconfined and horizontal;

3) Groundwater artificial recharge and withdrawal along the groundwater divide was negligible for all elements within the management area; consequently, the position of the groundwater divide was assumed stationary;

4) The numerical mass transport model was incorporated into a larger groundwater quality management model to view long term groundwater management schemes; therefore, simulated mass transport was predicated on information derived from a steady-state groundwater flow regime defined by future regional water demands, existing and future well locations, existing and potential municipal water distribution systems, and existing and potential sewer systems. This approach would be acceptable as long as deviations from the projected volumes of water exported and consumed did not induce significant changes in the original piezometric surface used in the construction of the groundwater quality management model.

Several specific assumptions were made regarding the physical and chemical mechanisms governing the fate and transport of nitrate in the Cape Cod aquifer:

1) Because nonpoint source groundwater pollution was being evaluated over long time and large length scales, vertical variations in groundwater quality become less important than horizontal variations (Bear, 1979); consequently, only depth-averaged concentrations were considered;

2) Since vertically averaged regional contaminant concentrations were being modeled, the scale of horizontal numerical discretization must be sufficiently large that sufficient vertical mixing through the entire depth of the aquifer could be assumed. Bear (1979) noted that contaminants entering the top of the saturated zone would occupy most of the saturated layer after travelling horizontal distances equal to 10-15 times the thickness of the saturated zone. The average thickness of the aquifer on Western Cape Cod is 72 meters. The discretization scale used with the management models (2000 meters) would be 28 times larger than the average thickness of the saturated zone; hence, complete vertical mixing was assumed between elements;

3) Because little mixing of waters occurs at the freshwater/saltwater interface and in the less permeable fine sand, silt, clay, and bedrock layers, it is assumed that no loss of nitrate occurs at these boundaries;

4) Because only regional scale groundwater quality modeling was pursued and because the sources of groundwater pollution (i.e., septic systems) to be modeled were assumed areally distributed, the influence of dispersive contaminant transport was considered negligible and was ignored;

5) Loss of nitrates in the saturated zone is considered minor (Freeze and Cherry, 1979); therefore, nitrate was treated as a conservative contaminant.

Finally there were specific modeling assumptions were made regarding nitrate sources and sinks:

1) Many sources of distributed groundwater pollution are semi-permanent and will have a lasting impact on the subsurface environment. For purposes of viewing the long term groundwater quality implications of semi-permanent nonpoint source pollution, only steady-state transport conditions were simulated;

2) Local scale descriptions of horizontal variations in nitrate levels were not considered. And for any element of discretization, all contaminant loadings were assumed evenly distributed throughout the element;

3) Source fluxes were based upon estimated per capita loading rates and assumed nitrate concentrations which actually reach the water table; transport through the unsaturated zone was not considered;

4) Because ponds on Western Cape Cod are generally phosphorous

limited, these bodies of water were not treated as sources or

sinks.


7.3.2 <u>Adaptation of the Management Models to Nitrate Sources on Cape</u>

<u>Cod</u>

Five sources of nitrate contamination were considered in the

modeling effort for Cape Cod. The first three sources were related land

use activities to be described in the following paragraph. The fourth

and fifth sources were, respectively, the land application of

wastewaters at the two sewage treatment facilities and the background

nitrate loads delivered to the subsurface flow system through natural

recharge.

The three land use activities were composite residential/commercial

sources of nitrate pollution which differed from each other in regard to

their source of water (municipal or on-site) and their method of

disposing residential/commercial wastewaters (through sewers or septic

systems). The first source was a composite land use activity which

embodied the combined domestic and commercial use of municipal well

water and septic systems. Domestic and commercial use of municipal

water contribute recharge to the aquifer from leaky water mains, and

septic system effluents. Nitrates pollution from this composite land

use activity are introduced to groundwaters through the application of

residential lawn fertilizers, from nitrates in municipal waters lost to

the aquifer through water distribution system leaks and from residential/commercial septic system effluents.

A second source was the combined residential and commercial activities which use municipal waters, but dispose wastewaters to sewers. For this source, leakage from water distribution systems and sewer exfiltration from the disposal of residential/commercial wastewaters produced artificial recharge. Applied residential lawn fertilizers, sewer exfiltration, and water distributions system losses added nitrates to the subsurface environment.

The third major source was also a form of residential/commercial land use but differed from the first two, because it did not contribute artificial groundwater recharge. This source represented the composite land use activity where commercial and domestic activities rely on water delivered from on-site wells. Domestic nitrate pollution evolved from the combined use of septic systems and fertilizers. Commercial nitrate loads were from septic systems alone.

Management models I and II were adapted to view the regional groundwater nitrate pollution problems on Cape Cod through additional specification of the general source terms in the continuity constraints, (Equations 63 and 64).

The general vector formulation of the continuity inequalities is

$$[G]\{C\} \geq [c_w]\{W\} + [c_s]\{S\} + [c_z]\{Z\}$$
$$+ [c_u]\{U\} + c_q \cdot [I]\{Q\} \tag{63}$$
$$[G]\{C\} \leq [c_w]\{W\} + [c_s]\{S\} + [c_z]\{Z\}$$

$$+ [C_u]\{U\} + C_q \cdot [I]\{Q\} \tag{64}$$

where in the Cape Cod problem, nitrate source vectors were defined as

$\{W\}$ = (n·m) x 1 vector of elemental recharge flows from the combined domestic and commercial use of waters from on-site wells, septic systems, and lawn fertilizers;

$\{Z\}$ = (n·m) x 1 vector of elemental recharges from the combined domestic and commercial use of municipal well water, septic systems, and lawn fertilizers;

$\{S\}$ = (n·m) x 1 vector of elemental recharge flows from the combined domestic and commercial use of municipal well water, sewers, and lawn fertilizers;

$\{U\}$ = (n·m) x 1 vector of elemental recharge flows from land application of secondary sewage;

$\{Q\}$ = (n·m) x 1 vector of elemental natural recharge flows;

$[C_w]$ = (n·m) x (n·m) diagonal matrix of nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of waters from on-site wells, septic systems, and lawn fertilizers;

$[C_z]$ = (n·m) x (n·m) diagonal matrix of nitrate concentrations in elemental recharges from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers;

$[C_s]$ = (n·m) x (n·m) diagonal matrix of nitrate concentrations in elemental recharge flows from the combined domestic and

commercial use of municipal well waters, sewers, and lawn
fertilizers;

$[C_u]$ = (n·m) x (n·m) diagonal matrix of effective nitrate
concentrations in the elemental recharge from land
application of secondary sewage;

$C_q$ = observed background nitrate concentration in groundwaters
(for convenience precipitation was treated as the source),
(mg/l).

## 7.3.3  Estimation of the 1980 Hydrologic Stresses and Nitrate Loads in Bourne and Falmouth

Recharge vectors ({W}, {S}, {Z} and {Q} and the source nitrate
concentrations were estimated from discrete natural recharge, artificial
recharge (from septic systems, leaky water distribution systems,
exfiltration flows, and secondary sewage), groundwater pumpage, and
estimated nitrate concentrations in all recharge flows.  Calculations of
nitrate concentrations in all recharge flows were accomplished after
assumptions were made regarding domestic nitrogen loads, commercial
nitrogen loads, lawn fertilizer application rates and percent nitrogen
losses in the vadose zone.  Nitrate concentration estimates for other
sources (i.e., secondary sewage recharge and sewer exfiltration) were
made from literature values.

Estimates of discrete hydrologic and contaminant stresses were made
after land use and population data for 1980 were compiled on Bourne and
Falmouth.

### 7.3.3.1 1980 Population Distribution

Nonpoint source pollution on Cape Cod has been affected by land use activities and population distribution. The average population in 1980 was calculated for each element of the discretized management area. The population could be categorized into two groups; one group represented the elemental population dependent on municipal well water, while the other group relied on water from on-site wells. The 1980 elemental population pattern of each group is presented in Figures 38 and 39.

The population of each group was calculated by multiplying the average number of people per housing unit times the number of housing units in each element connected to on-site wells or municipal water supplies. The number housing units of each type were obtained by superimposing the finite difference grid over 1980 population enumeration district maps (U.S. Census Bureau). The average number of people per housing unit was interpreted as the ratio of total year-round population for each town to the year-round occupied housing units. Table 7 summarizes the population data by town.

### 7.3.3.2 1980 Pumpage Pattern

Records on the 1980 municipal and private well pumpage were obtained from the pertinent water districts and the Cape Cod Planning and Economic Development Commission (CCPEDC). The CCPEDC identified the safe yields for each well. Total water pumpage by Falmouth was apportioned among the three wells in accordance with their relative safe yields. Long term pumpage at the Otis Air Force Base was calculated
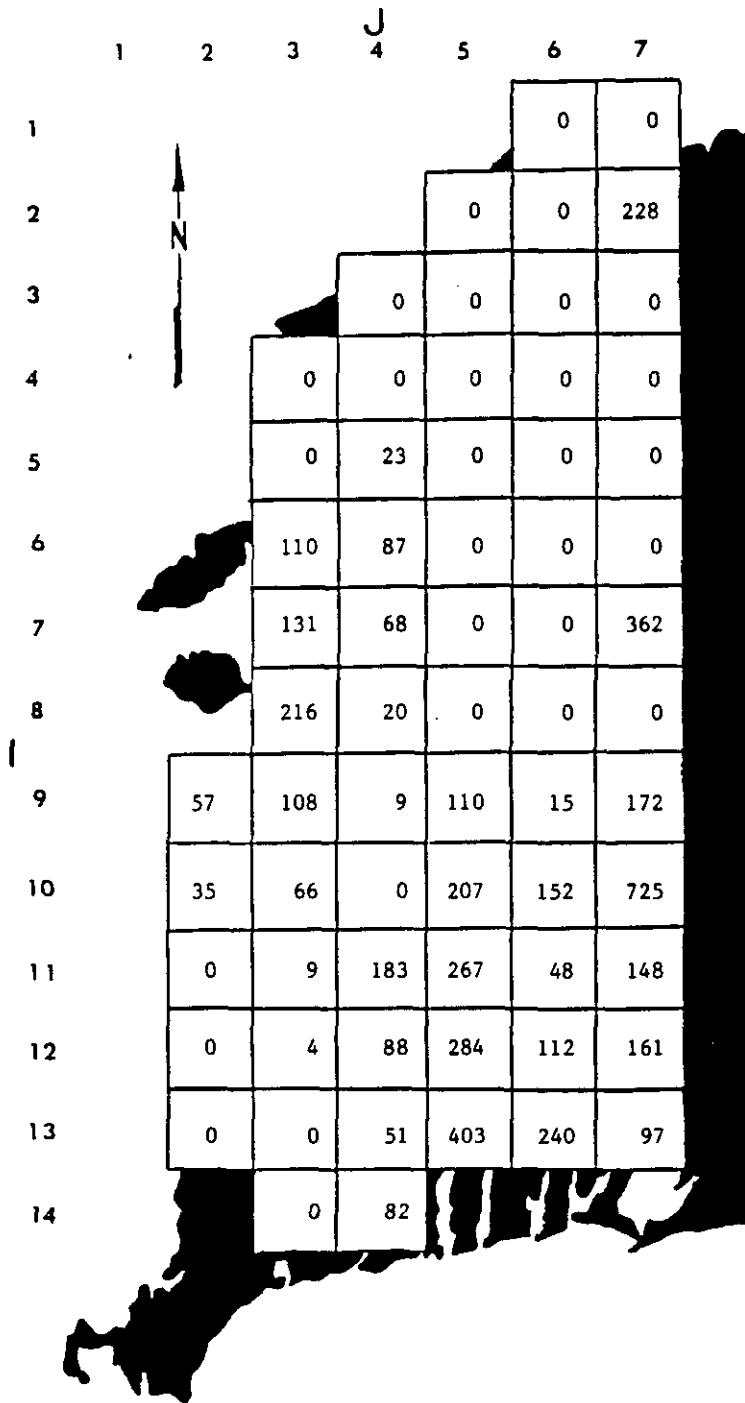
Figure 38.  Map illustrating the 1980 elemental population using on-site wells within the groundwater quality management study area.
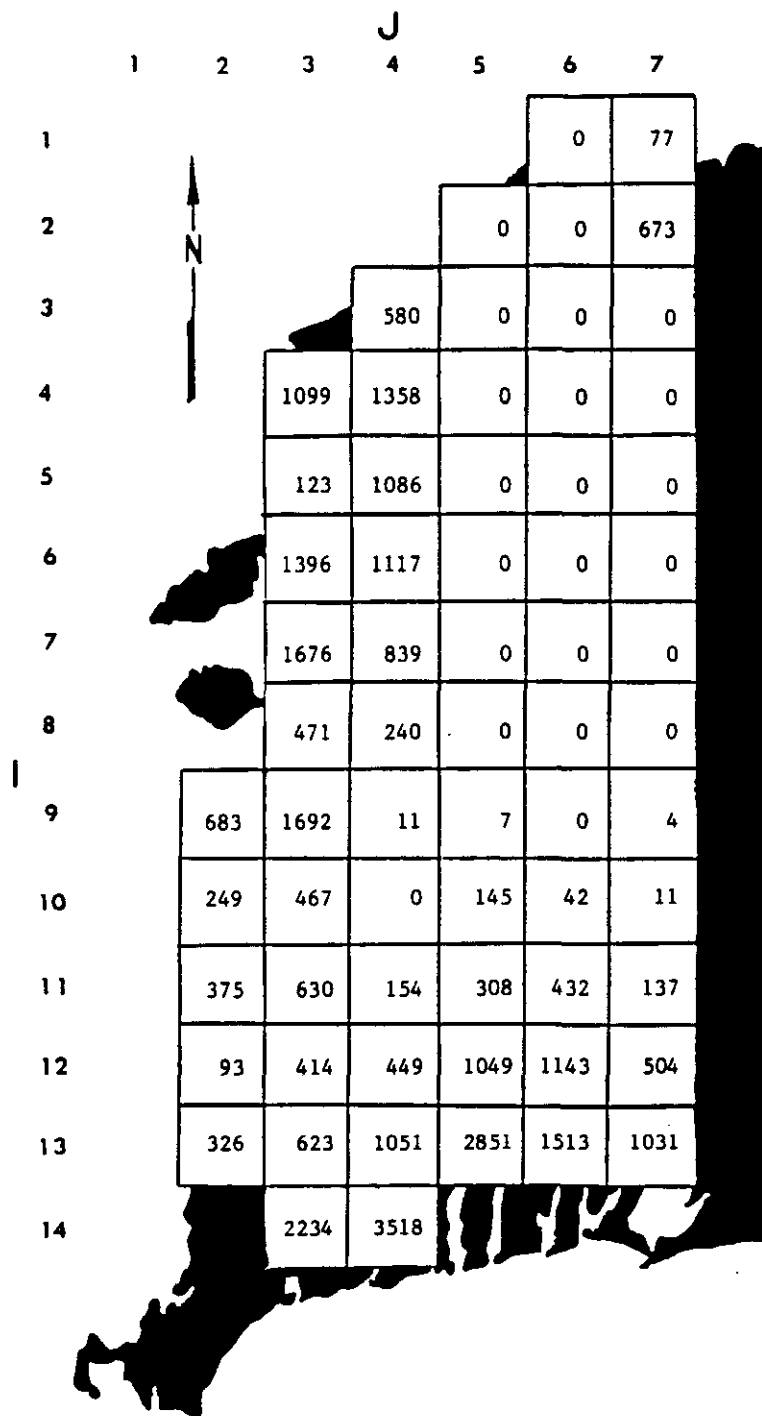
Figure 39. Map illustrating the 1980 elemental population using municipal water within the groundwater quality management study area.

Table 7.   1980 Population Data for the Portion of Each Town Within the
           Management Area.

|                                                      | Bourne | Falmouth |
|------------------------------------------------------|--------|----------|
| Total households connected to public supplies        | 4385   | 11658    |
| Total households connected to on-site                | 188    | 1756     |
| Persons/household                                    | 2.35   | 2.20     |
| Average daily population using public supplies       | 10304  | 27886    |
| Average daily population using on-site well water    | 442    | 3863     |

with 1975 records on population and groundwater pumpage and the long

term population projections given in the draft 208 plans developed by

CCPEDC (1978). The pumpage at the Sandwich well was conservatively

allowed to function at the maximum capacity. Table 8 summarizes the

locations and long term pumpage rates for each municipal or private well

within the boundaries of the management study area.

### 7.3.3.3  Water Usage Patterns

Data on municipal water use were obtained from available annual

water district reports and recent engineering studies. Records on the

Falmouth water distribution system were incomplete at the town, county,

and state levels. No records were available on the percent unaccounted

water. The town estimated commercial usage to be ten percent.

Estimates of the breakdown of water use in Bourne were obtained

from a recent water system study performed by Whitman and Howard, Inc.

(1984). The breakdown is shown in Table 9. Virtually all homes in

Bourne are connected to septic systems or cesspools. Based on the

Bourne study, the daily per capita domestic usage was 216 liters (57

gal) which compares well with the U. S. Environmental Protection Agency

(USEPA) (1980) estimate of 170/d (45 gal/d) for average septic system

flows. The USEPA estimate was the result of averaging several studies

where observed septic system flows ranged from 30.3 to 385 l/day.

In 1980 the fraction of households occupied year-round in Bourne

(81 percent) was only slightly higher than Falmouth (74 percent);

consequently, the per capita domestic usage rate from Bourne was

Table 8.   The 1980 and Long Term Pumpage Pattern in
Falmouth and Bourne.

| Water District and Well Name | Location (i,j) | Safe Yield [m³/d] | Fraction of Town Pumpage Capacity | 1980 Pumpage [m³/d] | Assumed Long term Pumpage Rate [m³/d] |
|---|---|---|---|---|---|
| **Bourne** | | | | | |
| Well #1 | 5,4 | 2241 | 0.332 | 1061 | 946 |
| Well #2 | 7,4-8,4 | 1254 | 0.167 | 850 | 476 |
| Well #3 | 5,4 | 1254 | 0.167 | 472 | 476 |
| Well #4 | 5,4 | 1254 | 0.167 | 466 | 476 |
| Well #5 | 7,4-8,4 | 1254 | 0.167 | 0 | 476 |
| **Falmouth** | | | | | |
| Long Pond | 12,3-12, 4-13,4 | 53429 | 0.989 | 10908 | 10911 |
| Fresh Pond | 11,7 | 3789 | 0.011 | 124 | 121 |
| Otis AFB (South of weeks) | 7,7 | >3309 | 1.000 | 3414 (1979) | 3309 |
| Sandwich Well #5 | 7,7 | 2554 | – | – | 2544 |

Table 9.  Municipal and Military Water Use Patterns.

| Total Pumpage for 1980 [$m^3/d$] | Bourne | Falmouth | Otis AFB |
|---|---|---|---|
| | 2850.5 | 11032 | 3309 |
| % Domestic usage | 78.2 | 54.5 | 86.7 |
| % Commercial | 4.4 | 15.0 (assumed) | 0.0 |
| % Unaccounted | 17.4 | 30.5 (assumed) | 13.3 |
| Total Population Served | 10304 | 27886 | 2000 |
| Per capita domestic usage | .216 | 0.216 (assumed) | 1.655 |

considered a reasonable estimate of the per capita domestic use in
Falmouth. Only 54.5 percent of all water pumped in Falmouth could be
attributed to domestic use (using the 216 l/day-person and a 1980
service population of 27886) (see Table 9). The amount of annual
pumpage devoted to commercial needs in Falmouth was therefore assumed to
be 15 percent. The assumed commercial fraction was similar to that
found in the contiguous town of Mashpee (14 percent in 1984) where the
1980 census showed the economic profile of the population to be the same
as Falmouth's. After commercial and domestic usage, 30.5 percent of
Falmouth's municipal water flows remained unaccounted-for. Water losses
through the Falmouth distribution system appeared quite high even
against observed losses in Yarmouth (20.4 percent, in 1984) and Cotuit
(23 percent in 1980 and 1984). Not all of the water lost could be
attributed to leakage; however, whether leaked or used by unmetered
users, all was assumed to recharge the aquifer.

Water withdrawn from the aquifer underlying the military
reservation fell into two water use categories: domestic and
unaccounted losses. The percent breakdown for each category was
estimated from pumpage records and wastewater flows for the year 1979
(LeBlanc, 1984). Wastewater flows at the Otis sewage treatment facility
amounted to 78 percent of the annual pumpage. Twenty-two percent of all
pumpage or 278 $m^3$/d of water was lost enroute to military housing units
through the Otis water distribution system and from homes to the Otis
wastewater treatment plant through sewer exfiltration. Using the ten
percent exfiltration rate reported by (Porter, 1980), the total loss of

flow through sewer leaks estimated 8.7 percent. The remaining 13.3 percent was assumed to be leakage from the water distribution system. The flow rate, listed in Table 8 as the 1980 pumpage for Otis AFB, was calculated from per capita usage in 197 and the long term population projection for the military reservation (CCPEDC, 1978). The quoted pumpage is only three percent lower than the observed 1979 flow (see Table 8); hence, Otis had attained the maximum projected development level as of 1979.

### 7.3.3.4  Artificial Recharge from the Combined Domestic and Commercial Use of Municipal Well Water and Septic Systems

Within the management study area, there are several sources of artificial recharge. Among those sources are flows from leaky water distribution systems and septic system effluents generated from domestic and commercial activities.

Estimates of the combined recharge from these sources were predicated on the following assumptions;

1) the ratio of flows associated with domestic and commercial use remained constant within a town regardless of source of water (i.e., municipal or on-site);

2) the ratio of flow associated with domestic and commercial use is uniform in space.

With the above assumptions, the combined recharge from domestic and commercial septic systems, and leakage from water distribution pipes was calculated as

$$Z = \left(1 + K_c + K_u\right) \cdot q_c \cdot P_z \tag{66}$$

where   $Z$   =   combined recharge from septic system effluent derived from

domestic and commercial activities plus recharge from

water distribution system leakage, $(m^3/d)$;

$q_c$   =   per capita domestic usage rate, which is .216 $m^3/d$;

$K_c$   =   ratio of commercial flows to domestic flows;

$K_u$   =   ratio of unaccounted water loss for the water distribution

system to domestic flows;

$P_z$   =   average daily population served by municipal water.

Table 10 listed, for each town and Otis AFB, the values of

constants used in the above equation.

## 7.3.3.5   Artificial Recharge from Sewered Residential Areas

Housing units overlying sewered areas have both public water and

wastewater services.   Artificial recharge from these areas originates

from sewer exfiltration and leakage from water distribution system.

Wastewater flows from housing units on Otis AFB are collected in

sewers and transported to the Base treatment plant.   In Falmouth

approximately 375 properties will be connected to a small sewer system.

The collected sewage will be conveyed north to a treatment facility

located between Route 28 North and the Falmouth Sanitary Landfill

between Blacksmith Shop Road and Thomas Landers Road.

Table 10.  Constants for the Distributed Artificial Recharge Equation.

| Constant $[m^3/d]$ | Bourne | Falmouth | Otis AFB |
|---|---|---|---|
| $q_c$ | 0.216 | 0.216 | 1.655 |
| $K_c$ | 0.056 | 0.275 | 0.000 |
| $K_u$ | 0.223 | 0.560 | 0.154 |

Recharge from households connected to sewers was calculated under the assumed conditions that; 1) unaccounted losses of water service flows result in direct groundwater recharge, and 2) the approximate sewer exfiltration rates are 10 percent of the flows discharged to the sewers (Flipse et al., 1984). Recharge fluxes were calculated using the equation

$$S = \left[0.10 \cdot \left(1 + K_c\right) + K_u\right] \cdot q_c \cdot P_s \tag{67}$$

where  $S$  =  combined recharge from sewer exfiltration and water

distribution leakage, $(m^3/d)$;

$q_c$  =  per capita domestic usage rate $(m^3/d)$;

$P_s$  =  average daily service population for a sewered area;

$K_c$ and $K_u$ were defined in equation (66).

Values for constants used the above equation are listed in Table 10. Actual recharge estimates appear in Table 11 along with the location of these flows in the management area. The flows for Falmouth were calculated by allowing the estimated service population to equal the product of the number of properties connected to the sewer and the number of people per household. Recharge flows for Otis AFB were estimated from the projected long term service population of 2000 people (CCPEDC, 1978).

### 7.3.3.6  Recharge Flows from Sewage Treatment Facilities

There are two sewage treatment facilities in the management area. The Otis plant is located in the south east corner of Bourne, while the

Table 11. Recharge from Sewered Neighborhoods and
Waste Water Treatment Facilities.

| Source | Element (i,j) | Flow [m$^3$/d] | Service Population within the Management Area |
|---|---|---|---|
| Otis Sewers | | | |
| | 8,5 | 364 | 1000 |
| | 8,6 | 364 | 1000 |
| Falmouth Sewers | | | |
| | 14,3 | 42 | 280 |
| | 14,4 | 81 | 546 |
| Otis Treatment Plant | | | |
| | 9,6 | 2581 | 2000 |
| Falmouth Plant | 11,3 | 1136+218 | 1106 |

Falmouth plant is located south of the Crocker Pond Watershed. The Otis plant discharges secondary effluent to 24 sand beds (each are .2 ha). The treated sewage percolates to the water table which lies six meters below the surface of the sand beds. Further details on the design of the treatment system can be obtained from LeBlanc (1984). Recharge from the plant was estimated at 258/$m^3$/d or 78 percent of the projected long term pumpage rate of 3309 $m^3$/d.

The Falmouth sewage treatment plant is a combination spray irrigation and rapid infiltration system. Upon completion of a two phase sewer construction project, the design flow capacity of the plant will be 4700 $m^3$/day (1.25 mgd). Phase I is just nearing completion; phase II may never begin. With the completion of phase I, the maximum capacity of the plant is projected at 2800 $m^3$/day (.75 mgd).

Recharge from the Falmouth plant was expected equal to the sum of 1137 $m^3$/d (300000 gpd) originating from Woods Hole and 218 $m^3$/d collected from the newly constructed sewers in Woods Hole, Falmouth Beach, Falmouth Center, Davis Straits, and Main Street East. The newly sewered section totals 400 property connections. Recharge was calculated from projected combined domestic and commercial flows that were reduced by ten percent to account for exfiltration.


7.3.3.7  Nitrate Concentrations in Septic System Effluents

On-site domestic and commercial waste disposal systems are designed to capture the solids and partially degrade influent wastewaters

anaerobically before a clarified effluent is distributed over a seepage field and allowed to percolate through the soils where further aerobic and anaerobic treatment of the wastewaters may occur. As the effluent percolates through the soils, organic nitrogen and ammonia are oxidized to nitrate. Approximately 40 percent of the nitrate is then reduced through denitrification to nitrogen gas and nitrous oxide (Porter, 1980). Of the total mass of nitrogen delivered to these waste disposal systems approximately 50 percent reaches the saturated zone as nitrate. Little if any degradation or adsorption of nitrate occurs in the saturated zone, unless conditions are anaerobic with sufficient dissolved organic substrate to support denitrifying organisms; thus, nitrate is often assumed to behave as a conservative soluble constituent in the subsurface environment. Groundwaters of Cape Cod aquifer are generally oxygenated and low in dissolved organics; therefore, significant reduction of nitrate is not expected.

For domestic waste flows, Porter (1980) estimated the per capita nitrogen production at 4.08 kilograms/year (9 pounds/y). Of the total nitrogen produced, the tank and the vadose should remove 50 percent. Thus the effective daily per capita nitrate load would total 5592 mg. Using the daily per capita flow rate from Bourne (216 l/d) the projected nitrate concentration at the top of the saturated zone was 25.9 mg/l as N. This concentration and flux appeared consistent with estimates presented by the USEPA (1980) and observed nitrate concentrations in effluent discharges (Dudley and Stephenson, 1973 in Porter, 1980).

Nitrate concentrations from nonresidential activities would vary greatly among commercial activities (i.e., hotels, swimming pools, restaurants); however, USEPA (1980) determined that many nonresidential wastewater generating sources produce effluents having water quality similar to residential sources. Because additional data to characterize the nitrate concentration from commercial septic system flows was unavailable, the domestic wastewater nitrogen levels were used.

### 7.3.3.8  Nitrate Concentrations in Otis Wastewater Flows

Estimates were needed for the total nitrogen in wastewater influent and effluent streams of the Otis sewage treatment facility. Total nitrogen concentrations were used as estimates of the potential nitrate concentration from streams of completely nitrified exfiltration flows and secondary effluent recharge flows. The total nitrogen load would enter the aquifer as nitrate, ammonia and organic nitrogen. If aerobic conditions exist the ammonia and organic nitrogen would be ultimately oxidized to nitrate as a groundwater plume moves down gradient from the recharge beds or the leaky sewer.

The inorganic fraction of secondary domestic wastewater effluent typically represents 79 percent of the total nitrogen and 56 percent of the total nitrogen in raw sewage (Metcalf and Eddy, Inc., 1979). From the total inorganic nitrogen concentrations reported by Vaccaro et al., (1979) in their study of Wastewater Renovation and Retrieval on Cape Cod, the total nitrogen in the Otis raw sewage was estimated $31.96 \pm 5.80$ mg/l-N. The calculated nitrogen in the effluent discharged to the

sand beds was 22.67 ± 4.12 mg/l-N. LeBlanc (1984) found an average total nitrogen level of 19 mg/l as N for Otis secondary effluent.

In a well situated close to the recharge beds LeBlanc (1984) reported total groundwater nitrogen at 24 mg/l as N. If this nitrogen concentration reflects the level of nitrogen removal as sewage percolates through the vadose zone, then nitrate concentrations in recharge beneath leaky sewers may have similar water quality. Nitrogen concentrations in the exfiltration flows and the secondary effluent recharge were assumed similar; hence, the value of 25 mg/l as N was used for both flows as an average of the extreme estimate derived from Vaccaro et al., (1979) and the observed level from LeBlanc (1984).

### 7.3.3.9 Nitrate Concentrations in Falmouth Wastewater Flows

Wastewater in Falmouth is primarily of domestic and commercial origin. Because exfiltration flows percolate through the vadose zone, the fate of nitrogen in those flows was presumed similar to that of nitrogen in septic system effluents. A fifty percent reduction of nitrogen occurs with the use of septic systems; therefore, under the equal nitrogen reduction assumption, the nitrate nitrogen level in exfiltration flows reaching the saturation zone was calculated as 26 mg/l.

The concentration of nitrate in recharge flows derived from the Falmouth sewage treatment facility could be as low as 2 mg/l as N. Research by Vaccaro et al., (1979) illustrated that proper operation of the facility could achieve groundwater nitrate concentrations of 1-3

mg/l as N.  In this work a 2 mg/l as N nitrate concentration was used in
the sewage recharge.


## 7.3.3.10  Lawn Fertilizer Loads

The application of fertilizers for lawn cultivation could be a
major source of groundwater nitrates on Cape Cod.  The Cape Cod Planning
and Economic Development Commission (1979) estimated that the average
home owner applies 1.46 kg of nitrogen (as N) per 100 $m^2$ of lawn area
per year.

The Suffolk County Department of Environmental Control (in Porter,
1980) investigated nitrogen loading from lawn fertilizers in a sewered
housing development in central Long Island, New York.  A nitrogen
loading rate of 1.07 kg/100 $m^2$ was determined from a survey of
homeowners.

The capacity for turf to assimilate nitrogen had been estimated at
0.5 kg/100 $m^2 \cdot y$ for lawns ten years or older (Porter, 1980).  For Cape
Cod, the consequence would be that as much as 60 percent of the nitrogen
applied leaches to the groundwater (Flipse, 1984 and CCPEDC, 1979).  In
an effort to calculate nitrate loading from lawn fertilizers, the CCPEDC
estimated that each household applied an average of 6.8 kg of nitrogen
per year.  Of the total mass of nitrogen applied, 60 percent (or 4.08
kg) was expected to leach below the root zone to the water table.  The
effective daily per capita nitrate loading rates were estimated for

Bourne and Falmouth by dividing the number of residents per household into the daily effective household rate (0.60 x gross application rate).

### 7.3.3.11  Background Nitrate Loading

The background nitrate concentration for Cape Cod groundwaters was reported by Frimpter and Gay (1979) to be 0.5 mg/l as N. The source of this nitrogen could be from precipitation, animal wastes, or possibly leaching of nitrates produced from the natural oxidation of animal and vegetable matter. Likens et al. (1977) reported a weighted annual mean of 1.47 mg/l nitrate-N in bulk precipitation from 1965-1974 in the Hubbard Brook Experimental Forest, New Hampshire. Other records of nitrates in precipitation revealed concentrations on the order of 1 mg/l or less (Frizzola in Flipse, et al., 1984). The actual source(s) of background nitrate were not ascertained for Cape Cod. In this modeling effort, background nitrate levels were generated through a fixed nitrate concentration of 0.5 mg/l as N in all natural recharge flows.

### 7.3.4  Calculation of the Components in the Recharge and Source Concentration Vectors

The components of the recharge and source concentration vectors, that appeared in the continuity constraints (see section 7.3.2), were calculated or directly obtained from the Cape Cod data presented above on the 1980 hydrologic stresses and nitrate pollution.

The equations used to calculate the various discrete recharge and discrete source concentration vector components are presented below.

Constants used in the equations that follow are given in Table 12. The results of calculations are summarized in Table 13.

### 7.3.4.1 Recharge from the Combined Domestic and Commercial Use of Municipal Well Water, Septic Systems, and Lawn Fertilizers

Several parameters were needed to calculate the effective nitrate concentration in artificial recharge derived from a composite land use activity using municipal well water, septic systems and lawn fertilizers. Those parameters included; 1) the volume of flow from septic system; 2) the recharge flows attributed to losses from water distribution systems; 3) the nitrate concentration at the water table for septic system effluents, 4) the nitrate concentration in leakage from the water distribution system; and 5) the effective per capita nitrogen loading rate from fertilizers.

The equation used to calculate discrete recharge from the use of municipal well water, septic systems, and fertilizer was

$$Z_{i,j} = \frac{\left[ 1 + K_{c_{i,j}} + K_{u_{i,j}} \right] \cdot q_{c_{i,j}} \cdot P_{z_{i,j}}}{\Delta x \cdot \Delta y} \qquad \forall \text{ i and j} \qquad (68)$$

where $Z_{i,j}$ = recharge in element i,j from septic system effluent derived from domestic and commercial use of municipal well water plus recharge from water distribution system leakage, (m/d);

$P_{z_{i,j}}$ = average population in element i,j using municipal water and septic systems;

Table 12.  Constants Used in the Source Recharge and Nitrate
Flux Equations.

| Constant | | Bourne | Falmouth | Otis AFB |
|---|---|---|---|---|
| $q_{c_{i,j}}$ | $[m^3/d]$ | 0.216 | 0.216 | 1.655 |
| $K_{c_{i,j}}$ | | 0.056 | 0.275 | 0.000 |
| $K_{u_{i,j}}$ | | 0.223 | 0.560 | 0.154 |
| $C_{ss}$ | (mg/l $NO_3$-N) | 26.000 | 26.000 | – |
| $F_{i,j}$ | (mg/d·person) | 4760 | 5070 | – |
| $C_{E_{i,j}}$ | | – | 26.000 | 25.000 |

Table 13.  Summary of Nitrate Source Concentrations (mg/l as N).

| Source (by Model Symbol) | Bourne | Falmouth | Otis AFB |
|---|---|---|---|
| $Z_{i,j}$ | 38.71 | 30.86 | - |
| $W_{i,j}$ | 46.96 | 44.42 | |
| $S_{i,j}$ | - | 38.90 | 9.87 |
| $U_{i,j}$ | - | 2.00 | 25.00 |
| $Q_{i,j}$ | 0.50 | 0.50 | 0.50 |

$q_{c_{i,j}}$ = elemental per capita domestic usage, $(m^3/day)$;

$K_{c_{i,j}}$ = ratio of commercial flows to domestic flows in element i,j;

$K_{u_{i,j}}$ = ratio of unaccounted water loss for the water distribution system to domestic usage in element i,j;

$\Delta x$ and $\Delta y$ = the x and y dimensions of the numerical element, (m).

Notice that no direct flows were attributed to lawn watering. If lawn watering was a significant component of domestic usage, than the recharge would be over estimated because calculated domestic recharge flows ignored losses due to evapotranspiration. The consequence would be underestimated nitrate concentrations in the combined recharge flows and in the predicted groundwater quality impacts, because the total nitrate load would remain the same regardless of consumptive water losses. The regional water quality impacts of the consumptive losses were considered minor, and consequently, were ignored. This conclusion was predicated on the assessment of ten percent consumptive losses (Quadri, 1984) in total pumpage, which amounted to less than five percent of total natural recharge; hence, a minor .5 percent loss of steady groundwater flow.

The equation used to calculate the nitrate concentration in the discrete recharge flow $Z_{i,j}$ for all i and j was

$$C_{z_{i,j}} = \frac{C_{ss} \cdot \left(1 + K_{c_{i,j}}\right) \cdot q_{c_{i,j}} + F_{i,j} + C_m \cdot K_{u_{i,j}} \cdot q_{c_{i,j}}}{\left(1 + K_{c_{i,j}} + K_{u_{i,j}}\right) \cdot q_{c_{i,j}}}$$

$$\forall \ i \ \text{and} \ j \qquad (69)$$

where $C_{z_{i,j}}$ = effective nitrogen concentration in all recharge flows

in element i,j derived from the combined septic

systems, and lawn fertilizers, (mg/l as N);

$C_{ss}$ = effective concentration of nitrate in recharge from

domestic and commercial use of septic systems, (mg/l);

$C_m$ = effective concentration of nitrate in recharge from

leaking municipal water distribution systems, (mg/l);

$F_{i,j}$ = effective per capita nitrate load from lawn fertilizer,

(mg/d as N).

Notice that nitrate loads associated with leakage in the water
distribution system were considered. The nitrate concentration in
municipal water supplies was low which meant that the expected
groundwater nitrate contributions from leaky water distribution systems
would be minor.

## 7.3.4.2  Recharge from the Combined Domestic and Commercial Use of On-site Wells, Septic Systems, and Lawn Fertilizers

If on-site wells were used in conjunction with septic systems and cesspools, little or no impact on the hydrologic balance of flows would be expected. The effective nitrate concentration from the use of on-site wells, septic systems, and lawn fertilizers was calculated from; 1) the volume of flow from the septic system, 2) the septic system effluent nitrate concentration after percolation to the water table, and 3) the effective per capita nitrogen loading rate from fertilizers.

The discrete recharge flows from the use on-site wells were estimated with the equation

$$W_{i,j} = \frac{\left[1 + K_{c_{i,j}}\right] \cdot q_{c_{i,j}} \cdot P_{w_{i,j}}}{\Delta x \cdot \Delta y} \qquad \forall \ i \ and \ j \qquad (70)$$

where $W_{i,j}$ = recharge in element i,j from septic system effluent derived from domestic and commercial use of on-site well water, (m/d);

$P_{w_{i,j}}$ = average daily population in element i,j using on-site wells.

Notice that consumptive losses were not considered.

The formula developed to calculate the nitrate concentrations at the water table from the combined influence of domestic and commercial activities plus lawn cultivation was

$$C_{w_{i,j}} = \left[\frac{C_{ss} \cdot \left(1 + K_{c_{i,j}}\right) \cdot q_{c_{i,j}} + F_{i,j}}{\left(1 + K_{c_{i,j}}\right) \cdot q_{c_{i,j}}}\right] \qquad \forall \ i \ and \ j \qquad (71)$$

where $C_{w_{i,j}}$ = effective nitrate concentration in all recharge flows

in element i,j derived from the combined domestic and

commercial use of on-site wells, septic systems, and

lawn fertilizers, (mg/l as N).

7.3.4.3  Recharge from the Combined Domestic and Commercial Use of

Municipal Well Water, Sewers, and Lawn Fertilizers

The effective nitrate concentration in recharge from the use of

municipal water, sewers, and lawn fertilizers was evaluated with

information on; 1) volume of flow associated with exfiltration, 2)

recharge flow attributable to water distribution system losses, 3)

nitrate concentration at the water table for exfiltration flows, 4) the

nitrate concentration in leakage from the water distribution system, and

5) the effective per capita nitrogen loading rate from fertilizer.

Discrete recharge flows from the use of municipal well water,

sewers, and fertilizer were calculated with the equation

$$S_{i,j} = \frac{\left(0.10 \cdot \left(1 + K_{c_{i,j}}\right) + K_{u_{i,j}}\right) \cdot q_{c_{i,j}} \cdot P_{s_{i,j}}}{\Delta x \cdot \Delta y} \quad \forall \; i \; \text{and} \; j \quad (72)$$

Notice that sewer exfiltration was limited ten percent of the domestic

and commercial flow collected from each household and business as

discussed in section 7.3.3.5.

The effective nitrate concentration in the recharge flow ($C_{s_{i,j}}$)

was calculated with the equation

$$C_{s_{i,j}} = \frac{C_{E_{i,j}} \cdot 0.1 \cdot \left(1 + K_{c_{i,j}}\right) \cdot q_{c_{i,j}} + C_m \cdot K_{u_{i,j}} \cdot q_{c_{i,j}} + F_{i,j}}{\left[0.1 \cdot \left(1 + K_{c_{i,j}}\right) + K_{u_{i,j}}\right] \cdot q_{c_{i,j}}}$$

$$\forall\ i\ \text{and}\ j \qquad (73)$$

where $C_{s_{i,j}}$ = effective nitrate concentration in all recharge flows in

element i,j derived from the combined domestic and

commercial use of municipal water, sewers, and lawn

fertilizers, (mg/l as N);

$C_{E_{i,j}}$ = effective nitrate concentration in element i,j for sewer

exfiltration recharge, (mg/l as N).


### 7.3.4.4 Recharge from Land Application of Secondary Sewage and Background Loads

Estimates of recharge from the land application of secondary sewage
at Falmouth and Otis amounted to 90 percent of the sum of domestic and
commercial flows from the sewered areas. Thus,

$$U_{i,j} = 0.9 \sum_{i=1}^{n} \sum_{j=1}^{m} \left(1 + K_{c_{i,j}}\right) \cdot q_{c_{i,j}} \cdot P_{s_{i,j}} \cdot \omega_{i,j} \quad \forall\ i\ \text{and}\ j$$

$$(74)$$

where $U_{i,j}$ = the recharge in element i,j from land application of

secondary sewage collected from elements where

underlying sewers convey flows to site i,j;

$\omega_{i,j}$ = integer variable which is equal to one if sewers

underlying an element convey flows to land application

site i,j.

Sections 7.3.3.8 and 7.3.3.9 discussed the effective nitrate concentration in recharge flows from the land application of secondary sewage at both sewage treatment facilities.

The final nitrate flux to be incorporated in the model was the nitrogen load responsible for observed background nitrates of 500 µg/l as N. The source or sources of background nitrates were not elucidated but their groundwater quality impacts were approximated by specifying a nitrate concentration in natural recharge equal to 500 µg/l nitrate as N.

## 7.3.5 Constructing and Solving Management Models I and II

Fortran programs were developed that create computer files containing the objective functions and continuity constraints for the two nonpoint source groundwater pollution management models. Each Fortran program employed the groundwater flow model to define necessary fluid velocity coefficients used to construct the continuity constraints. The data required to create the computer files included: 1) the area and dimensions of the elements, 2) the elemental recharge rates of all sources, 3) the elemental pumpage rates, 4) the piecewise aquifer transmissivities, 5) the hydraulic model and contaminant model boundary conditions, and 6) the nitrate concentrations in all recharge flows.

Model I was constructed to accommodate all the numerical elements in Bourne and Falmouth. A total of 79 decision variables and 145 constraints comprised the body of the model. Thirteen constraints equated contaminant decision variables from 13 imaginary elements east of the groundwater divide to 13 real elements across the divide and inside the bounded study area. These 13 constraints imposed zero contaminant gradients across the groundwater divide; hence, zero flux conditions were attained when central finite differencing schemes were used on boundary nodes. One hundred thirty-two continuity constraints were employed; two for each of the 66 elements within the discretized area of Bourne and Falmouth. The objective function was cast as a summation of the 66 contaminant decision variables.

Model II contained the same decision variables and constraints as Model I. This model was used to elucidate regional nitrate impact isopleths around Long Pond; Falmouth's major municipal water supply. Long Pond intersects three elements ($i,j$ = 12,3 - 12,4 - 13,4). Equal pumpage at each node was used to approximate the aquifer stress produced when water was withdrawn from this pond. The objective function, formulated as a summation of contaminant decision variables from the pond elements, appeared as

$$\text{Maximize } \frac{1}{3} \left[ c_{12,3} + c_{12,4} + c_{13,4} \right]$$

where the value of the objective function represents the average concentration of nitrate in the three elements intersected by Falmouth's major municipal water supply. Model I and II were solved using the regular simplex algorithm made available through the Multi Purpose

Optimization System package (Northwestern University, 1973). Files

representing the linear programming formulations of Models I and II were

submitted as input data to the optimization package.


7.3.6  Results of Models I and II

The results form solving Model I included the steady-state

groundwater nitrate predictions for each element of the discretized

management area for 1980 levels of development and the optimum values of

dual variables. Regional groundwater quality changes effected through

alternative nitrate disposal stresses were interpreted through the dual

variables.

Figure 40 presents the predicted steady-state nitrate concentration

contours for groundwaters underlying Bourne and Falmouth. For the

entire management area, the average simulated water quality was 1.5 mg/l

nitrate nitrogen. The extent of groundwater nitrate pollution reflected

the location, intensity and type of land use activity in 1980. In

general, the highest levels of simulated groundwater nitrates occurred

in areas of the highest density of residential/commercial land use

(i.e., South Falmouth). Elemental nitrate concentrations were predicted

under 5 mg/l as N everywhere, except in the element (i=9, j=6), where

the Otis wastewater treatment facility is located.

The concentration of nitrate nitrogen in the center of the Otis

plume was 5.9 mg/l, which was much lower than the observed 20 mg/l total

nitrogen reported by LeBlanc (1984). Part of this discrepancy is

explained through the exaggerated predicted width of plume near the
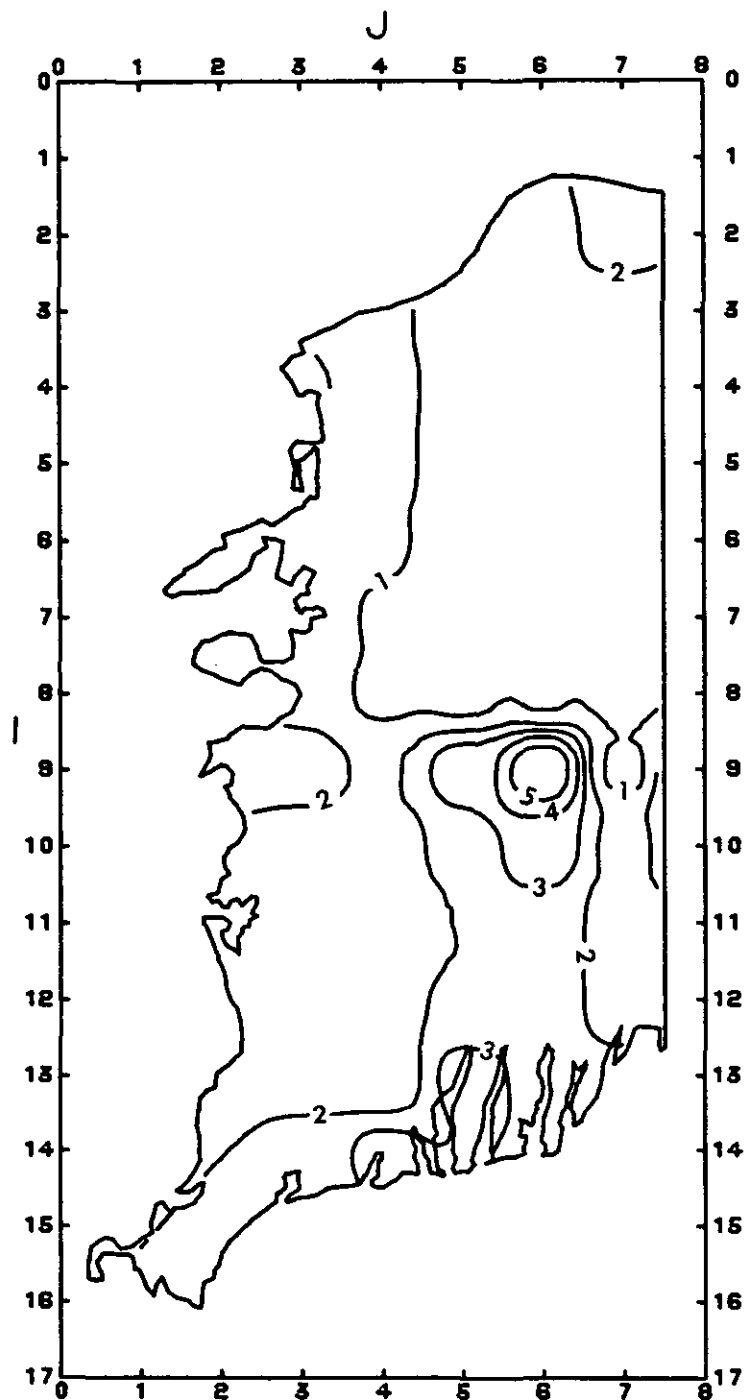
Figure 40. Map depicting the steady-state nitrate nitrogen concentration (mg/l) contours predicted by Model I from 1980 development patterns.

sewage treatment facility: LeBlanc estimated the plume width at 1000
meters whereas the Model I predicted the plume width at 2000 meters.
Nitrogen contour lines drawn by LeBlanc (1984) were integrated to obtain
the nitrogen mass per unit longitudinal length of the plume; this
figure, when divided by the 2000 meter width, produced an average plume
concentration of 5.5 mg/l as N which was similar to the above estimate
from the model. The length of the simulated Otis plume was 3000 meters
(using the 3 mg/l contour as a boundary). Elevated groundwater nitrates
found in observation wells (LeBlanc, 1984) indicated that the plume was
at least that long.

For the elements which contain water supply wells and ponds, the
water quality ranged from .8 to 1.4 mg/l as Nitrate nitrogen. The
highest steady state nitrate levels were predicted at Long Pond and
Fresh Pond which were both in Falmouth (see Table 14).

Postoptimality analysis retrieved the optimum values of dual
variables associated with the less-than-or-equal-to continuity
constraints. The values of the duals were used to interpret regional
water quality impacts caused by unit changes in nitrate nitrogen loading
at each element. The values of the dual variables give the increases in
the value of the objective function (which is the average nitrate
concentration for the whole management area) obtained from unit
relaxations of the less-than-or-equal-to continuity constraints (i.e.,
from a unit increase in the nitrate loading rate). Discrete dual
variable values were plotted and contour lines were drawn. These
represent regional water quality impact isopleths.

Table 14. Model I and II Predicted Regional Steady-state Nitrate
Concentrations at Wells from 1980 Land Use Activities.

| Water District and Well Name | Location (i,j) | Predicted Nitrate Concentration (mg/l as N) |
|---|---|---|
| Bourne | | |
| Well #1 | 5,4 | 1.3 |
| Well #2 | 7,4-8,4 | 0.8 |
| Well #3 | 5,4 | 1.3 |
| Well #4 | 5,4 | 1.3 |
| Well #5 | 7,4-8,4 | 0.8 |
| Falmouth | | |
| Long Pond | 12,3-12,4-13,4 | 1.4 |
| Fresh Pond | 11,7 | 1.3 |
| Otis AFB | | |
| (south of Weeks) | 7,7 | 1.0 |
| Sandwich | | |
| Well #5 | 7.7 | 1.0 |

The iso-impact contour plot is portrayed in Figure 41. The numbers represent the increased nitrogen concentration ($\mu$g/l as N) resulting from an increased nitrate nitrogen loading rate of one kilogram per day per square kilometer along the contour line. This change in the loading rate is equivalent to increasing the year-round resident population by 100 per square kilometer.

The iso-regional-water quality impact map indicated that increased nitrate loadings over the interior region would lead to greater increases in the regional nitrate concentrations than would equivalent increases in nitrate loading along the coast. The iso-impact contours followed a pattern similar to the water table contours (see Figure 31).

The differences between coastal and interior regions are explained when consideration is given first to the cumulative impacts of pollution occurring in the interior regions and secondly to the small marginal changes in local contaminant levels effected by nitrate loading near the coast. Sources located at interior nodes have a cumulative effect on regional contaminant levels because they affect nitrate concentrations in all down gradient nodes. Sources located nearer to discharge zones have a small cumulative contaminant impact because dissolved solutes do not travel through of the groundwater flow system to the same extent. Furthermore, sources located near recharge areas have a greater marginal impact on local groundwater quality than sources located near discharge zones; this is because the available groundwater flows to dilute the pollution are much less than the cummulative flows found near discharge zones.
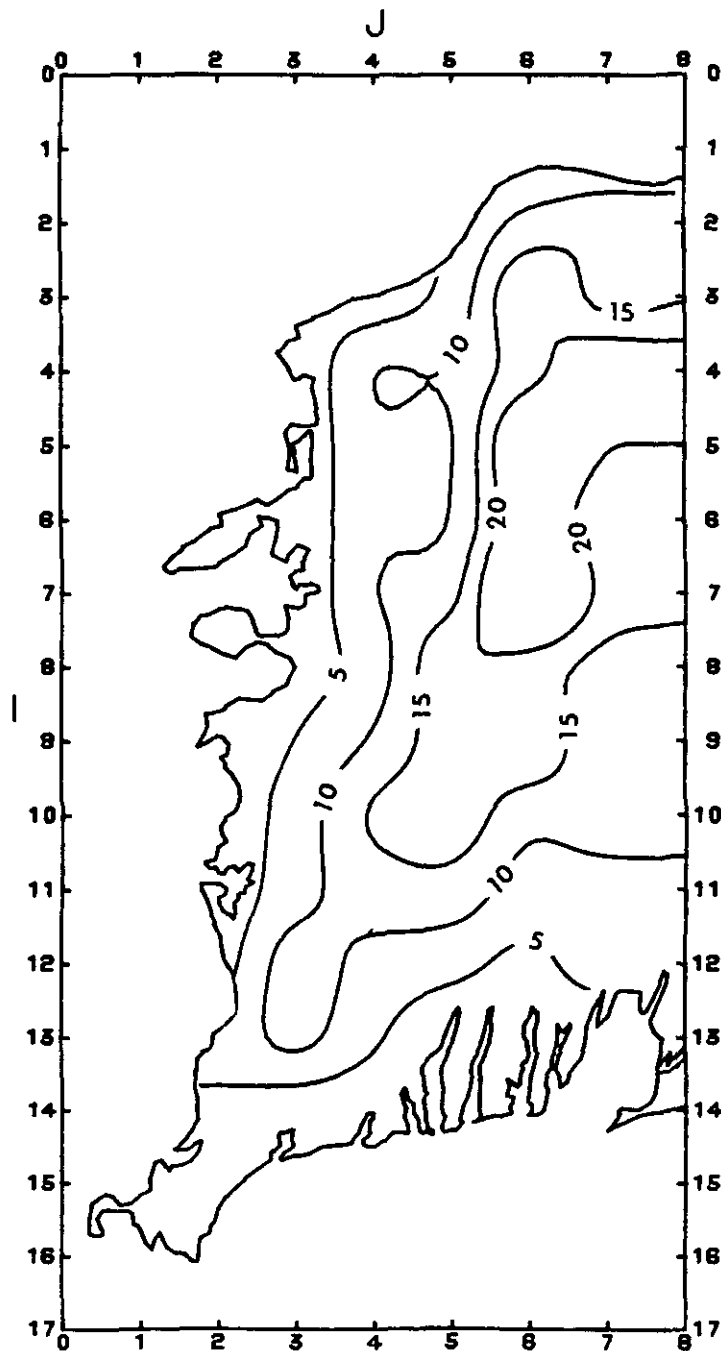
Figure 41.  Map depicting the global nitrate nitrogen loading impact
            isopleths constructed from the optimum values of dual
            variables associated with continuity constraints of Model
            I.  Numbers represent the ug/l increase in average
            groundwater nitrate nitrogen over the region for an
            increase nitrate nitrogen load of 1 kg/day·sqkm.

A primary implication of the iso-regional-water-quality-impact

contour map for Bourne and Falmouth is that certain regions of the area

(especially areas occupied by the Military Reservation) are more

important with regard to the preservation of regional water quality.

The utility of iso-impact maps is founded on the ease of using these

maps to predict the relative significance of sources distributed inside

the bounded study area to long term regional groundwater quality.

The results from solving Model II included the same steady-state

groundwater nitrate predictions over the towns of Bourne and Falmouth.

Values of the dual variables associated with the less-than-or-equal-to

continuity constraints were plotted. As with the last model, the values

of the dual variable are used to identify increases in the value of the

objective function (which is the average nitrate concentration for the

three elements intersected by Long Pond) obtained from unit relaxations

of the less-than-or-equal-to continuity constraints. Plotting the

values of the dual variables revealed a pattern of values around the

three target elements ($i,j$ = 12,3 and 12,4 and 13,4). Iso-water-

quality-impact contours were drawn between discrete values of the dual

variables (see Figure 42).

The numbers associated with each contour represent the approximate

$\mu g/l$ increase in groundwater nitrate nitrogen observed over groundwaters

equally withdrawn from elements $i,j$ = 12,3 and 12,4 and 13,4 from an

increase nitrate load of one kilogram nitrogen per day per square

kilometer in elements along that contour. The configuration of contours

in Figure 42 illustrate the areal extent to which water quality around
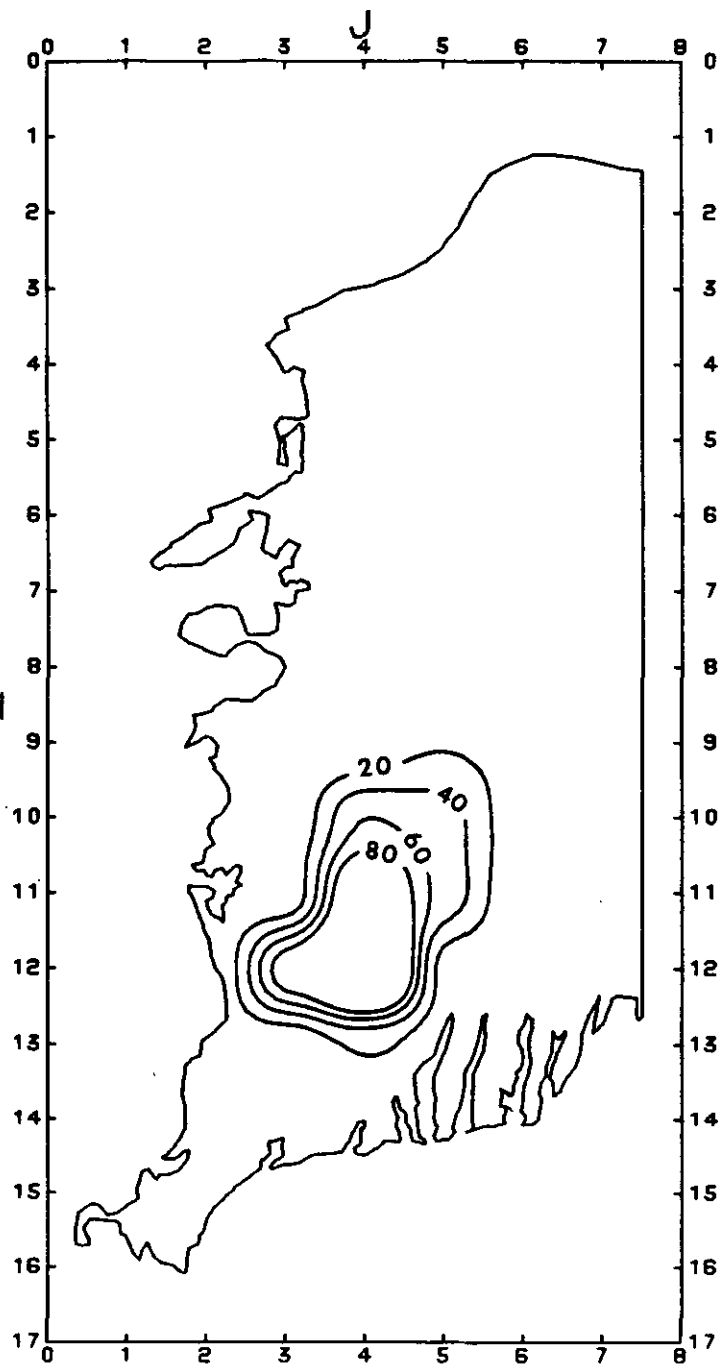
Figure 42.   Map depicting the nitrate nitrogen loading impact isopleths
around Long Pond constructed from the optimum values of
dual variables associated with continuity constraints of
Model II.  Numbers represent the ug/l increase in
groundwater nitrate nitrogen around Long Pond for an
increase nitrate nitrogen load of 1 kg/day·sqkm.

Long Pond is determined by upgradient sources. The contours extend for several kilometers into the regional water table contours (See Figure 31). The shape of the iso-impact contours is as one would expect for nodes which lie down gradient of a groundwater mound or near discharge zones. After nitrate is introduced to the aquifer, the fraction reaching the target nodes is determined by the change in the magnitude and direction of the Darcian velocity field. As the assumed source moves toward the target elements the iso-impact contours increase because a larger fraction of the contaminant loaded to the aquifer reaches the these elements. The contours delineate zones around Long Pond. Zones close to Long Pond are most important, especially if they are upgradient. The contours map out the steady-state response of the groundwater system at elements containing Long Pond due to increased nitrate loading anywhere within the bounded study area. The long term water quality impact of modified subsurface disposal activities can be evaluated with Figure 42 if the source location and nitrate loading rate are known and the effects on groundwater hydraulics are minor.

## 7.4 Conclusions

It must be remembered that the objective of these management model is to relate regional nonpoint source pollutant loadings to regional water quality. Thus the model's strength is in explaining the relative impacts of alternative development patterns (i.e., alternative placement and strength of sources) and at evaluating areally averaged water quality. The concentrations predicted by the model may not correspond

to concentrations found at a given well in the assigned element. The concentrations at a specific well may be greater or less than the predicted concentrations depending on the location of the well relative to the local sources of pollution. The regional contaminant concentrations indicate that if a given well in a element has pollutant concentrations greater than that predicted, then there must also be another well (i.e. alternate well location and screen depth) within that element which will produce water with contaminant concentrations less than that predicted (this is because the regional contaminant predictions represent average elemental contaminant levels). Model I and Model II are both nonpoint source groundwater pollution management models. Both models were applied to a specific area, Western Cape Cod, to examine regional nitrate pollution from land development. The objective function for Model I was a linear summation of the contaminant decision variables (i.e., nitrate concentrations) from the entire management area. The primary constraints of the model were continuity constraints constructed from a defined groundwater flow field. Model II is a simplified version Model I that used an objective function which was a summation of contaminant decision variables of elements that were targeted for investigation.

Approximations of hydraulic stresses and nitrate loads were predicated on the 1980 population estimates and information gathered from the literature. Solutions were obtained using available computer algorithms.

Models I and II were used to identify regions in the study area which are critical to the long term protection of regional groundwater quality availability over the study area and in a specific subarea of the region (i.e., the area of the Falmouth municipal well field). Results from Model I and II included steady-state nitrate distributions for Bourne and Falmouth for 1980 development conditions. The nitrate nitrogen in elements containing municipal wells was predicted to be below 1.5 mg/l. Other results included the optimal values of dual variables associated with relaxing the continuity constraints (i.e., investigating the impacts of marginal increases in nitrate loadings from each element). Plotting contours around the values of the dual variable created iso-water quality impact isopleths.

For Model I, the iso-water quality impact isopleths related changes in nonpoint source nitrate loads to regional changes in groundwater quality. A primary implication of the iso-regional-water-quality-impact contour map for Bourne and Falmouth was that certain regions of the area (especially areas occupied by the Military Reservation) should be ranked higher with regard to their importance to the preservation regional groundwater quality.

With Model II the isopleths could predict changes in water quality in a few elements as a result of increased nitrate loadings anywhere in Bourne and Falmouth. The iso-water quality impact plots developed from Model II can be used to identify critical recharge zones: areas which are most important to the long term preservation of groundwater quality at target elements. In this case the elements containing Falmouth's

municipal water supply (Long Pond) were chosen as the target elements.

As expected, the implication of the Model II results is that activities

situated close to the pond have the greatest impact on water quality at

the pond. The isopleths give an indication of the extent of areas

having significant water quality impact (note the increased areal extent

of the contours in the upgradient direction). In addition, the

isopleths define the relative significance of separate zones within the

recharge area around Long Pond which are critical to the preservation of

water quality at the pond. The water quality impacts of placing sources

within the region containing a municipal water supply can be evaluated

in terms of the approximate water quality impacts on target elements.

The actual long term water quality at municipal wells could be more or

less than the elemental nitrate concentration shown depending on the

actual positions of the well.

CHAPTER 8


MODELING TO ELUCIDATE MAXIMUM DEVELOPMENT OF

MULTIPLE POLLUTING LAND USE ACTIVITIES


This chapter describes the development and application of a third
nonpoint source groundwater pollution management model to determine the
simultaneous maximum feasible development of multiple land use
activities which are known to contribute to the nonpoint source
contamination of aquifers. The optimum pattern and combination of
surface activities is identified from given restrictions on available
resources (i.e., land and water), imposed water quality standards,
specified land use density regulations, and existing land use patterns.

Model III is applied to the section of Western Cape Cod that
constitutes the town Falmouth, Massachusetts. The general formulation
was adapted to ascertain the regional patterns of three slightly
different forms of residential/commercial land use which in combination
accommodate a maximum year-round population. An optimum pattern is
identified from a large set of feasible residential/commercial land use
development patterns which incorporate present (1980) development in
Falmouth. The feasible development patterns satisfy constraints on
desirable nitrate nitrogen levels at municipal water supplies and for
the rest of the region, constraints on available water supplies, and
constraints representing imposed regulations on specific land use
activities.

Model results include:  1) contour plots of the steady-state regional groundwater nitrate distribution from the maximum potential residential/commercial development; 2) maximum feasible population predictions for the area in each element, and for each residential/commercial land use type; 3) maps illustrating optimal locations of residential/commercial development; 4) maps showing which numerical elements have land use densities approaching zoning restrictions and which elements have predicted nitrate concentrations on the verge of violating standards; and 5) figures depicting the optimal values of dual variables associated with constraints on source densities and water quality.

## 8.1  Formulation of Management Model III

Model III incorporates all five components of the general management model (recall Chapter 3).  The formulation discussed below is presented to evaluate the management of a combination of three composite land use activities known to contribute areal contamination of groundwaters; however the formulation is general and the variety of surface activities considered could be expanded.  Details on model components are presented below.

## 8.1.1  Decision Variables

Two groups of decision variables are used in this model.  The first is the contaminant concentration variables $(C_{i,j})$ for each discrete element i,j.  The second group embodies the elemental subsurface

recharge flows ($Z_{i,j}$, $W_{i,j}$, and $V_{i,j}$) contributed by three specific land use activities being evaluated with the model.

## 8.1.2 Objective Function

The objective is a summation of all elemental subsurface recharge flows generated from the three land use activities. The optimum value of the objective function is the maximum total population in the management area. This is the same population engaged in the three land use activities. The objective function is written

$$\text{Maximize} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \left[ D_{W_{i,j}} \cdot W_{i,j} + D_{Z_{i,j}} \cdot Z_{i,j} + D_{V_{i,j}} \cdot V_{i,j} \right] \quad (76)$$

where $D_{W_{i,j}}$ = the reciprocal of the per capita generation of recharge

flow $W_{i,j}$, $\left( \frac{t \cdot person}{L^3} \right)$;

$D_{Z_{i,j}}$ = the reciprocal of the per capita generation of recharge

flow $Z_{i,j}$, $\left( \frac{t \cdot person}{L^3} \right)$;

$D_{V_{i,j}}$ = the reciprocal of the per capita generation of recharge

flow $V_{i,j}$, $\left( \frac{t \cdot person}{L^3} \right)$.

## 8.1.3 Continuity Constraints

The continuity constraints tie together elemental contaminant decision variables with the variables for the contaminant laden recharge

flows $Z_{i,j}$, $V_{i,j}$ and $W_{i,j}$ for all i and j in the management area. The constraints are the discrete approximation of the governing equation for steady-state contaminant transport (see Equation 45). The form of the continuity constraints is

$$[G]\{C\} - [C_w]\{W\} - [C_z]\{Z\} - [C_v]\{V\} =$$

$$[C_s]\{S\} + [C_u]\{U\} + C_q[I]\{Q\} \qquad \forall \ i \ and \ j \qquad (77)$$

where all vectors are defined generically as in the objective function above or as in Equation (49). Model variables appear in vectors on the left side of the equation sign while the terms on the right reduce to matrices of constants. The terms on the right hand side of the equation represent sources of groundwater contamination not subject to control or other constant sources.

## 8.1.4 Management Constraints

Several types of management constraints are employed which bound the feasible values of decision variables. One form of management constraint establishes minimum values on recharge decision variables. For example minimum levels reflect the existing intensity of polluting land use activities.

Another form of management constraint imposes upper bounds on allowable values of contaminant and recharge decision variable; these constraints respectively ensure the attainment of water quality standards and ensure the satisfaction of various land use regulations. Finally there is a type of management constraint which is applied to

restrict the upper limit on the summed values of recharge decision variables to correspond to available resources (i.e., land, water, etc.).

### 8.1.4.1 Constraints to Incorporate Present Levels of Land Use Activities

The purpose of these management constraints is to ensure that minimum feasible values of decision variables are set to reflect the existing intensity of permanent land use activities in each element, while allowing some surface activities to change from one type to another. Generally, two constraints are used in each element to reflect the combinations and intensities of existing land use activities contributing to the nonpoint source pollution of groundwaters. The formulation allows for two initial land use activities which produce recharge flows $W_{i,j}$ and $Z_{i,j}$. A third type of recharge decision variable ($V_{i,j}$) can enter the solution set if surface activities in any element change from those contributing flows $W_{i,j}$, to surface activities generating recharge flows $V_{i,j}$. The two constraints for each element are written as

$$D_{z_{i,j}} \cdot Z_{i,j} \geq \text{(total existing population engaged in the}$$

$$\text{land use activity generating recharge } Z_{i,j} \quad (78)$$

$$\text{in element i,j)} \quad \forall \text{ i and j}$$

$$D_{W_{i,j}} \cdot W_{i,j} + D_{V_{i,j}} \cdot V_{i,j} \geq \text{(total existing population}$$

$$\text{in the land use activity} \qquad (79)$$

$$\text{generating recharge } W_{i,j} \text{ in}$$

$$\text{element } i,j) \qquad \forall \text{ i and j}$$

Constraint (78) sets minimum values for the decision variables $Z_{i,j}$

to reflect permanent discrete populations responsible for these

elemental recharge flows. Opportunities to switch from a land use

activity producing recharge flow $W_{i,j}$ to one generating flow $V_{i,j}$ are

feasible through constraint (79) as long as the total long term

combination of polluting activities reflect a population greater than or

equal to the original number of people contributing to the elemental

flows $W_{i,j}$.


## 8.1.4.2  Constraints Limiting Maximum Levels of Land Use Activities

Constraints reflecting maximum limits on land use activities

take two forms which either prohibit specific surface activities, or

place limits on maximum allowable levels on land use activities per unit

area of polluting activities. The first constraint type prohibiting

activities is expressed simply as

$$W_{i,j} \leq 0 \qquad \forall \text{ i and j where appropriate} \qquad (80)$$

$$Z_{i,j} \leq 0 \qquad \forall \text{ i and j where appropriate} \qquad (81)$$

$$V_{i,j} \leq 0 \qquad \forall \text{ i and j where appropriate} \qquad (82)$$

The use of constraint (80) and constraint (79) in the model will change existing surface activities producing flows $W_{i,j}$ to those generating flows $V_{i,j}$. Constraint (82) is applied only where the option does not exist to switch land use activities from those generating flow $W_{i,j}$ to those producing flows $V_{i,j}$. Constraint (81) occurs for elements where the land use activity producing flows $Z_{i,j}$ is not permitted.

The second form of management constraint to regulate land use activities is implemented if the combination of surface polluting activities compete for resources which cannot be transported between elements (i.e., land). The formulation of this constraint is

$$L_z \cdot Z_{i,j} + L_w \cdot W_{i,j} + L_v \cdot V_{i,j} \leq L_{s_{i,j}} \qquad \forall \text{ i and j} \qquad (83)$$

where $L_z$ = the resource requirement per unit flow of $Z_{i,j}$;

$L_w$ = the resource requirement per unit flow of $W_{i,j}$;

$L_v$ = the resource requirement per unit flow of $V_{i,j}$;

$L_{s_{i,j}}$ = supply of resource in element i,j.

## 8.1.4.3 Constraints on Transferable Resources

If a combination of land use activities compete for common transferable resources, then these constraints ensure that the optimal combination of surface activities does not require more resources than are presently available. Constraints of this type are formulated as a

summation of all elemental demands for a resource which is set less-than-or-equal-to the available supply of that resource. An example formulation is

$$\sum_{i=1}^{n} \sum_{j=1}^{m} R_z \cdot Z_{i,j} + R_w \cdot W_{i,j} + R_v \cdot V_{i,j} \leq R_s \qquad (84)$$

where $R_z$ = the unit resource requirement per unit recharge flow $Z_{i,j}$;

$R_w$ = the unit resource requirement per unit recharge flow

$W_{i,j}$;

$R_v$ = the unit resource requirement per unit recharge flow $V_{i,j}$;

$R_s$ = resource supply.

## 8.1.4.4  Water Quality Constraints

The last management constraints incorporated in Model III are the water quality constraints. These constraints ensure the optimum pattern of land use activity (which also accommodates the maximum year-round population) which will satisfy desired steady-state groundwater quality standards. The formulation is

$$C_{i,j} \leq Std_{i,j} \qquad \forall \; i \; and \; j \qquad (85)$$

where $Std_{i,j}$ = the discrete steady-state water quality standard for

element i,j.

## 8.1.5 The General Formulation of Model III

The complete formulation of Model III is

Maximize $\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} \left[ D_{w_{i,j}} \cdot W_{i,j} + D_{z_{i,j}} \cdot Z_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \right]$

s.t.

Continuity Constraints:

$$[G]\{C\} - [C_w]\{W\} - [C_z]\{Z\} - [C_v]\{V\}$$

$$= [C_s]\{S\} + [C_u]\{U\} + C_q [I]\{Q\}$$

Management Constraints:

$D_{z_{i,j}} \cdot Z_{i,j} \geq$ (total existing population engaged in the land

use activity generating recharge $Z_{i,j}$ in element

i,j)    ∀ i and j

$D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \geq$ (total existing population engaged

in the land use activity generating

recharge $W_{i,j}$ in element i,j)    ∀ i

and j

$W_{i,j} \leq 0$    ∀ i and j where appropriate

$Z_{i,j} \leq 0$    ∀ i and j where appropriate

$V_{i,j} \leq 0$    ∀ i and j where appropriate

$L_z \cdot Z_{i,j} + L_w \cdot W_{i,j} + L_v \cdot V_{i,j} \leq L_{s_{i,j}}$        ∀ i and j

$\sum\limits_{i=1}^{n} \sum\limits_{j=2}^{m} R_z \cdot Z_{i,j} + R_w \cdot W_{i,j} + R_v \cdot V_{i,j} \leq R_s$

Nonnegativity Constraints:

$$C_{i,j}, \ W_{i,j}, \ Z_{i,j} \ \text{and} \ V_{i,j} \geq 0 \qquad \forall \ i \ \text{and} \ j$$

## 8.2  Application of Model III to Falmouth, Massachusetts

It was planned that Model III would be used to investigate the management of nonpoint source nitrate contamination of groundwater over the towns of Falmouth and Bourne, Massachusetts (depicted in Figure 35). However, to reduce the size of the problem, the optimal pattern of land use activities (using decision variables $Z_{i,j}$, $W_{i,j}$ and $V_{i,j}$ for each element) was determined for Falmouth alone. The continuity constraints and contaminant decision variables ($C_{i,j}$) were retained for both Bourne and Falmouth. Treating the management of nonpoint source groundwater pollution in Falmouth separately from Bourne was determined to be acceptable because of the presence of strong east-west hydraulic gradients (between the military reservation and the coast) which preclude north-south exchange of groundwaters between Falmouth and Bourne.

The optimum placement and intensity of the three composite land use activities were evaluated over the study area. All three surface activities contribute nitrate pollution of groundwaters in Falmouth. The first land use activity generated contaminated recharges flows from the combined domestic and commercial use of municipal well water, septic systems, and lawn fertilizers. Nitrate loads from this type of residential/commercial land use were introduced to the aquifer through

elemental flows represented by decision variables $Z_{i,j}$ for all i and j

elements in Falmouth.

The second source of dispersed nitrate pollution was also
residential/commercial land use; however recharge flows were produced
from the combined domestic and commercial use of on-site well water (as
opposed to municipal water), septic systems, and lawn fertilizers.
Nitrate loads were delivered to the subsurface environment by recharge
flows represented by decision variables $W_{i,j}$ for all i and j elements in

Falmouth.

The third and final groundwater polluting surface activity is
equivalent to the first land use type, but represents sources
transformed from the second type to the first type; commercial and
domestic activities that have abandoned on-site wells for municipal
water. Recharge from these converted sources was represented in values
of decision variables $V_{i,j}$ for all i and j elements in Falmouth. Note

that the recharge from the first and third land use activities
represents artificial recharge of imported water originating from the
municipal water supply source; whereas, the second land use activity
receives its water from on-site wells and recharges it to groundwater
via on-site septic systems.

Other sources of groundwater nitrate contamination in Falmouth
(i.e, sewer exfiltration, natural recharge, and subsurface disposal of
secondary sewage) were treated as constant fluxes in the continuity
constraints (terms on the right-hand-side). Because land use activity

in Bourne was not evaluated with the model, all sources of nonpoint source groundwater nitrate contamination in Bourne were treated as constant nitrate fluxes in the continuity constraints.

### 8.2.1 Data Requirements for Model III

The underlying hydrologic and contaminant transport conditions applied in Model III were the same as those used in Models I and II; consequently, Model III required the same basic input data as Models I and II. Beyond this data, Model III required additional information to define coefficients appearing in the model components.

### 8.2.1.1 Data for the Objective Function

The objective function is a summation of decision variables representing the elemental recharge flows from the three land use activities under evaluation. Associated with each decision variable is a coefficient that converts recharge flows into the population equivalent; hence the value of the objective function is the total population contributing to the sum of the recharge from the three types of land use. The coefficients $D_{w_{i,j}}$, $D_{z_{i,j}}$ and $D_{v_{i,j}}$ from the objective function (76) were calculated using the equations

$$D_{w_{i,j}} = \frac{\Delta x \cdot \Delta y}{\left(1 + K_{c_{i,j}}\right) \cdot q_{c_{i,j}}} \cdot \quad \forall \ i \ \text{and} \ j \qquad (86)$$

$$D_{z_{i,j}} = \frac{\Delta x \cdot \Delta y}{\left(1 + K_{c_{i,j}} + K_{u_{i,j}}\right) \cdot q_{c_{i,j}}} \qquad \forall \ i \ \text{and} \ j \qquad (87)$$

$$D_{v_{i,j}} = \frac{\Delta x \cdot \Delta y}{\left(1 + K_{c_{i,j}} + K_{u_{i,j}}\right) \cdot q_{c_{i,j}}} \qquad \forall \; i \text{ and } j \qquad (88)$$

where all coefficients and constants appearing on the right side of the equal sign are defined in Chapter 7, Equation (68), and the values of the constants and coefficients are specified for the field problem in Table 12.

## 8.2.1.2 Data for the Continuity Constraints

The continuity constraints of Model III embody the same components as those constructed for Model I and II with the following additions: first the recharge vectors {Z} and {W} are brought over to the left side of the equal sign because they are now treated as variable vectors; secondly, the variable recharge vector {V} and source concentration vector $[C_v]$ are added to the constraint set. The components of the source concentration vector are the same as those appearing in the source concentration vector $[C_z]$. The value of the decision variable $V_{i,j}$ represents the elemental flows created from switching domestic and commercial usage from onsite well water to municipal well water; consequently the nitrate concentration in recharge flows are expected to be the same as flows from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers.

8.2.1.3  Data for Present-land-use-activity Constraints

Management model constraints (78) and (79) also require the

coefficients calculated above for the objective function.  These

constraints restrict the lowest feasible values of the recharge decision

variables to reflect the existing elemental population engaged in the

two land use activities; i.e., the elemental population $(P_{z_{i,j}})$, using

municipal water and  septic systems and the elemental population

$(P_{w_{i,j}})$, relying on on-site well water and septic systems.

The right-hand-sides of the constraints (78) and (79) are

respectively $P_{z_{i,j}}$ and $P_{w_{i,j}}$ for all i and j.  The respective

populations were calculated from 1980 U.S. census data as described in

section 7.3.3.1.


8.2.1.4  Data for Maximum-level-land-use-intensity Constraints

Constraints which prohibit specific land use activities were used

only in two elements; those down gradient from the Falmouth sewage

treatment facility the use of on-site well water was prohibited  to

protect the health of local residents; hence, $W_{i,j} \leq 0$  for elements i,j

= 11,2 and 11,3.

All the land use activities under evaluation compete for available

land.  Constraints similar to equation (83) were written to ensure that

optimal values of recharge decision variables reflect feasible

intensities of combined land use activities given real restrictions on

available land. The land use density (or source density) constraints were written for each element in Falmouth in the form of constraint (83). The constraint coefficients represent the number of housing units required per unit flow of recharge ($m^3/d$), and the constant on the right-hand-side is set equal to the permissible number of housing units in each element. The land use density constraints require data on the number of households presently situated in each element plus a priori specification of the residential zoning regulations, and the area of each element.

### 8.2.1.5 Data for Available-resources Constraints

The CCPEDC (1986) provided data for a single constraint to limit total elemental municipal water use ($V_{i,j}$ and $Z_{i,j}$ for all i and j) in Falmouth to the present capacity of the town's water supply (55132 $m^3/d$). With regard to the coefficients used in the general constraint equation (84) $R_z$ and $R_v$ equal one, and $R_w$ equals zero.

### 8.2.1.6 Data for Water Quality Constraints

Water quality constraints were constructed for each element of Falmouth. Data from the steady-state nitrate nitrogen predictions produced in Model I (or Model II) were used with information on municipal well locations and a specified global groundwater nitrate standard.

The elemental nitrate standards varied among elements. For all elements having predicted steady-state nitrate nitrogen concentrations (from 1980 development) in excess of the arbitrary global regional standard, and for all elements containing municipal water supplies, the imposed elemental nitrate standard was the predicted long term concentration produced from Model I for 1980 development patterns. These constraints operated as nondegradation constraints. They precluded the placement of activities in those elements or in other elements if such additional activities would cause further degradation of groundwaters at municipal supplies or in areas already unable to meet the global nitrate nitrogen standard due to existing development. For all other elements, groundwater quality was allowed to be degraded to a prespecified level. This was accomplished by using the global nitrate nitrogen standard in their respective discrete water quality constraints.

8.2.2  Constructing and Solving Management Model III

A Fortran program was written to create a computer file containing the objective function and constraints of Model III. The Fortran program used the groundwater flow model to define fluid velocity coefficients necessary for the construction of the continuity constraints. Data required by the Fortran program included: 1) the regional nitrate standard, 2) the residential housing density regulation applied uniformly over the region in these runs, 3) the steady-state predicted nitrate concentrations from 1980 development (obtained from

solving Model I or II), 4) the area and dimensions of the elements, 5) the elemental recharge rates of all sources, 6) the elemental pumpage rates, 7) the piecewise aquifer transmissivities, 8) the hydraulic and contaminant model boundary conditions, and 9) the nitrate concentrations in all recharge flows.

The assembled model was a linear program containing 79 contaminant decision variables and 102 recharge decision variables. The problem constraint set included 66 continuity constraints, 68 constraints specifying present levels of land use activities, 36 constraints establishing maximum levels of on land use activities, one constraint establishing a limit on available municipal water, and 34 water quality constraints (6 nondegradation and 28 global water quality). Thirteen imaginary contaminant decision variables and thirteen constraints were employed to ensure a no flux condition along the groundwater divide when the central difference molecules were used on boundary nodes. The total number of constraints was 218 and the total number of decision variables was 181.

Several executions of the general model were performed using different global nitrate standards and residential/commercial density regulations. Source density regulations and water quality standards are two recognized methods of controlling the water quality impacts of nonpoint source groundwater pollution. The multiple model runs were intended to reveal insights into the effectiveness of source density restrictions and water quality standards in protecting groundwater resources, plus characteristics of the relationship between source

density restrictions and water quality standards when used together to achieve optimal land use development.

### 8.2.3  Model III Results:  Effects of Water Quality Standards

Multiple runs of Model III were performed to investigate the effects of water quality constraints on the predicted maximum feasible development of the three residential/commercial land use activities in Falmouth.  The development of each composite land use activity was expressed in terms of the population engaged.

During each model run a constant housing density regulation was imposed through constraint equation type (83).  The zoning regulation limited total residential/commercial land use to no greater than 500 households per square kilometer.  For elements containing municipal water supplies (and also elements having a predicted nitrate concentration from Model I which exceed the global standard) the elemental long term nitrate nitrogen predictions from 'present' (1980) land use activity were used as the standards; these are referred to as nondegradation constraints.  In all remaining water quality constraints the imposed right-hand-side was the specified global nitrate nitrogen standard.

Several Model III runs were performed where only the global nitrate standard was changed.  The solutions to the linear programs revealed the influence of global and nondegradation groundwater nitrate standards on residential/commercial development and on protection of groundwater resources in Falmouth.

8.2.3.1  Effects of Nitrate Standards on Land Use Development

Figure 43 shows the maximum feasible population of Falmouth (that is within the boundaries of the modeled area) under different global nitrate standards. The distribution of this population will be discussed later. When the global nitrate standard was varied between 5 and 8 mg/l as N, the capacity of the study area to accommodate more people increased. As the global standard was relaxed, land use activity expanded to take advantage of an apparent increase in the assimilative capacity of the aquifer. Under global groundwater nitrate standards from 5-8 mg/l as N, maximum feasible development (maximum combined residential/commercial land use activity) was constrained by nondegradation for elements containing municipal wells, global water quality, and land use density constraints. The land use density constraints precluded additional growth in areas where existing (1980) development exceeded the zoning restriction (500 households/$km^2$). In addition these source density constraints assured that new development when combined with existing development was never greater than the specified zoning regulation.

When a global nitrate nitrogen standard greater than 8 mg/l was used, the capacity of Falmouth to include more people remained essentially constant (see Figure 43). Maximum feasible development of Falmouth was no longer dependent on the global nitrate standard because additional town growth was restricted by binding water quality constraints (of the nondegradation type) and binding land use density constraints.
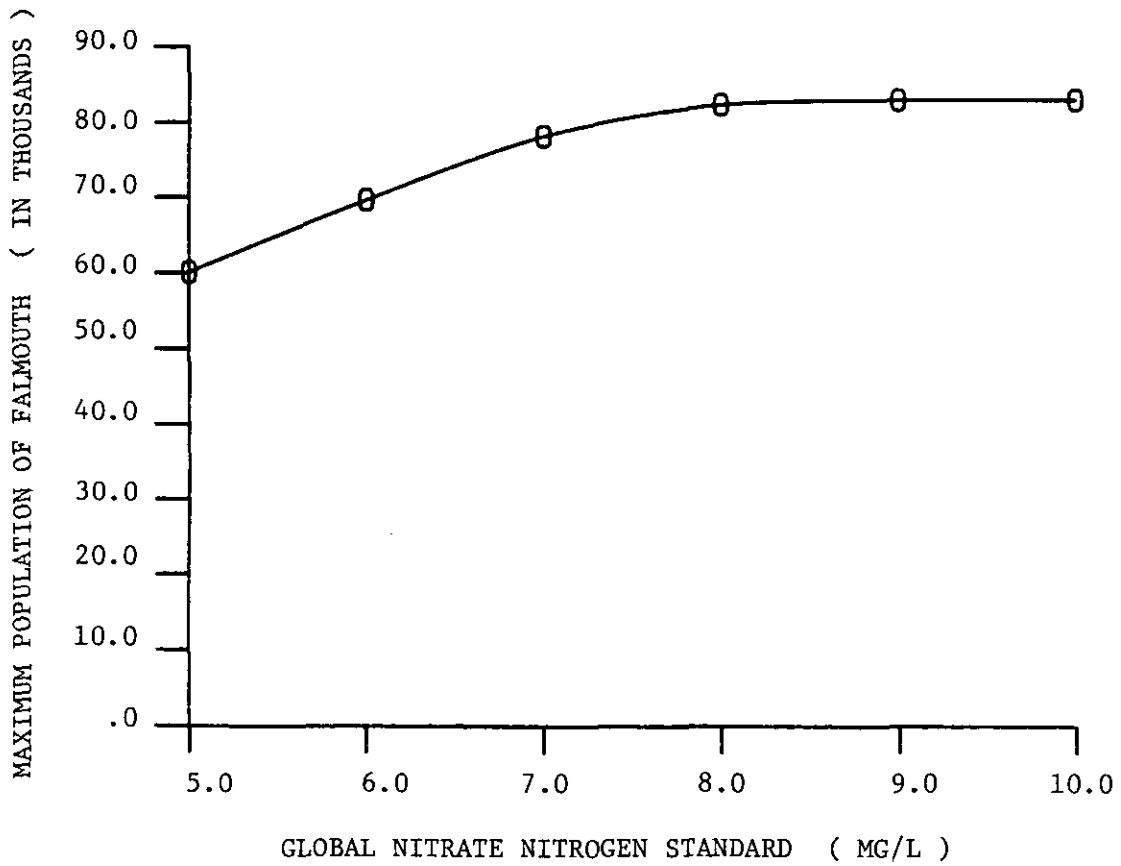
Figure 43. Predicted maximum population of Falmouth as a function of global nitrate nitrogen standards (for development scenarios under the constant land use density limit of 500 houses/sqkm).

From Figure 44, it is evident that as the global nitrate standard is relaxed, the number of water quality constraints which are binding decreases, from a total of 16 down to the five nondegradation water quality constraints embodied in the problem (one at each element containing a municipal water source). Simultaneously, the number of binding land use density constraints doubled. Figure 44 shows that when higher global nitrate standards are used nitrate levels in fewer regions (elements) approach the global standard before development is curtailed by the land use density regulations.

The optimal patterns of residential/commercial development for Falmouth under a range of global nitrate nitrogen standards are depicted in Figures 45-50. Elements which are lightly shaded are designated areas where additional growth (above 1980 levels) is allowed. The presence of a letter 'D' indicates development proceeded to the maximum feasible level allowed by the land use density constraints. Elements with a letter 'N' are predicted to have steady-state nitrate levels which just satisfy water quality constraints for their elements if the optimal development plan is implemented. Municipal water supplies are located in elements containing a letter 'W'.

Figure 45 shows the optimal pattern of development under a land use density restriction of 500 households per square kilometer, a global nitrate nitrogen standard of 5 mg/l, and six nondegradation constraints associated with elements containing municipal water supplies and the Otis wastewater treatment plant $(i,j = 8,4 - 11,7 - 12,3 - 12,4 - 13,4)$. The maximum population associated with this residential/commercial land
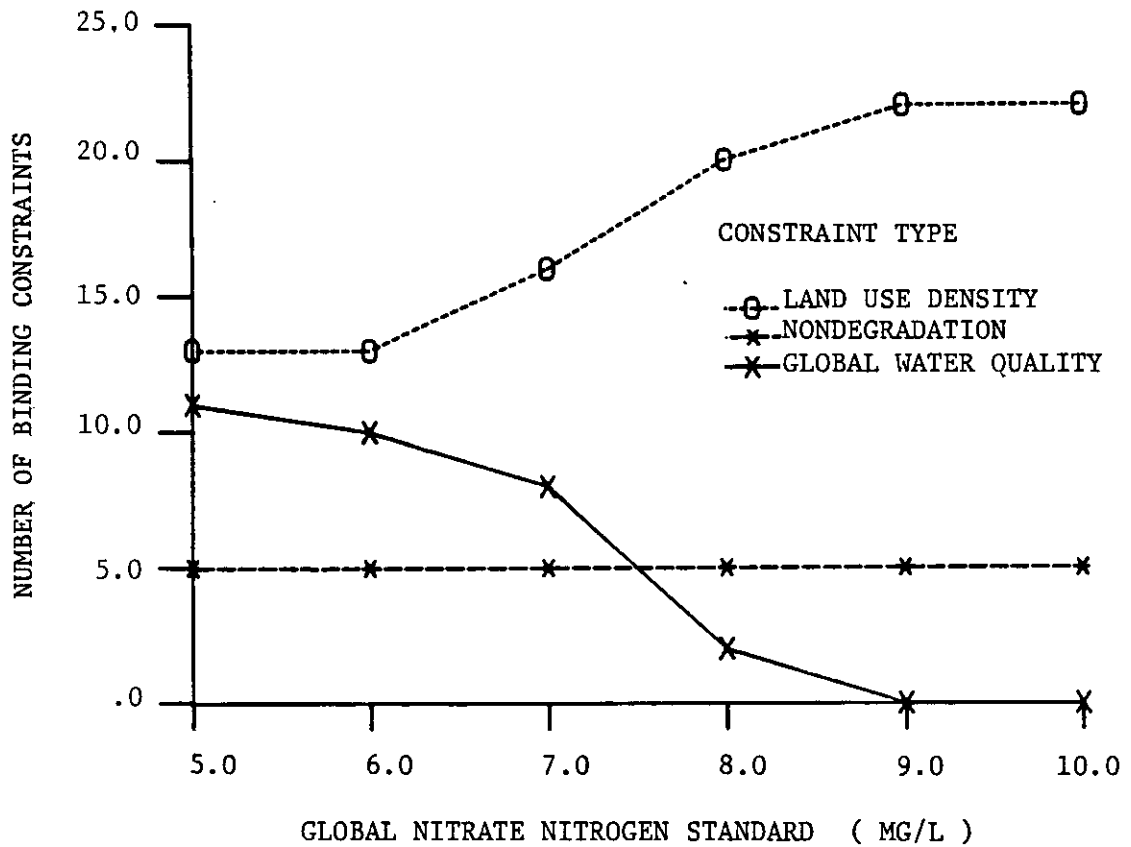
Figure 44.   Number of binding land use density, nondegradation, and
global water quality constraints as a function of the
global nitrate nitrogen standard (in the optimal solutions
of Model III using a constant land use density limit of 500
houses/sqkm).
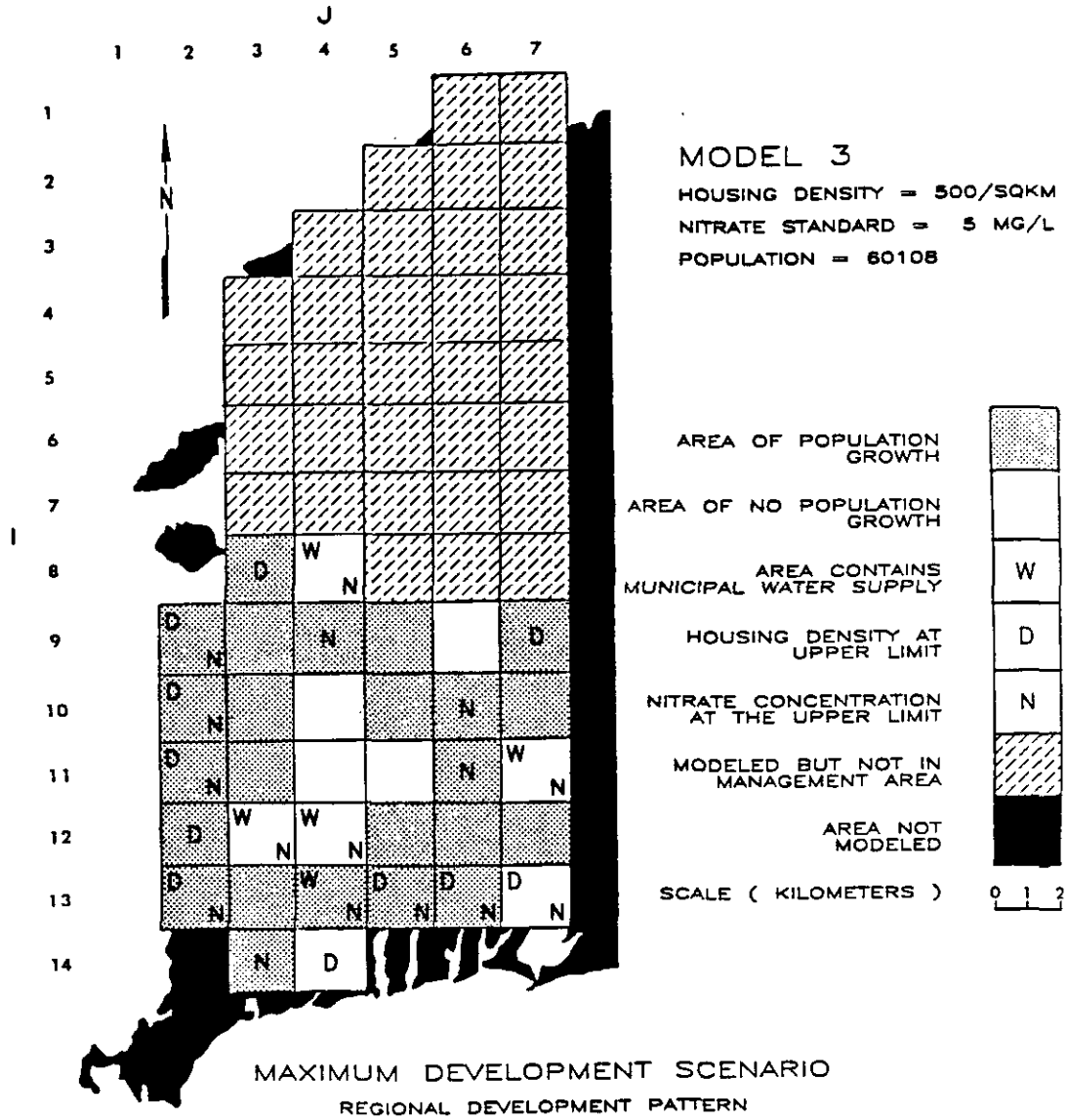
**Figure 45.** Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

Figure 46. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 6 mg/l).

Figure 47. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 7 mg/l).

MODEL 3

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 8 MG/L
POPULATION = 82242

AREA OF POPULATION GROWTH

AREA OF NO POPULATION GROWTH

AREA CONTAINS MUNICIPAL WATER SUPPLY

HOUSING DENSITY AT UPPER LIMIT

NITRATE CONCENTRATION AT THE UPPER LIMIT

MODELED BUT NOT IN MANAGEMENT AREA

AREA NOT MODELED

SCALE ( KILOMETERS )    0  1  2

MAXIMUM DEVELOPMENT SCENARIO
REGIONAL DEVELOPMENT PATTERN

Figure 48.    Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 8 mg/l).

MODEL 3

HOUSING DENSITY = 500/SQKM

NITRATE STANDARD = 9 MG/L

POPULATION = 82838

AREA OF POPULATION GROWTH

AREA OF NO POPULATION GROWTH

AREA CONTAINS MUNICIPAL WATER SUPPLY

HOUSING DENSITY AT UPPER LIMIT

NITRATE CONCENTRATION AT THE UPPER LIMIT

MODELED BUT NOT IN MANAGEMENT AREA

AREA NOT MODELED

SCALE ( KILOMETERS )

MAXIMUM DEVELOPMENT SCENARIO

REGIONAL DEVELOPMENT PATTERN

Figure 49.    Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 9 mg/l).

MODEL 3

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 10 MG/L
POPULATION = 82838

AREA OF POPULATION GROWTH

AREA OF NO POPULATION GROWTH

AREA CONTAINS MUNICIPAL WATER SUPPLY

HOUSING DENSITY AT UPPER LIMIT

NITRATE CONCENTRATION AT THE UPPER LIMIT

MODELED BUT NOT IN MANAGEMENT AREA

AREA NOT MODELED

SCALE ( KILOMETERS )

MAXIMUM DEVELOPMENT SCENARIO
REGIONAL DEVELOPMENT PATTERN

Figure 50.    Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 10 mg/l).

use pattern is 60,108; this population represents a 123 percent increase over the 1980 population for the modeled area of Falmouth.

From Figure 45 it is obvious that additional residential/commercial growth above the 1980 levels is feasible in most elements. Pre-existing land use densities precluded new development in only two elements (i,j = 13,7 and 14,3). For most coastal nodes, development occurred up to the maximum feasible level, and nitrate nitrogen concentrations in the coastal areas increased to the maximum allowable under the global standard. Land use activity for interior nodes (nodes upgradient of the coastal elements) did not reach maximum allowable levels, otherwise nitrogen concentrations in the coastal elements would have exceeded the global nitrate standards. In most elements containing municipal water supplies, no new residential/commercial land use appeared because the nondegradation constraints were binding. Elements located up gradient from Long Pond (i,j = 12,3 - 12,4 - 13,4) were identified by the model to remain at the 1980 development levels to satisfy nondegradation constraints on elements around Long Pond: this region corresponds to the critical recharge zone of Long Pond as delineated by Model II (see Figure 42).

When the global nitrate nitrogen standard was raised to 6 mg/l, Model III results (see Figure 46) showed more residential/commercial development occurring in more elements and more frequently to the maximum level. Again, development in the interior stopped below the maximum allowable level as water quality in most down gradient coastal nodes was just meeting global nitrate standards.

With model runs using a global nitrate nitrogen standard of 7 mg/l, changes in the regional development pattern began to appear (see Figure 47). Under the existing land use density regulations few coastal nodes attained nitrate concentrations of 7 mg/l as N even though with several interior nodes developed to the maximum level allowed by the zoning regulation. Nondegradation constraints continued to be binding at all elements containing municipal supplies because more pollution was being allowed.

The land use development impacts of relaxing the global nitrate standard to 8 mg/l were dramatic (See Figure 48). Only two global nitrate constraints remained in the linear programming solution (i.e., were binding). Land use activities in most elements increased to the limits allowed by the zoning. It would appear, that under the present land use density restriction, development could proceed in most regions to the maximum extent without causing groundwater nitrate concentrations to exceed the 8 mg/l global standard.

When the global nitrate standard was raised to 9 and 10 mg/l as N, model results were the same. Figures 49 and 50 show that the global water quality constraints were not binding anywhere. Development proceeded to the maximum feasible level (82,838 people) in most areas. The only binding water quality constraints were those protecting municipal water supplies from further degradation.

Through the sequence of model runs (Figures 45-50), 71-76 percent of Falmouth model elements received added residential/commercial land use above 1980 levels (see Figure 51). Of these elements, the
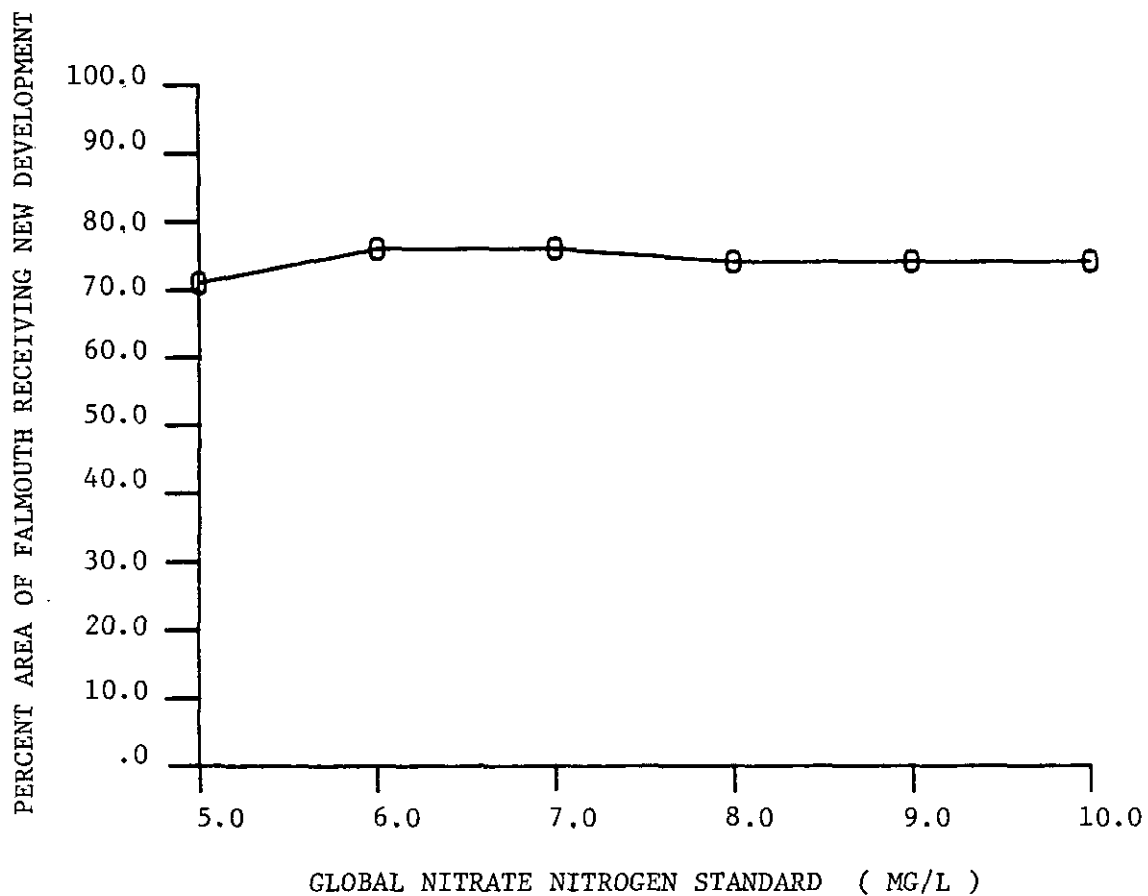
Figure 51.    The percent area of Falmouth to receive additional growth
as a function of global nitrate nitrogen standards (for
maximum development of Falmouth under the constant land use
density limit of 500 houses/sqkm).

percentage in danger of violating water quality standards decreased as the global nitrate level was relaxed (see Figure 52). But, as shown in Figure 53, increasing the allowable groundwater nitrate level encouraged expansion of residential/commercial land use activities to maximum limits.

The discussion to this point has focused on the total combined population due to the three residential/commercial land use types. The model was designed to identify the optimal combinations of the three sources (land use types) that would emerge under various global nitrate standards and residential/commercial land use density restrictions.

The most obvious trend describing a relationship between preferred land use types and global water quality standards is presented in Figure 54. This figure shows that with the relaxation of the global groundwater nitrate standard there was an increase in the percentage of town area where municipal water was selected as the sole form of water used. The model selected land use activities which require on-site well water over equivalent land use types requiring municipal water wherever water quality constraints were binding; that is, when both water quality and housing density constraints defined the boundary, the model selected the source (on-site well water users) which delivered the lowest per capita nitrate loadings.

Use of municipal water generates higher nitrate loads to the aquifer than usage of on-site well water because of the nitrates in the imported municipal water. The management model perceived only that the mass loadings were higher with municipal water and not that the nitrate

Figure 52.    The percent new-growth area of Falmouth polluted to
               allowable nitrate levels as a function of global nitrate
               nitrogen standards (for maximum development of Falmouth
               under the constant land use density limit of 500
               houses/sqkm).

Figure 53.  The percent new-growth area of Falmouth to reach the
allowable land use density limit as a function of global
nitrate nitrogen standards (for maximum development of
Falmouth under the the constant land use density limit of
500 houses/sqkm).

Figure 54.    Percent area of Falmouth entirely dependent on municipal
              water as a function of global nitrate nitrogen standards
              (for maximum development of Falmouth under the constant
              land use density limit of 500 houses/sqkm).

concentration in the recharge was lower. Consequently, the model was unable to distinguish a diluted source from a concentrated source because the linear continuity equations could not incorporate the nonlinear hydraulic effects induced from disposing large volumes of dilute wastewater to the aquifer. The result of this modeling limitation was a consistent selection of the most conservative land use development patterns wherever binding water quality constraints precluded additional growth.

The general pattern of the three evaluated residential/commercial land use activities are illustrated in a series of figures (55-60) which span the multiple solutions of Model III using various global nitrate standards ranging from 5-10 mg/l as N. The figures show in which elements municipal water was the principal source of water. On-site well development was prohibited in elements $i,j$ = 11, 2 and 11,3 because these nodes were situated down gradient from the Falmouth wastewater treatment facility. In a few elements, residential/commercial land use activities requiring on-site well water were introduced where they never existed before (i.e., in elements $i,j$ = 13,2 and 14,3); this occurred when the optimum development pattern depended on satisfying global nitrate standards. Element $i,j$ = 12,2 never allowed land use activities needing on-site well water. Land use changes occurred in a few elements where existing residential/commercial activities dependent on on-site well water were converted to the municipal water supply (i.e., elements $i,j$ = 8,3 - 9,3 - 10,3 - 12,5 - 12,6 - 12,7 and 14,4). Most conversions from land use activities generating recharge flows $W_{i,j}$ (from on-site

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN
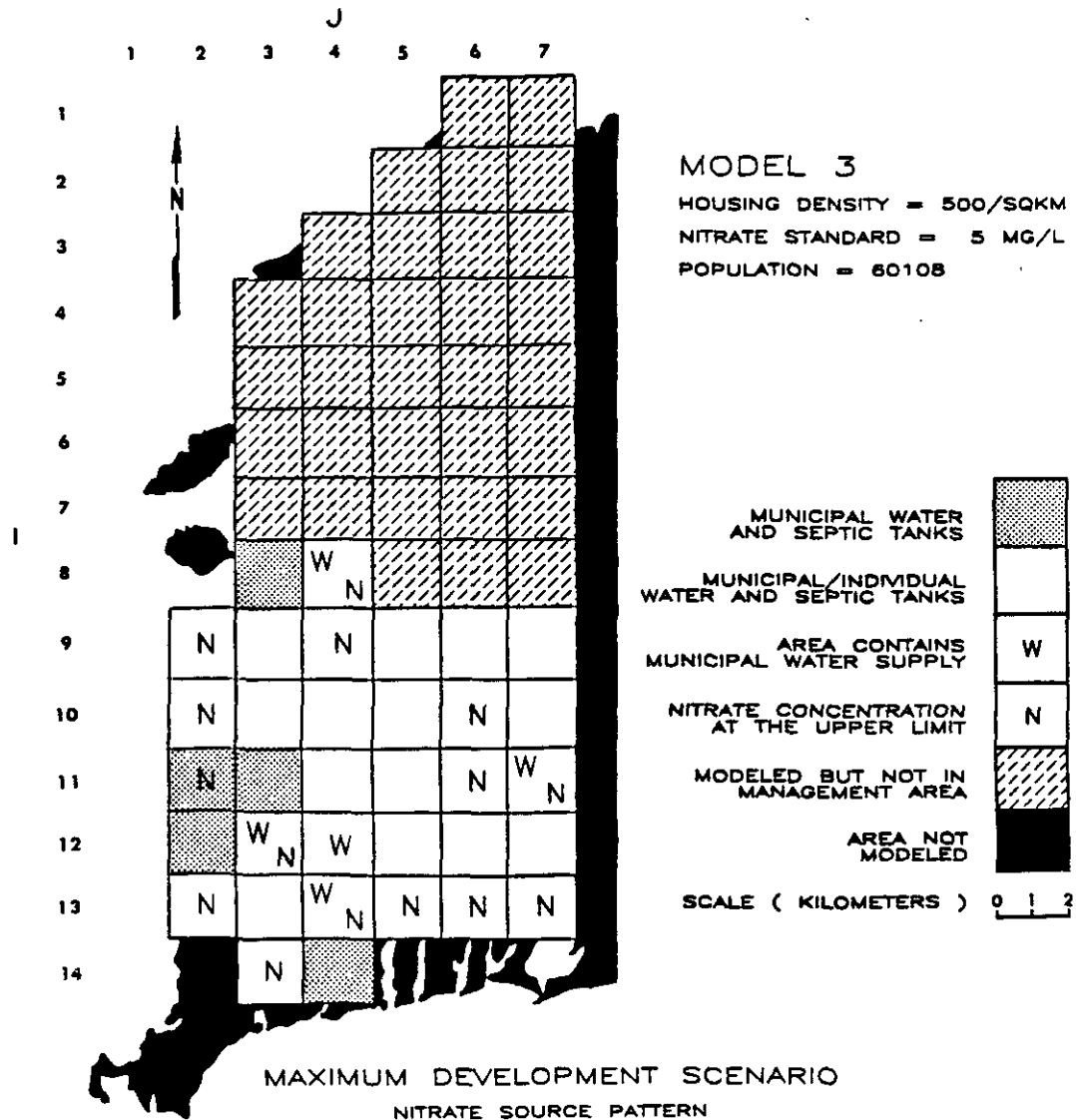
Figure 55.    Water usage patterns for residential/commercial land use
in Falmouth (for a specified land use density limit of 500
houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).

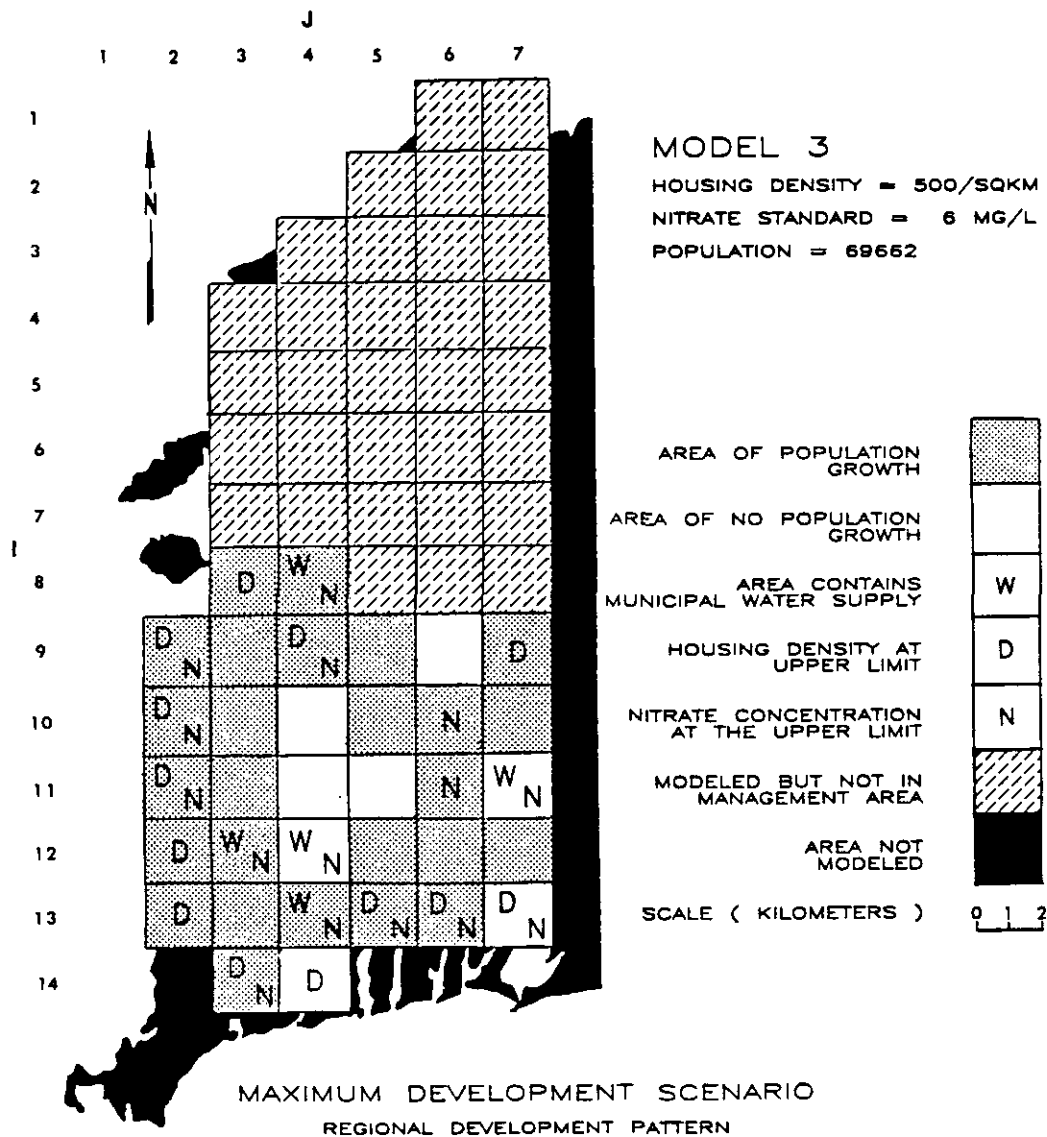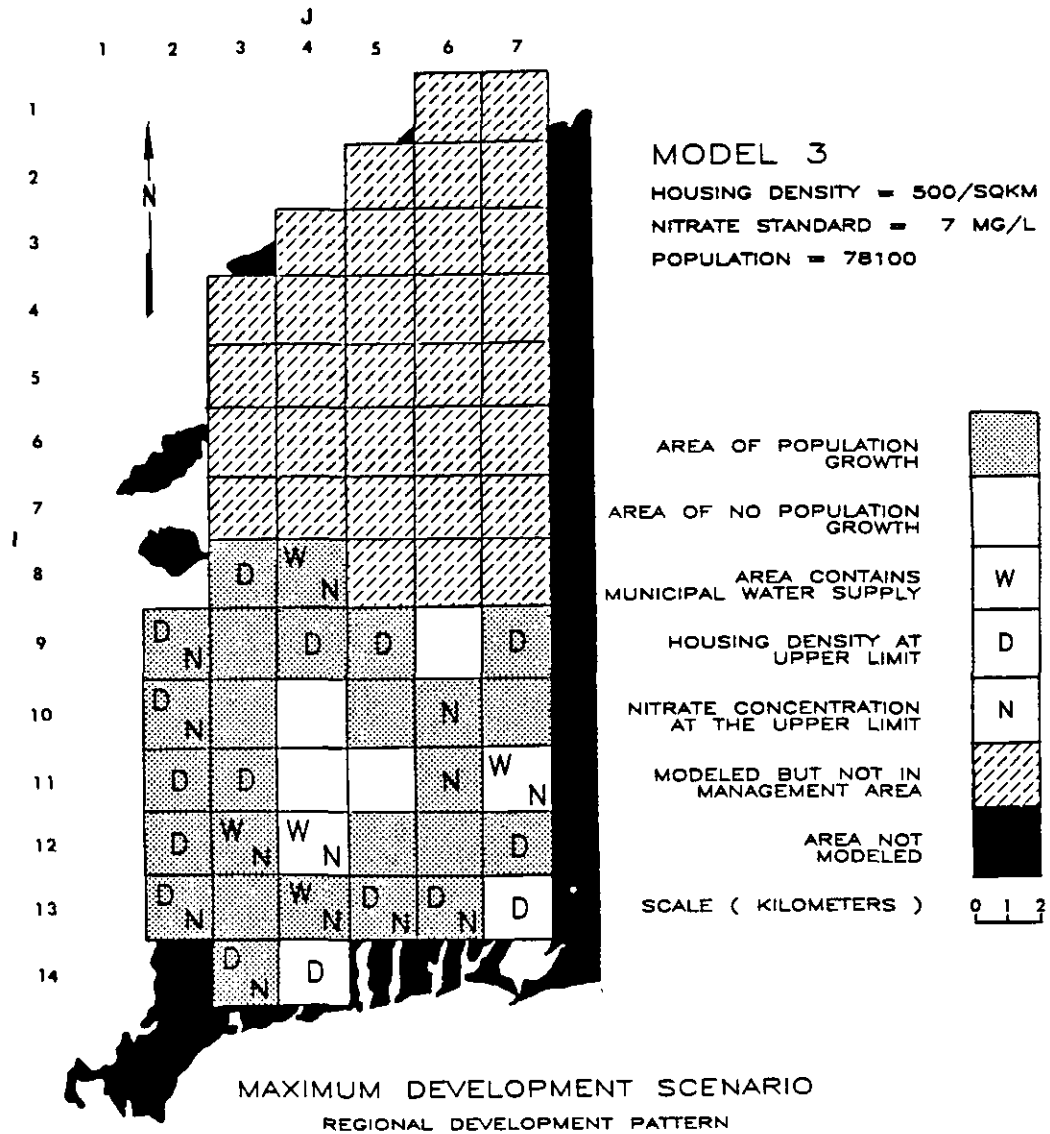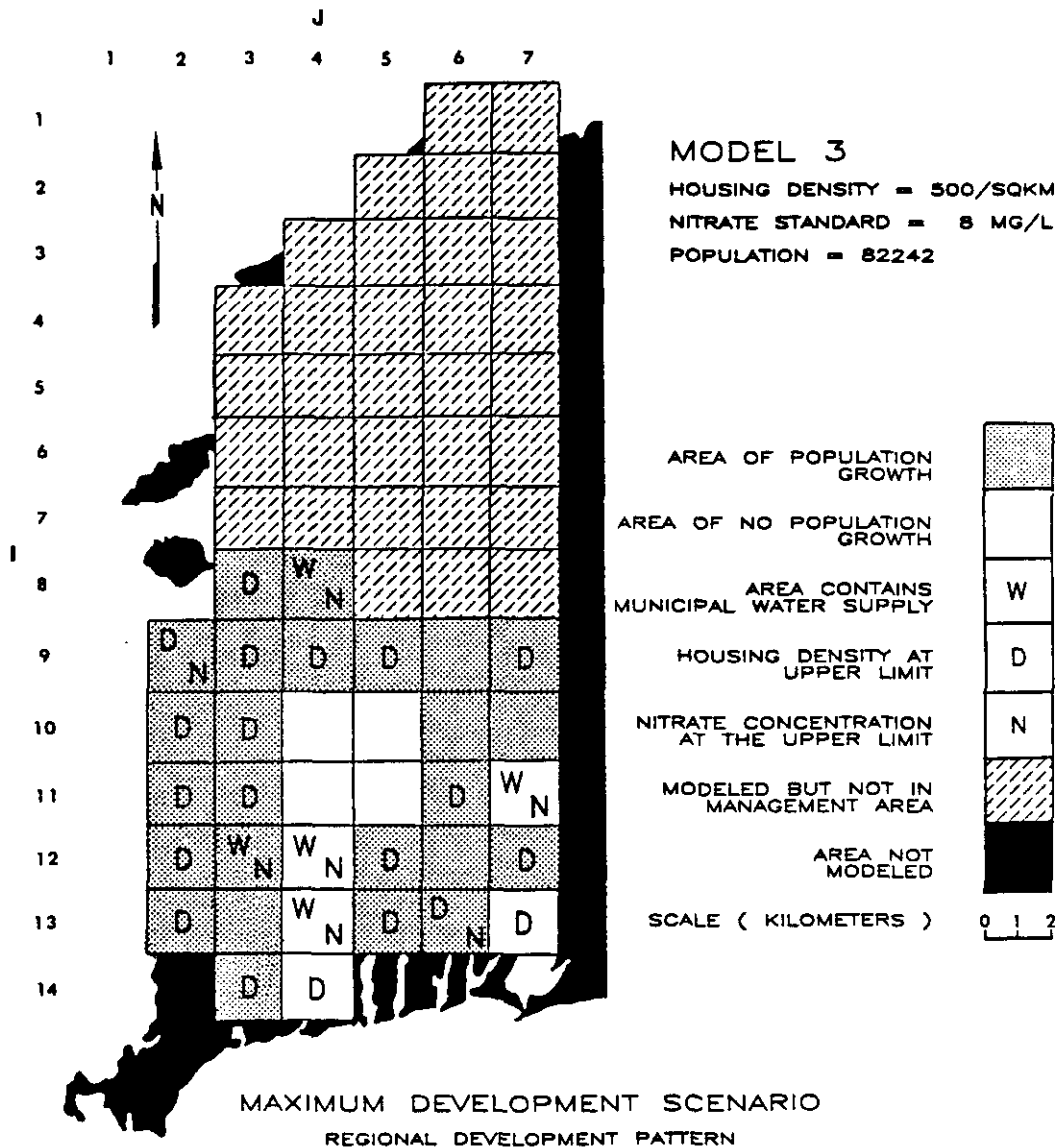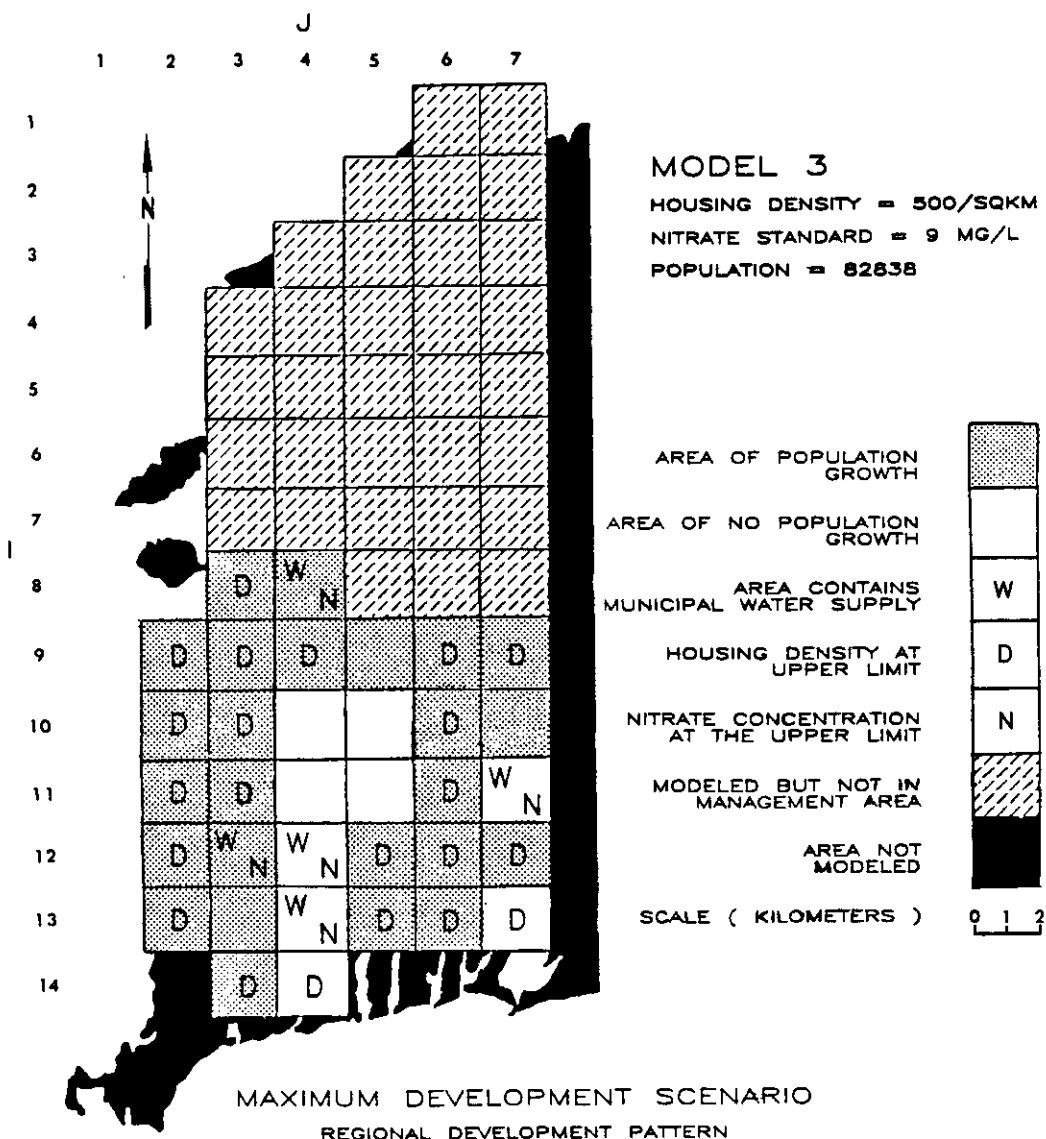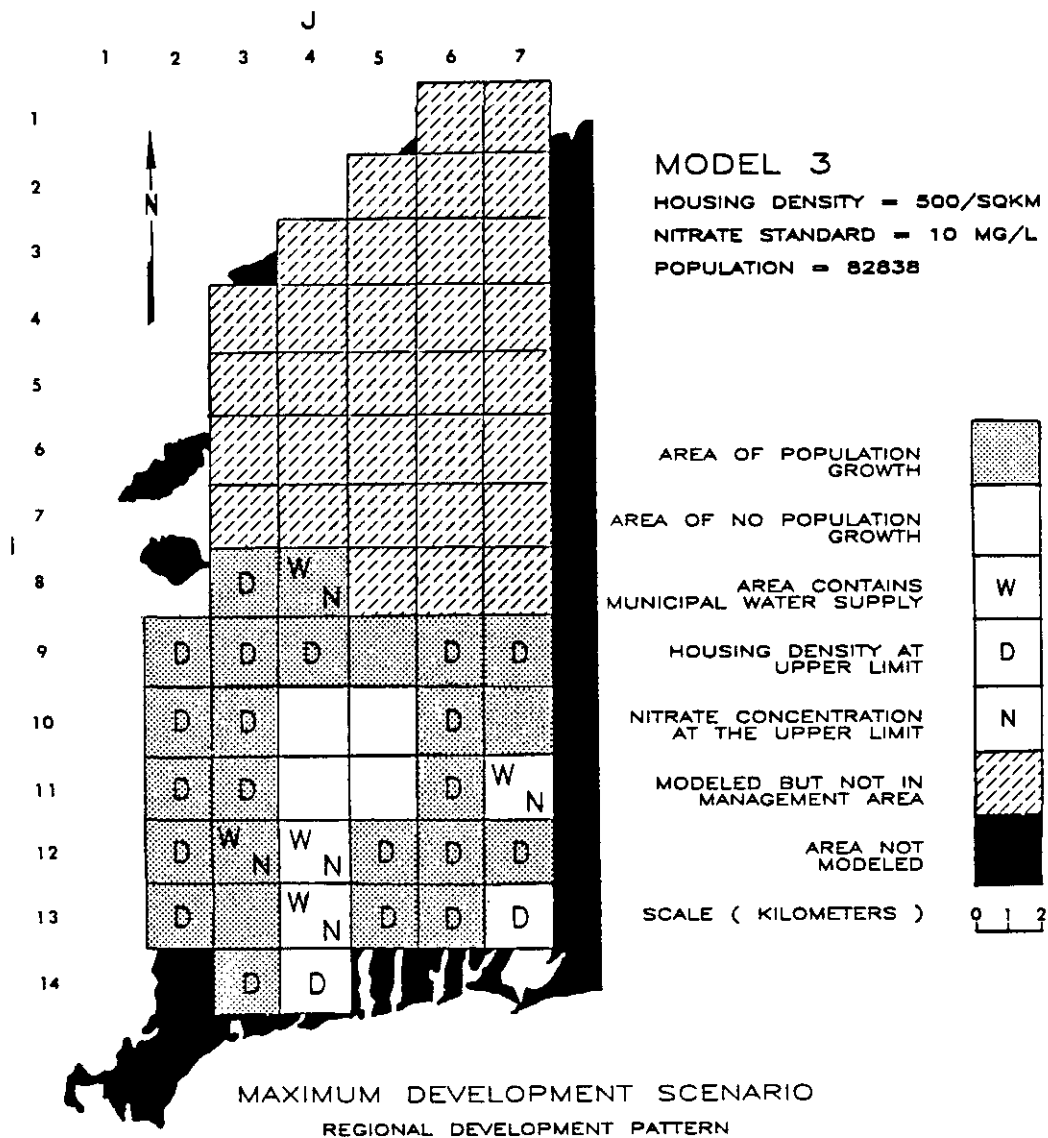Figure 56.    Water usage patterns for residential/commercial land use in
             Falmouth (for a specified land use density limit of 500
             houses/sqkm and a global nitrate nitrogen standard of 6
             mg/l).

MODEL 3

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 7 MG/L
POPULATION = 78100

MUNICIPAL WATER
AND SEPTIC TANKS

MUNICIPAL/INDIVIDUAL
WATER AND SEPTIC TANKS

AREA CONTAINS
MUNICIPAL WATER SUPPLY

NITRATE CONCENTRATION
AT THE UPPER LIMIT

MODELED BUT NOT IN
MANAGEMENT AREA

AREA NOT
MODELED

SCALE ( KILOMETERS )  0  1  2

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN

Figure 57.    Water usage patterns for residential/commercial land use in
Falmouth (for a specified land use density limit of 500
houses/sqkm and a global nitrate nitrogen standard of 7
mg/l).

**MODEL 3**

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 8 MG/L
POPULATION = 82242

MUNICIPAL WATER
AND SEPTIC TANKS

MUNICIPAL/INDIVIDUAL
WATER AND SEPTIC TANKS

AREA CONTAINS
MUNICIPAL WATER SUPPLY    W

NITRATE CONCENTRATION
AT THE UPPER LIMIT    N

MODELED BUT NOT IN
MANAGEMENT AREA

AREA NOT
MODELED

SCALE ( KILOMETERS )    0  1  2

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN

Figure 58.    Water usage patterns for residential/commercial land use in
Falmouth (for a specified land use density limit of 500
houses/sqkm and a global nitrate nitrogen standard of 8
mg/l).
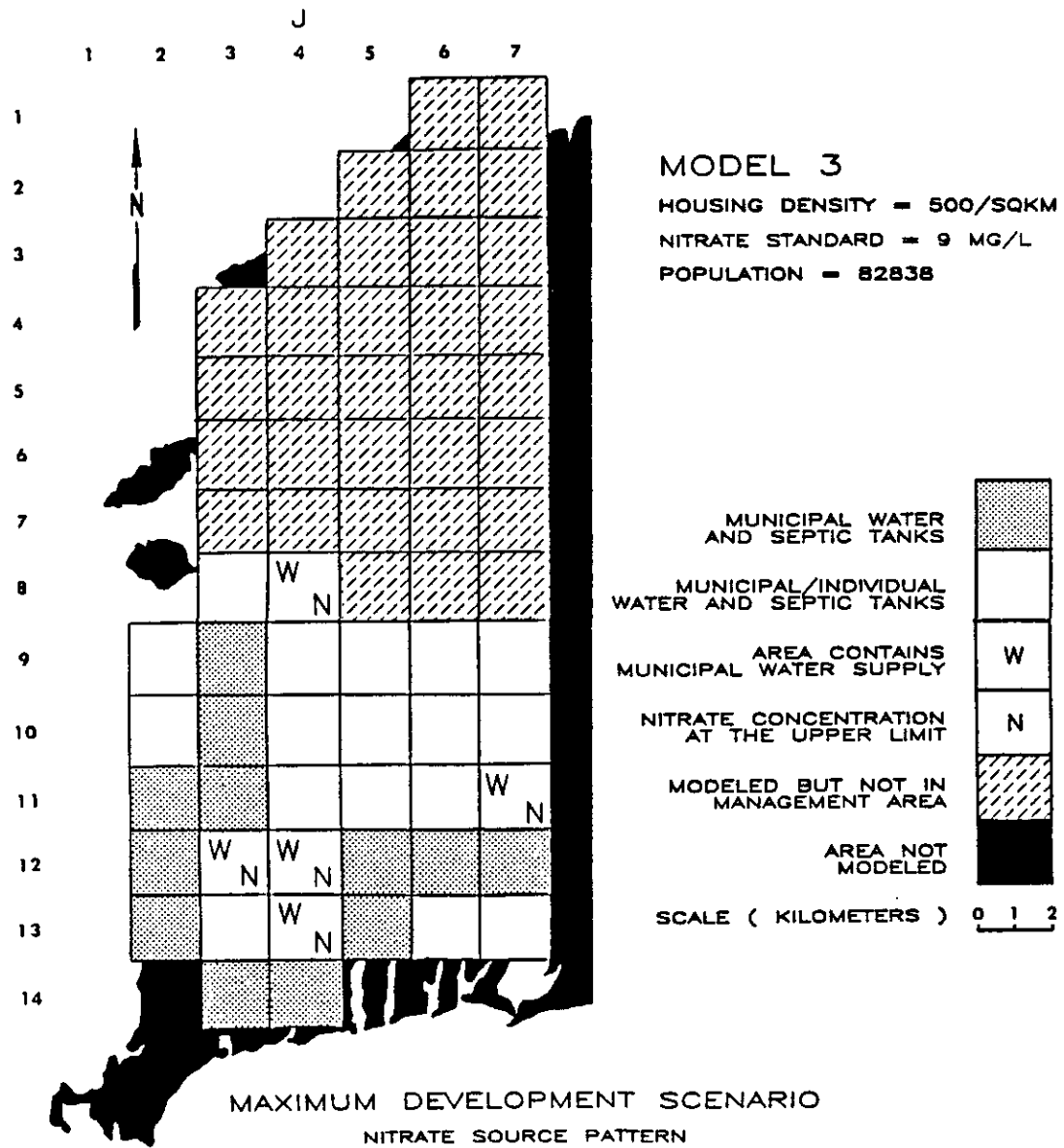
Figure 59. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 9 mg/l).

MODEL 3

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 10 MG/L
POPULATION = 82838

MUNICIPAL WATER
AND SEPTIC TANKS

MUNICIPAL/INDIVIDUAL
WATER AND SEPTIC TANKS

AREA CONTAINS
MUNICIPAL WATER SUPPLY

NITRATE CONCENTRATION
AT THE UPPER LIMIT

MODELED BUT NOT IN
MANAGEMENT AREA

AREA NOT
MODELED

SCALE ( KILOMETERS )   0  1  2

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN

Figure 60.   Water usage patterns for residential/commercial land use in
Falmouth (for a specified land use density limit of 500
houses/sqkm and a global nitrate nitrogen standard of 10
mg/l).

well water usage) to activities producing flows $V_{i,j}$ (from municipal

water usage), occurred after the global nitrate nitrogen standard was

increased above 7 mg/l.

The model never increased use of municipal water in an element

without first connecting all on-site well water users to the town water

distribution system; two observation were drawn from this.  First, any

areas depicted in Figures 55-60 as having only municipal water users

were also the only elements where residential/commercial use of

municipal water increased.  The second observation was that all other

elements known to acquire new development saw an increase in only on-

site well water use.  Therefore, wherever development occurred either

on-site well water or municipal water usage increased but not both.

The constraint specifying an upper limit on available town water

(see equation 84) was never binding in any of the model runs.  At most,

maximum municipal water use never amounted to more than 36 percent of

the available supply.

8.2.3.2  Effects of Nitrate Standards on Groundwater Protection

Groundwater protection can be effected through the enforcement of

groundwater quality standards and land use zoning regulations.  This

section discusses the effects of nitrate nitrogen standards on efforts

to protect the long term availability of groundwaters and the

development tradeoffs associated with relaxing these standards.

The protection of groundwater was evaluated in terms of groundwater

nitrate changes effected through additional development above 1980

levels. The regional preservation of groundwater was achieved through limited land use development over Falmouth in a pattern indirectly determined by binding land use density constraints and directly through binding nondegradation water quality constraints. Figure 61 shows the average (i.e., average overall Falmouth elements) steady-state nitrate nitrogen concentration in Falmouth under maximum feasible development conditions for a specified global nitrate standard and a residential land use density restriction of 500 households per square kilometer. Average nitrate nitrogen concentrations slightly exceeded the 5 mg/l goal set by the CCDEPC (1978) after the global nitrate standard was elevated to 8 mg/l as N. As expected, the percent area of Falmouth polluted to allowable nitrate levels decreases as the global nitrate nitrogen standard is elevated (see Figure 62). However, 15 percent of Falmouth corresponds to regions protected with nondegradation constraints (which are presently at the permissible limit of nitrate contamination and are consequently, not affected by relaxing the global nitrate nitrogen standard).

Nondegradation constraints for elements containing municipal water supplies preserved water quality at steady-state levels predicted from present (1980) land use activities. Additional development was all but precluded near municipal water supplies which forced most development to occur down gradient and between Falmouth's water supplies. The nondegradation constraints were a dominant factor affecting the pattern of optimal development in the study area.
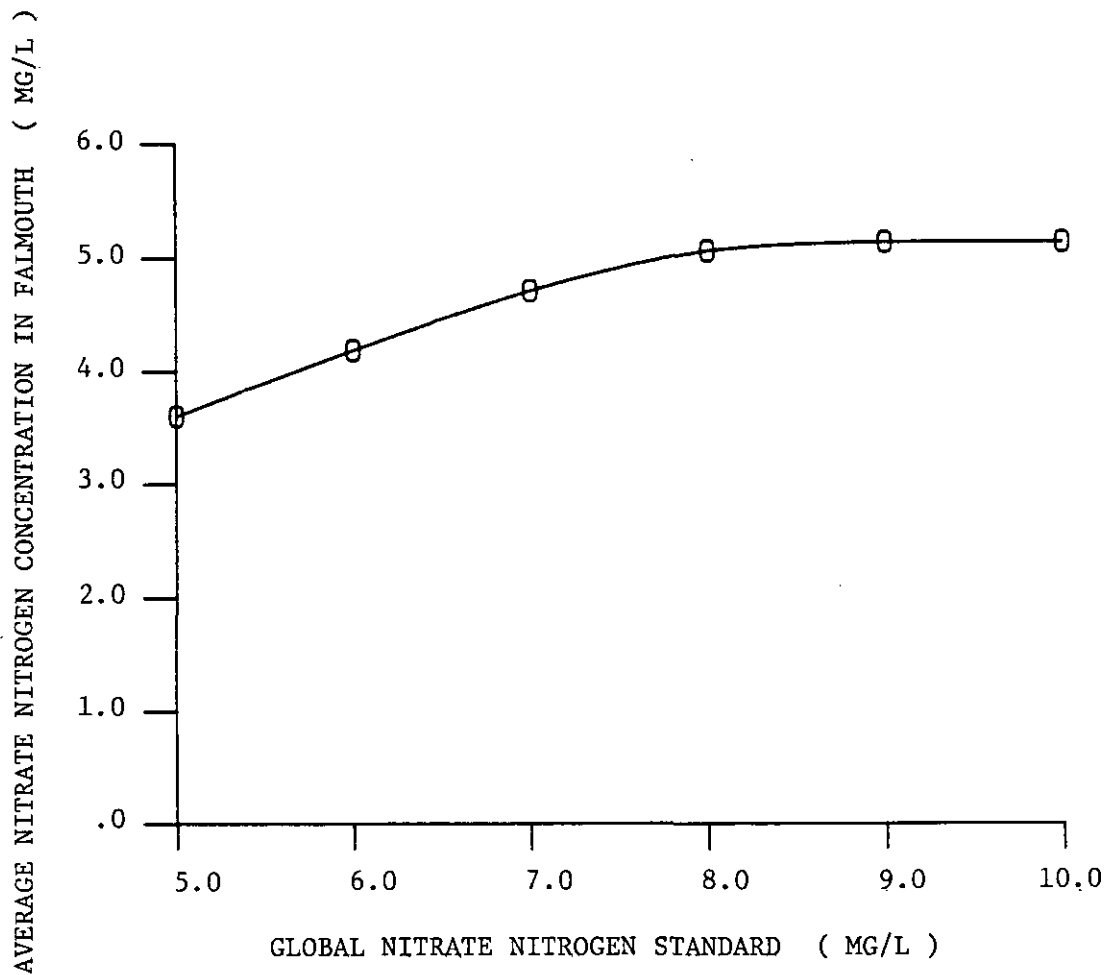
Figure 61. Average nitrate nitrogen concentration in Falmouth as a
function global nitrate nitrogen standards (for maximum
development scenarios under the constant land use density
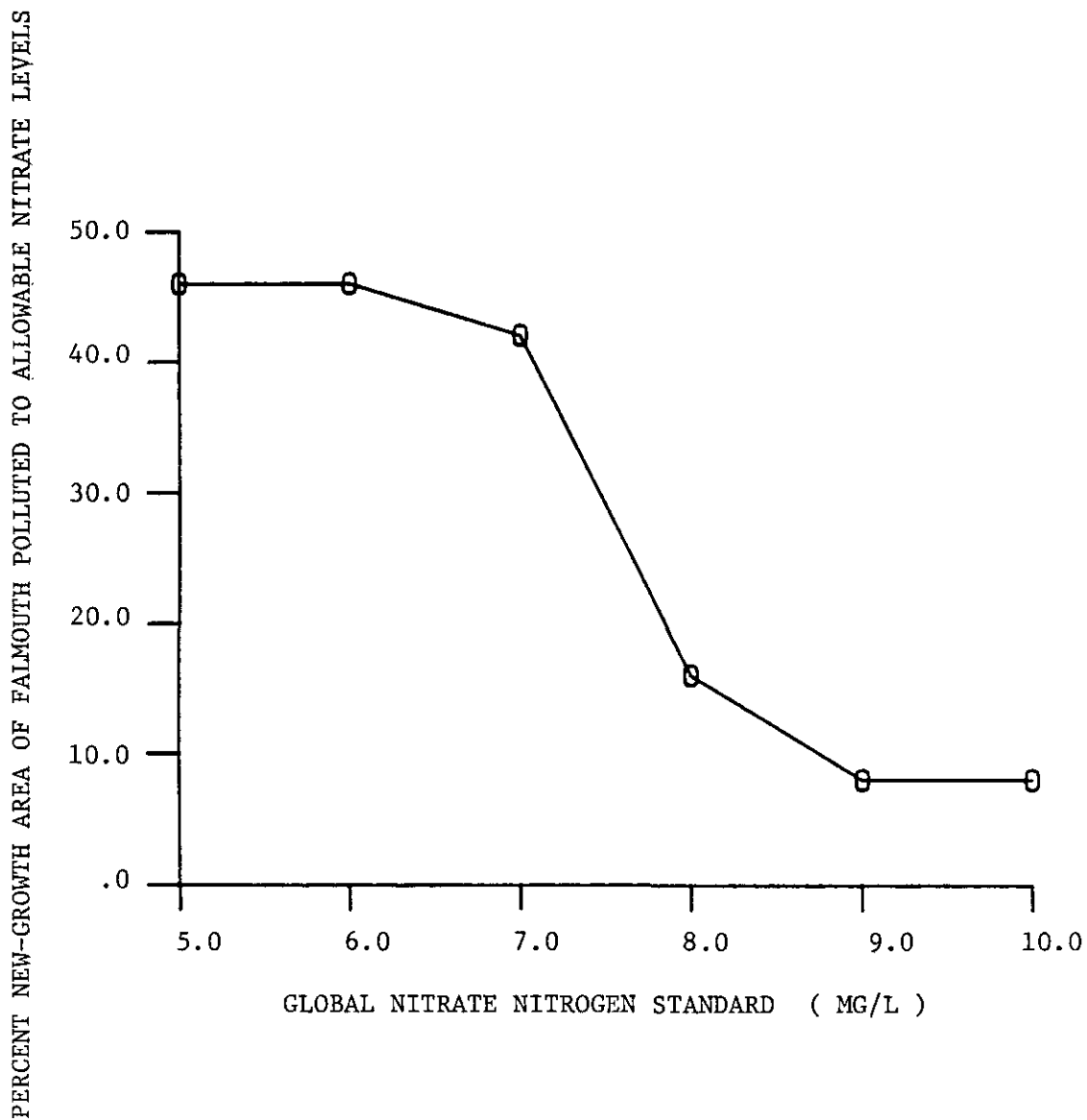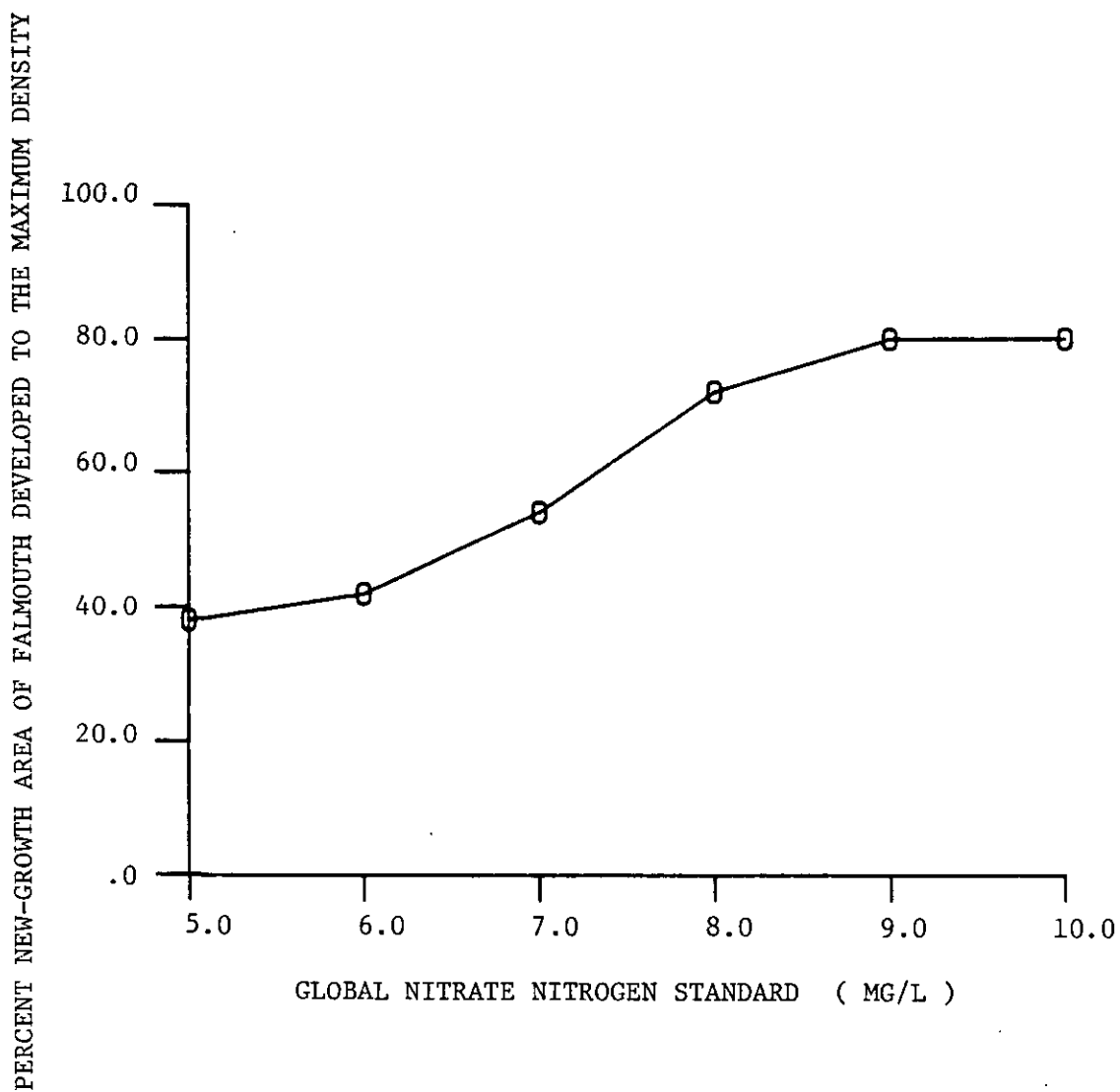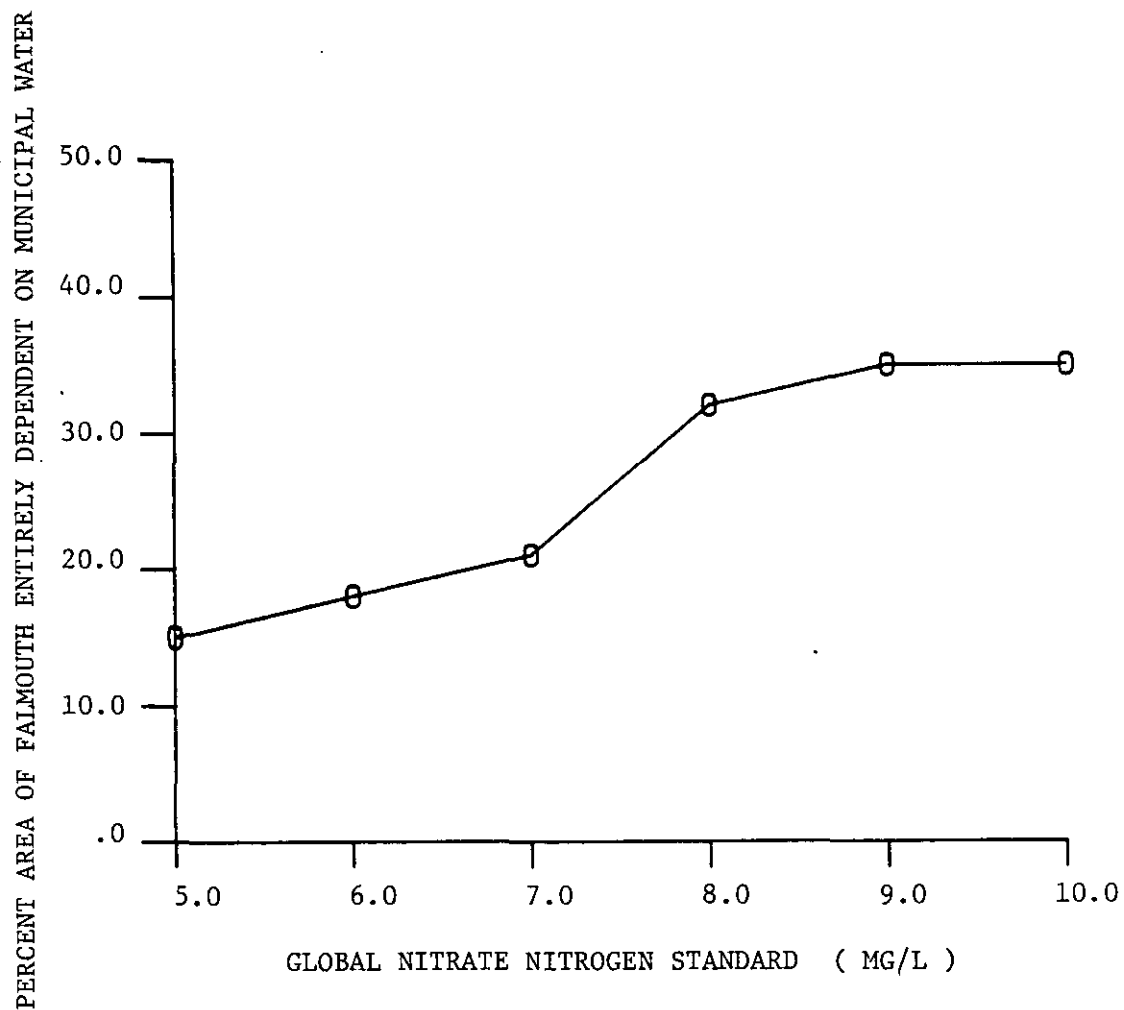limit of 500 houses/sqkm).
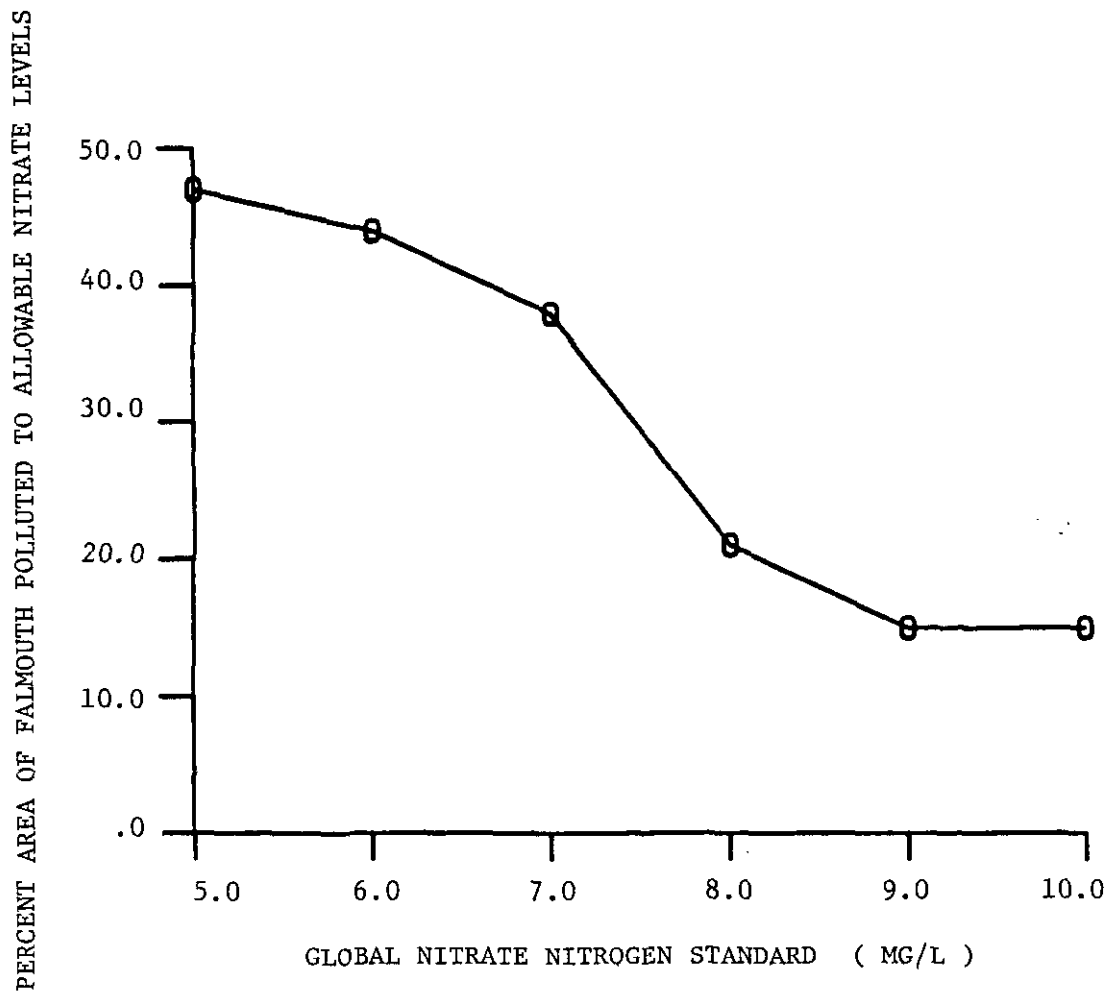
Figure 62.   Percent area of Falmouth polluted to allowable nitrate
             levels as a function of global nitrate nitrogen standards
             (for maximum development scenarios under the constant land
             use density limit of 500 houses/sqkm).

Global water quality constraints protected groundwater quality near
the coasts by limiting development upgradient where expanded
residential/commercial activities were possible without affecting
nitrate changes in municipal water supplies.

Figures 63-68 illustrate the steady-state nitrate nitrogen
distributions in Falmouth from maximum development under various global
nitrate nitrogen standards. Most evident from leafing through the
figures are the preservation of water quality around municipal supplies
(centered at nodes 8,4-12, 3-12, 4-13,4 and 11, 7) and extensive nitrate
pollution between supplies. Notice that in every figure elemental
groundwater nitrate levels were less than the specified global standard.

The development tradeoffs associated with relaxing land use density
and water quality constraints were investigated through the optimal
values of the dual variables. Each model run produced values for dual
variables associated with each model constraint. For the binding
constraints, the values of the dual variables were used to interpret
marginal changes in the maximum population of Falmouth for unit
relaxations of tne constraints. Figure 69 presents the values of the
dual variables associated with binding land use density constraints.
These were obtained from the solution of Model III with a housing
density restriction of 500 household per square kilometer and a regional
nitrate nitrogen standard of 5 mg/l. The number appearing in each
element represents the additional people, which could be located in
Falmouth, if one more household were added to that element. To relax
the housing density constraints of elements which contain no number,
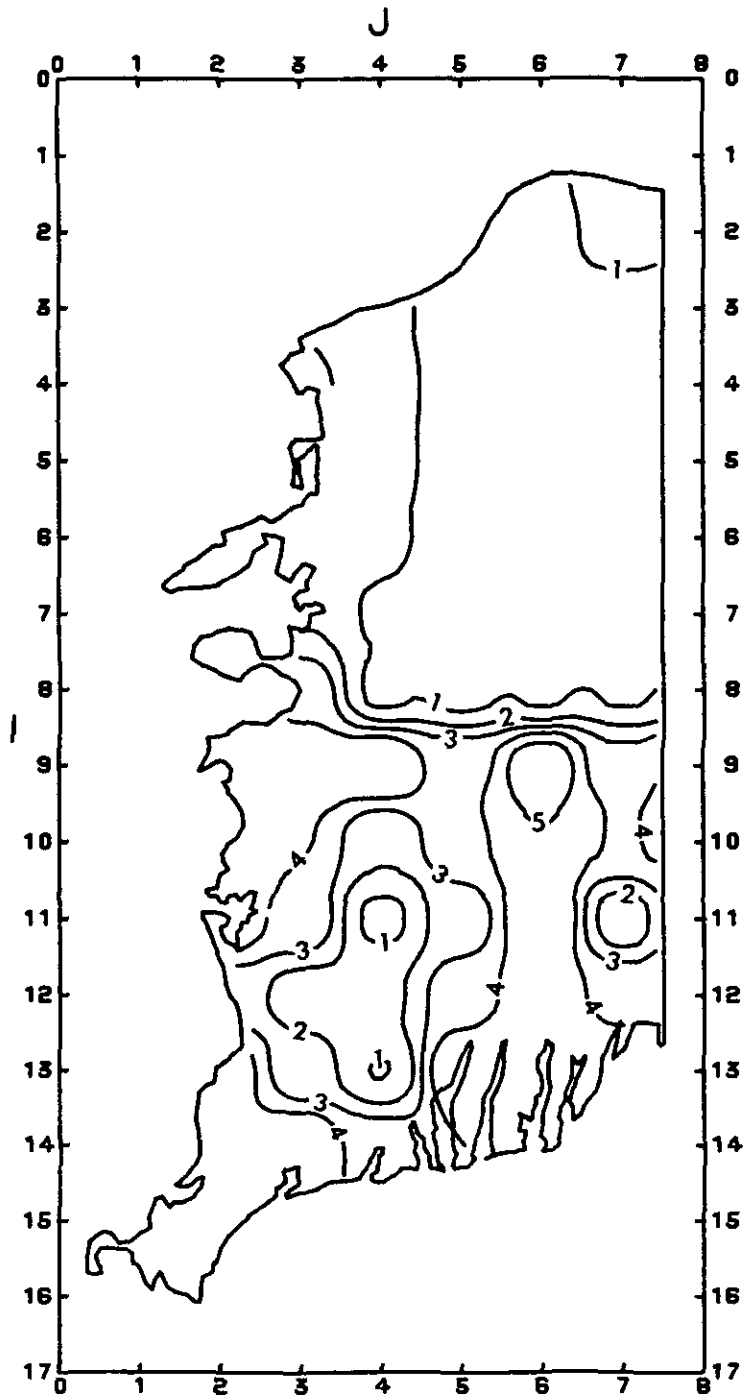
Figure 63.    Predicted steady-state nitrate nitrogen concentration
             (mg/l) contours over Bourne and Falmouth (at maximum
             feasible development of Falmouth for a specified land use
             density limit of 500 houses/sqkm and a global nitrate
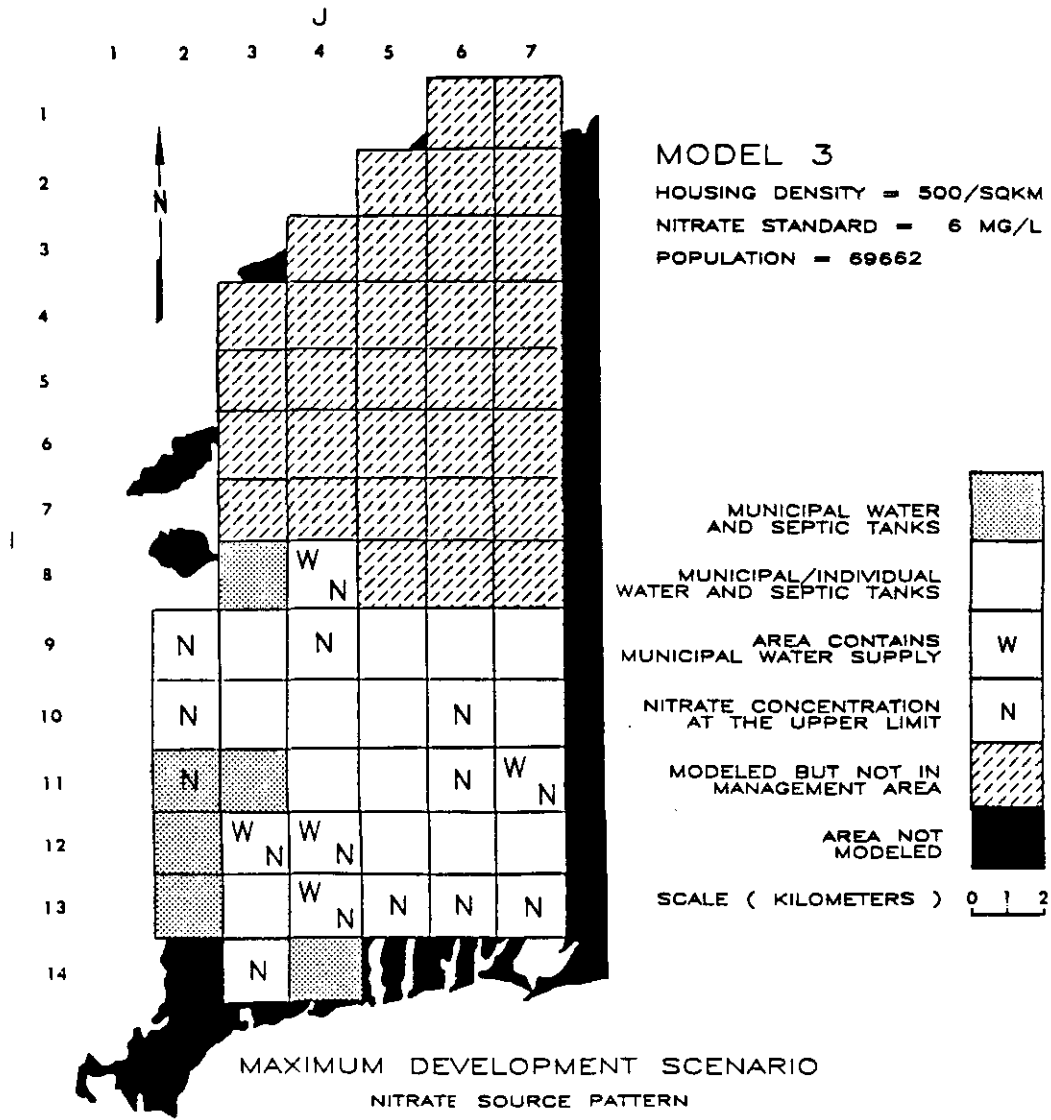             nitrogen standard of 5 mg/l).
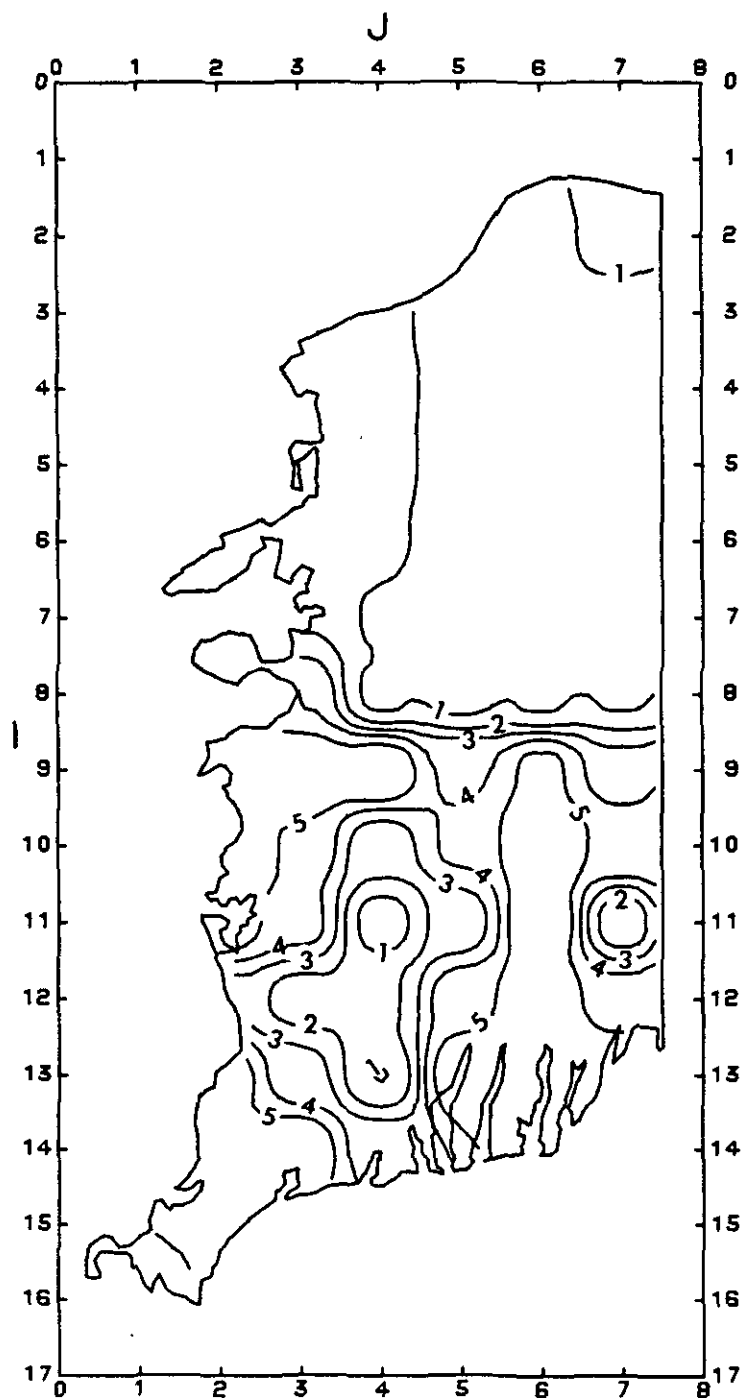
Figure 64.    Predicted steady-state nitrate nitrogen concentration
              (mg/l) contours over Bourne and Falmouth (at maximum
              feasible development of Falmouth for a specified land use
              density limit of 500 houses/sqkm and a global nitrate
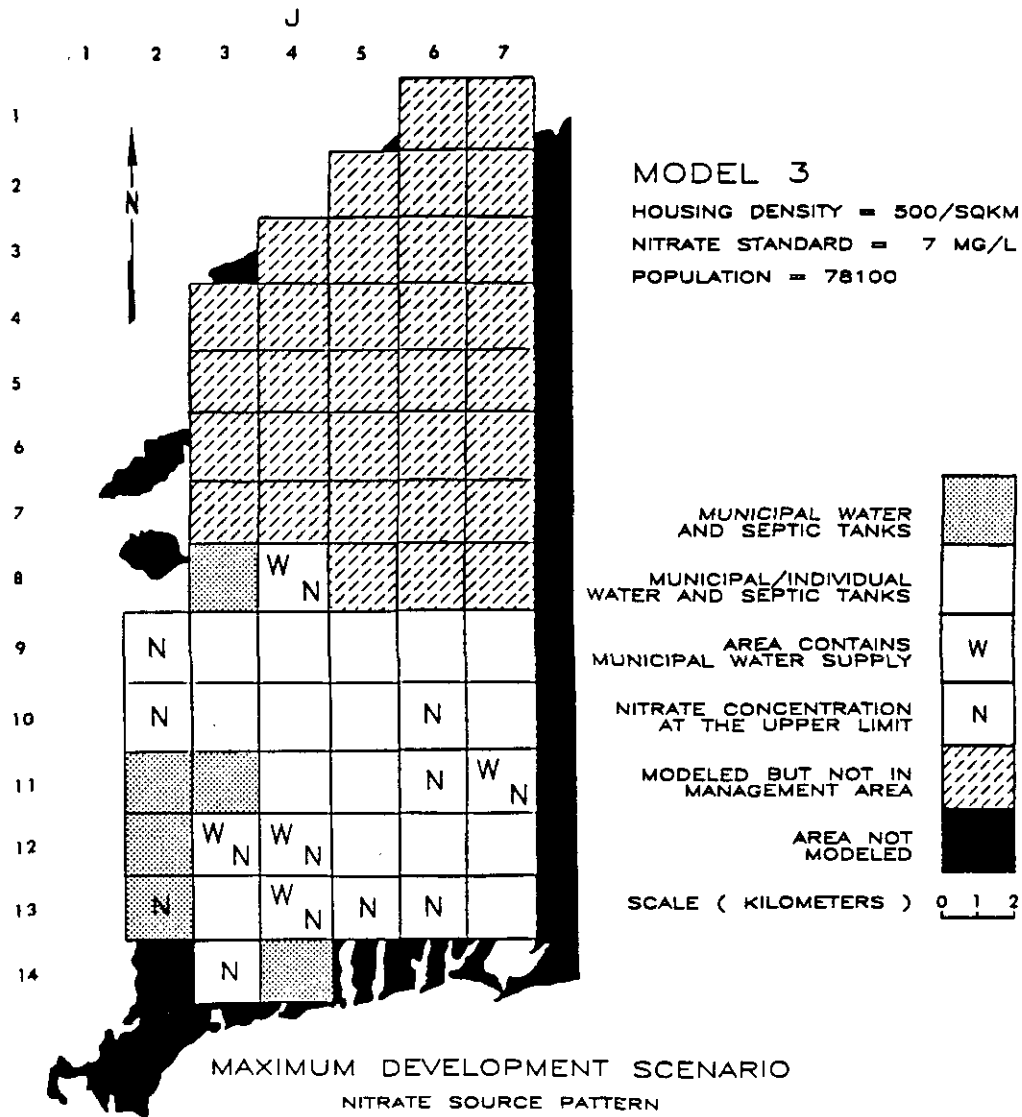              nitrogen standard of 6 mg/l).
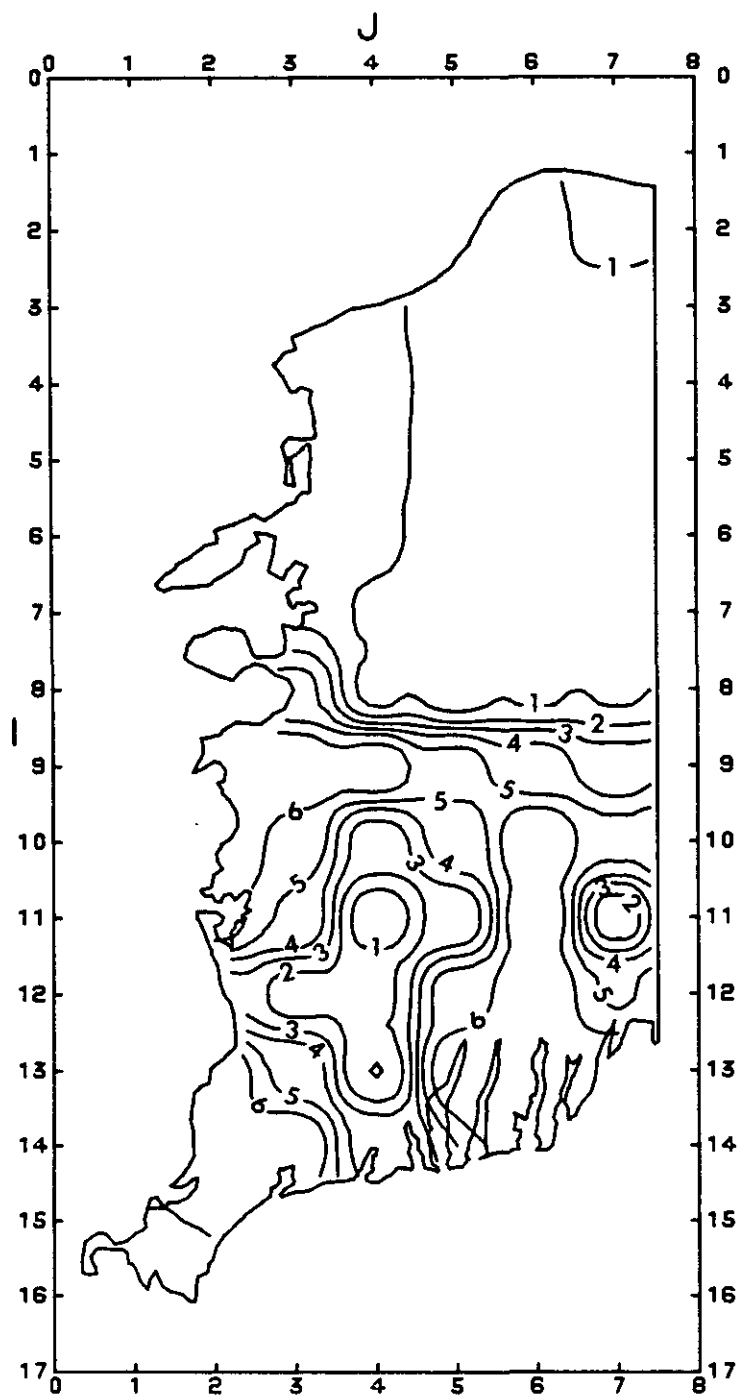
Figure 65. Predicted steady-state nitrate nitrogen concentration
(mg/l) contours over Bourne and Falmouth (at maximum
feasible development of Falmouth for a specified land use
density limit of 500 houses/sqkm and a global nitrate
nitrogen standard of 7 mg/l).
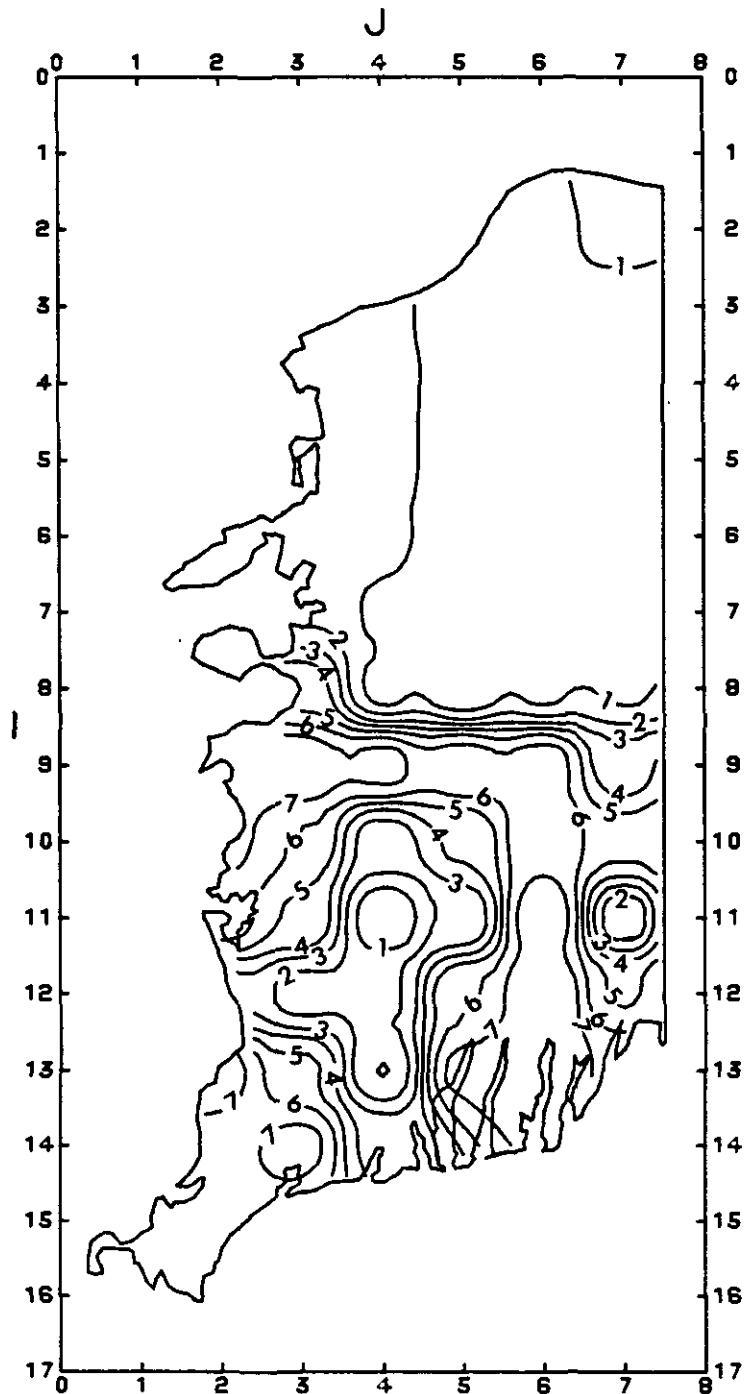
Figure 66.   Predicted steady-state nitrate nitrogen concentration
(mg/l) contours over Bourne and Falmouth (at maximum
feasible development of Falmouth for a specified land use
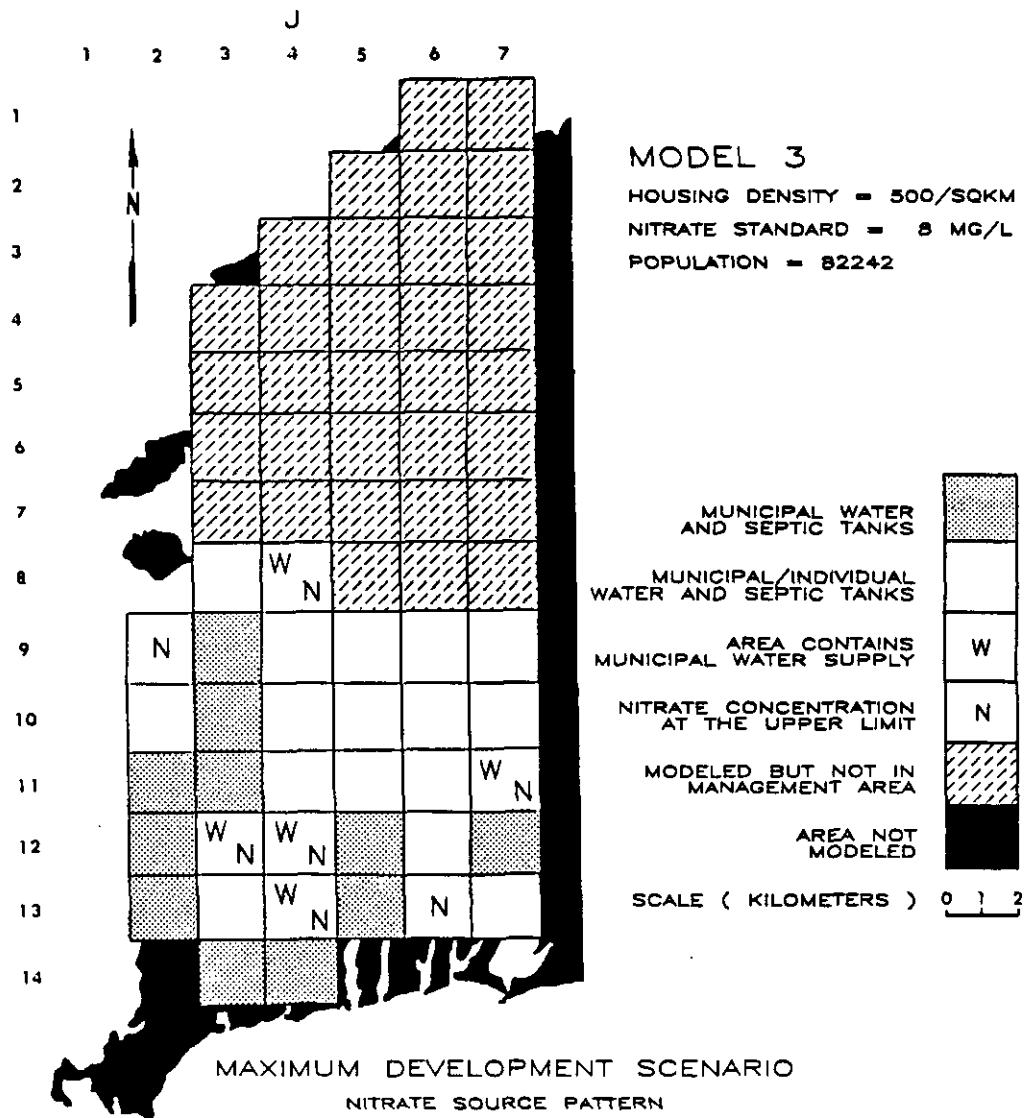density limit of 500 houses/sqkm and a global nitrate
nitrogen standard of 8 mg/l).
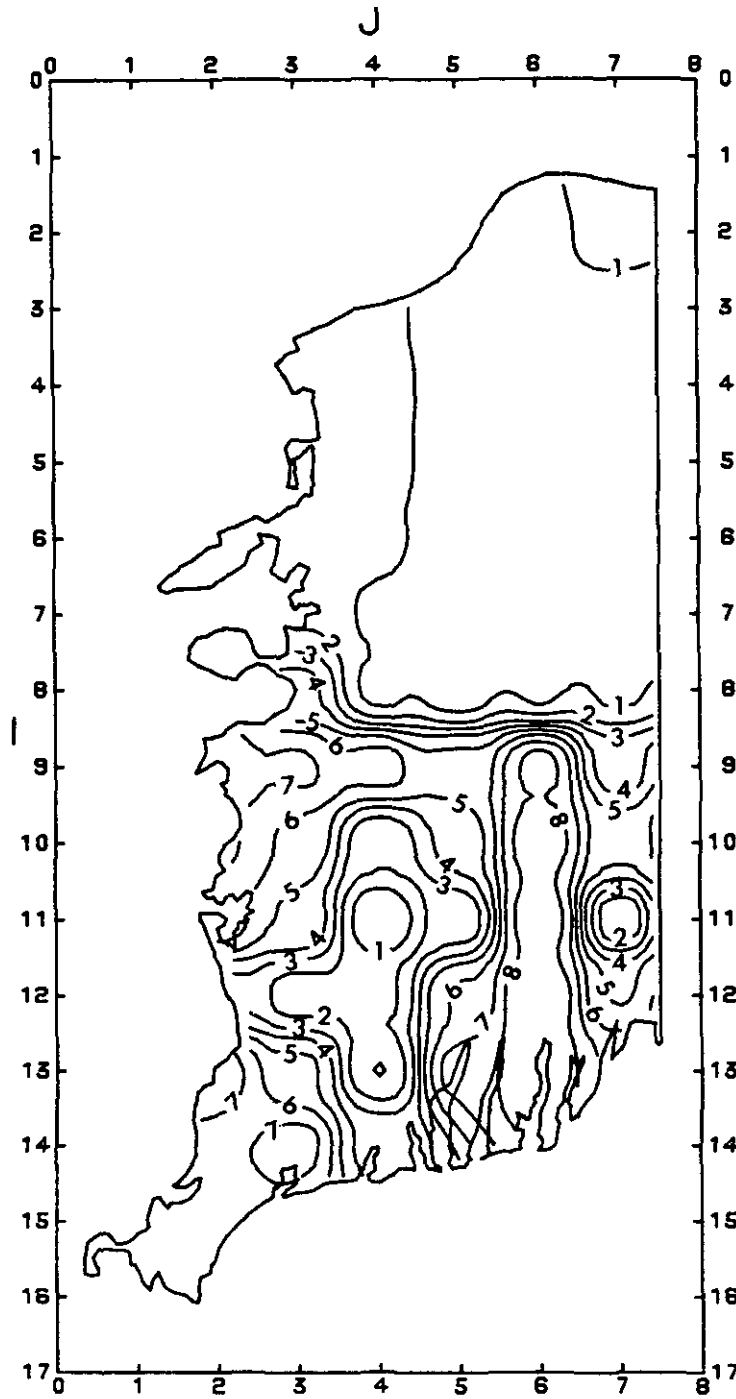
Figure 67. Predicted steady-state nitrate nitrogen concentration (mg/l) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 9 mg/l).
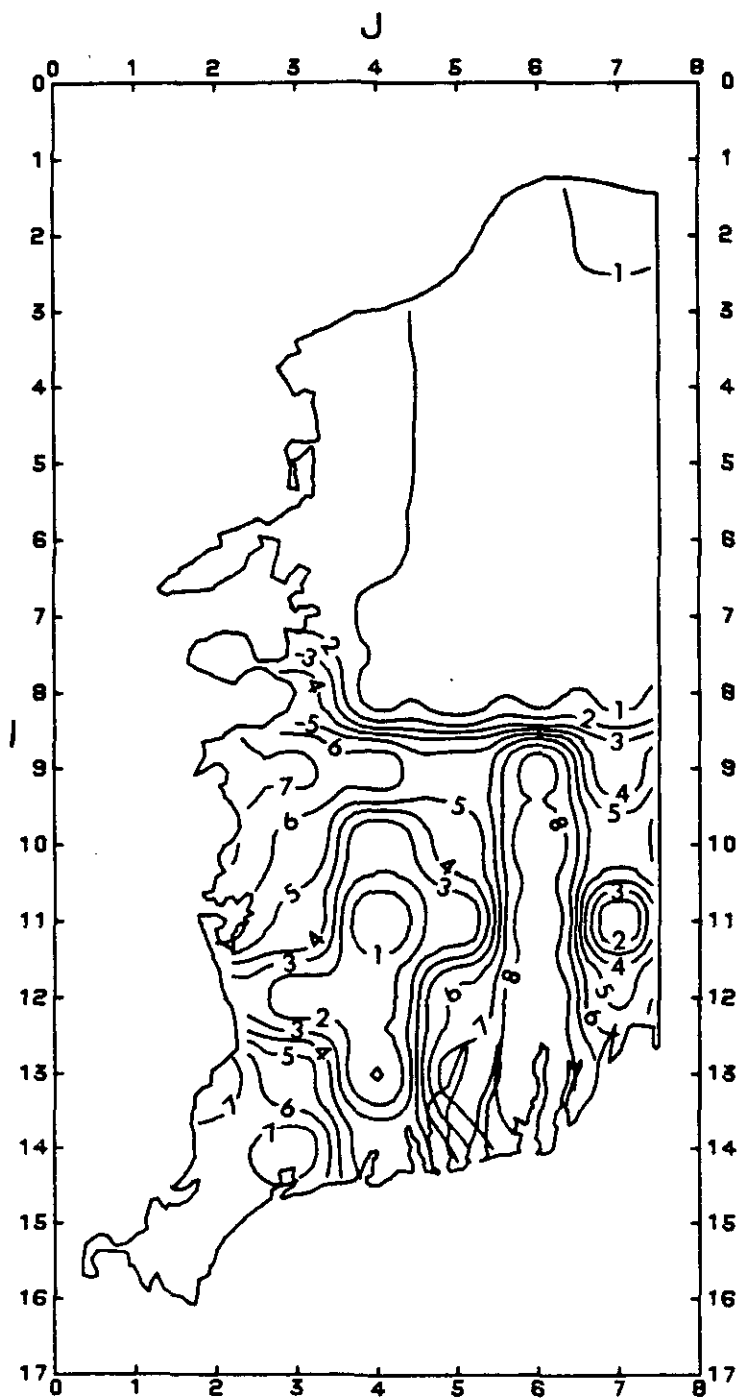
Figure 68.  Predicted steady-state nitrate nitrogen concentration
(mg/l) contours over Bourne and Falmouth (at maximum
feasible development of Falmouth for a specified land use
density limit of 500 houses/sqkm and a global nitrate
nitrogen standard of 10 mg/l).

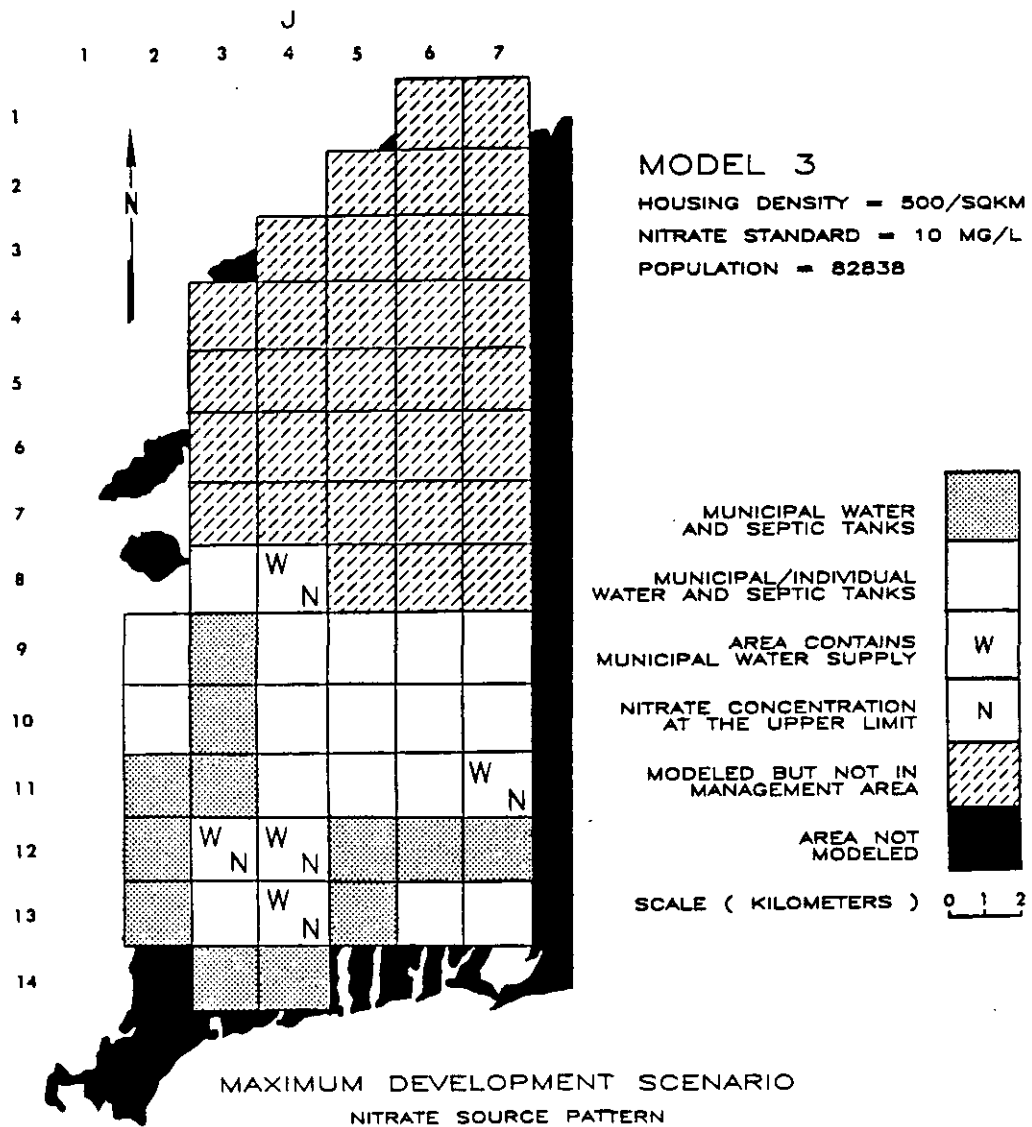Figure 69.  Optimum values of dual variables associated with binding
elemental land use density constraints (from the solution
of Model III under a land use density limit of 500
houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).  Numbers represent the additional population growth
from allowing one more housing unit in appropriate
elements.

would not permit more people to live in Falmouth, because the land use activity in these elements has not yet exhausted available land for development.

The values of the dual variables for the land use density constraints did not change with the relaxation of the regional nitrate standard. Dual variable values less than the residential occupancy rate (2.203 people/household) indicated an added household in one place would require a reduction in residential/commercial activity elsewhere to ensure other model constraints remain satisfied. When the values of dual variables were larger than the occupancy rate the effect of adding one more housing unit allowed a shift in upgradient land use activities which in turn permitted additional residential/commercial development elsewhere; hence, the net population gain was greater than the average occupancy rate.

The development tradeoffs generated from relaxing water quality constraints are displayed in Figure 70. The numbers appearing in specific elements represent the marginal increase in the number of people which could be located in Falmouth if the water quality constraints for those elements were relaxed by 1 mg/l as N.

For binding global water quality constraints the values of dual variables decreased as the global standard increased from 5 to 10 mg/l as N. The fall in values is probably indicative of the extent to which feasible development would assume a pattern determined less by global water quality constraints and more as by nondegradation and land use density constraints.
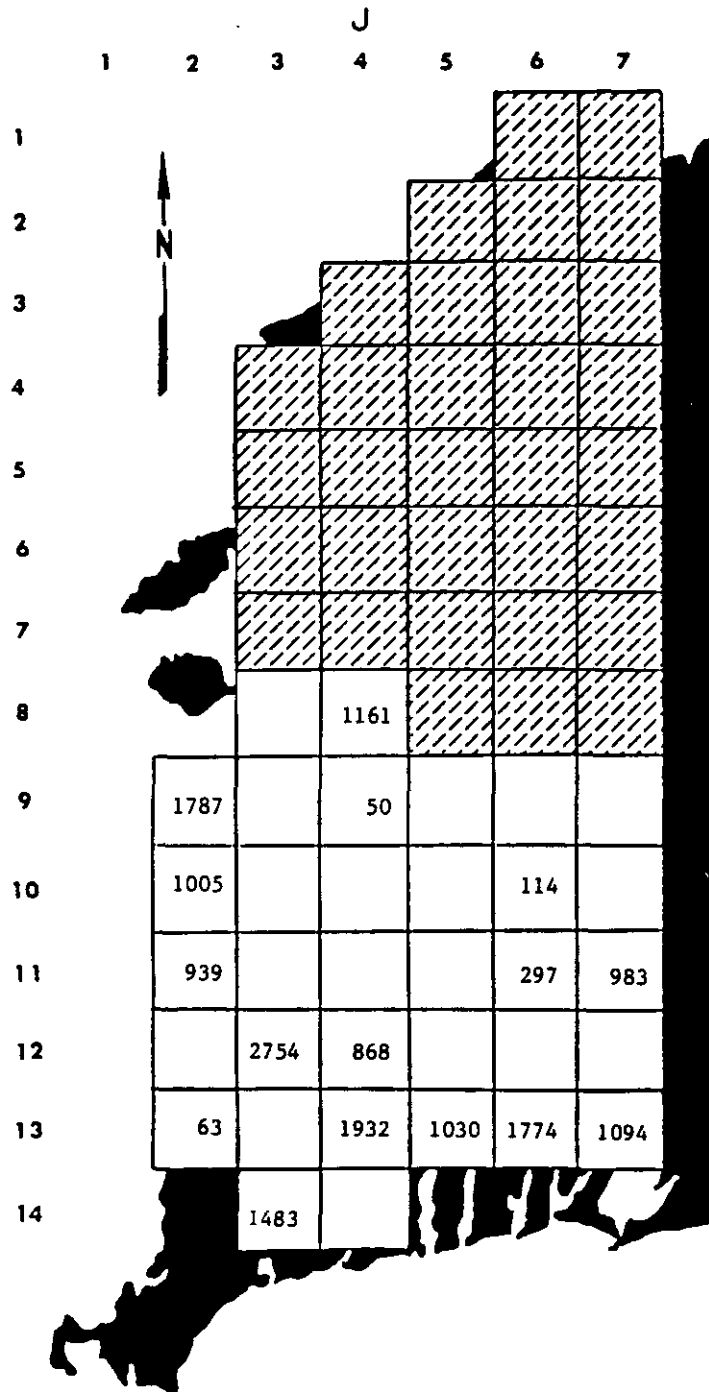
Figure 70. Optimum values of dual variables associated with binding elemental water quality constraints (from the solution of Model III under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l). Numbers represent the additional population growth from allowing one more mg/l of nitrate nitrogen in appropriate elemental groundwaters.

Values of dual variables generated from binding nondegradation water quality constraints increase as the global nitrate standard is increased. That is, as the development in surrounding areas increases, the potential development in areas with more stringent standards increases. The potential growth is not realized until the stringent water quality standards are relaxed. Hence, the costs (in terms of development foregone) of having stringent water quality constraints in downgradient elements increases as the opportunity for development elsewhere in the region increases (i.e., as when global nitrate nitrogen standards are increased).

## 8.2.4 Model III Results: Effects of Land Use Density Constraints

An investigation was made, with multiple runs of the third nonpoint source groundwater pollution management model, to elucidate the effects of source density constraints (also known as land use density constraints or residential/commercial zoning constraints) on the maximum feasible residential/commercial development and the protection of groundwater resources in Falmouth. During each model run a different land use density regulation was imposed. Nondegradation and global water quality standards remained constant between model executions. The global nitrate nitrogen standard was always 5 mg/l. Nondegradation water quality constraints were constructed for each element containing a municipal water supply; the other nondegradation nitrate standards were generated as before from the long term nitrate predictions from Model I.

8.2.4.1  Effects of Land Use Density Constraints on Land Use Development

A range of specified land use density limits was used in Model III to ascertain optimal magnitudes and deployment patterns of the three residential/commercial surface activities in Falmouth. Residential/commercial density limits were expressed in units of allowable houses/km$^2$.  In Figure 71 the maximum population projections are plotted against density restrictions varying from 200-500 houses/km$^2$ (one house per 48,900 to 16,500 ft$^2$).  The figure exhibits a curve showing increased maximum development potential (expressed in population) for Falmouth under successively more relaxed regulations on allowable commercial/residential densities.

The shape of the curve in Figure 71 reflects a change in the type of constraints which become boundary equations when more sources are permitted per unit area.  The numbers and types of binding water quality and land use density constraints associated with each solution of Model III are illustrated in Figure 72 as a function of the different land use density regulations.  For source density limits under 200 houses/km$^2$, maximum potential residential/commercial development was independent of global nitrate nitrogen standards.  Development occurred to the maximum allowable density, which was sufficiently low that global water quality constraints were never binding.  The nondegradation constraints, however, were either binding or were on the verge of becoming binding.

When density regulations of no less than 375 houses/km$^2$ were used, Model III yielded optimal development patterns which were determined by
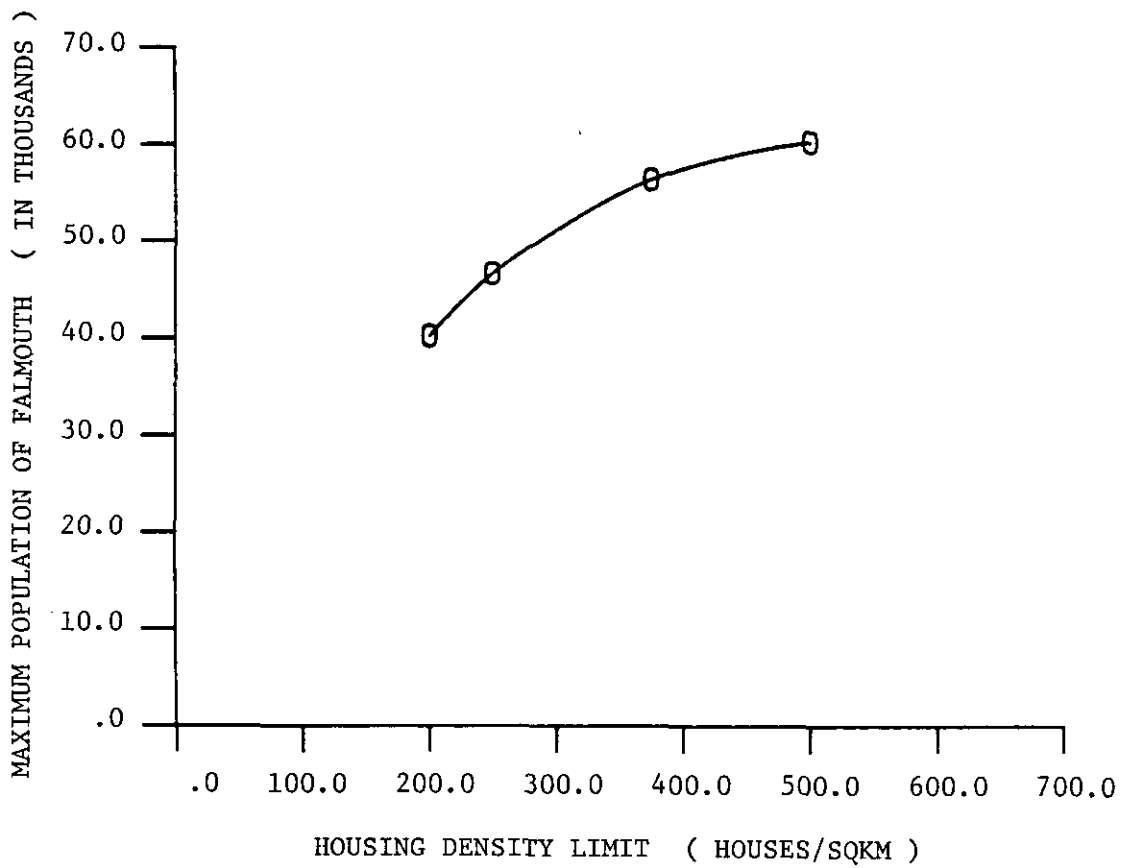
Figure 71.    Predicted maximum population of Falmouth as a function of
              land use density limits (for development scenarios under
              the constant global nitrate nitrogen standard of 5 mg/l).
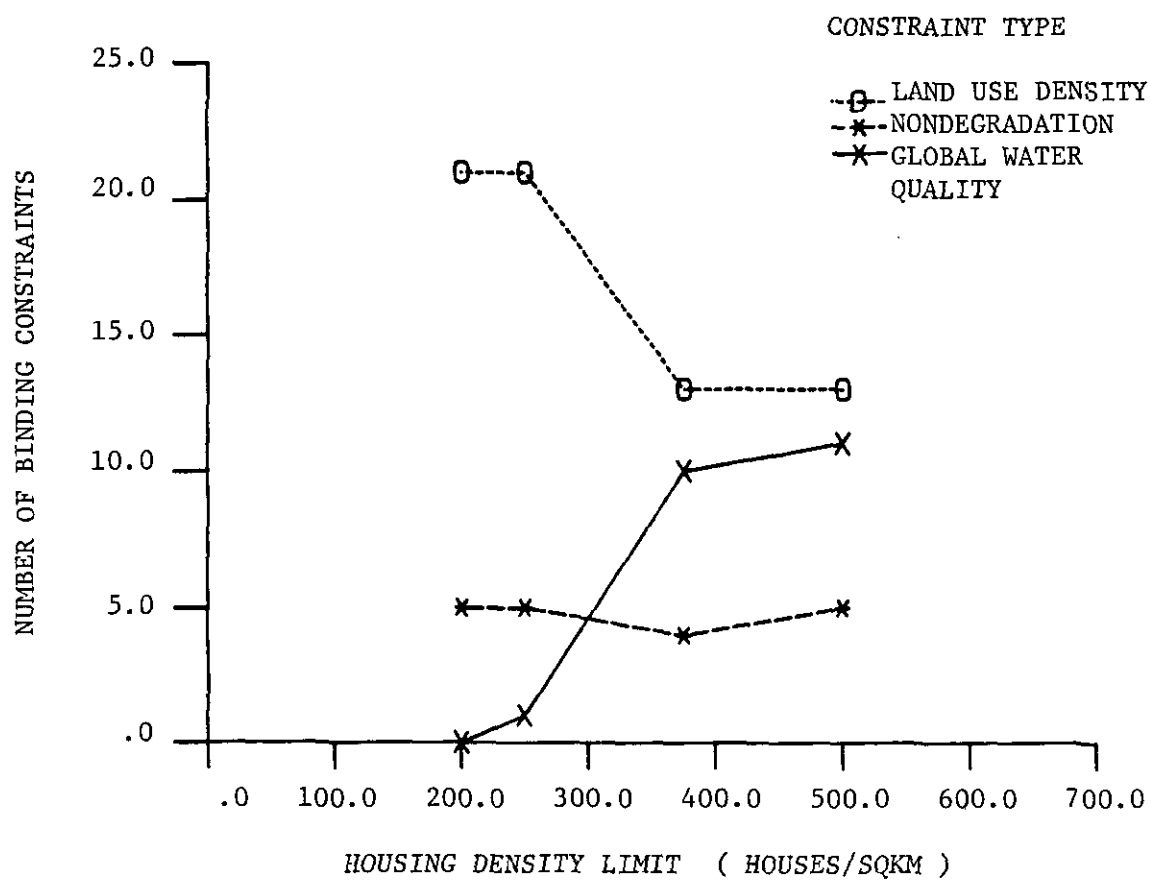
Figure 72.    Number of binding land use density, nondegradation, and
global water quality constraints as a function of land use
density limits (in the optimal solutions of Model III using
a constant global nitrate nitrogen standard of 5 mg/l).

binding global water quality, nondegradation, and land use density

constraints. Wherever development was feasible it occurred at maximum

density levels when stringent land use density limits were used. Under

more relaxed density regulations maximum allowable growth occurred

primarily among coastal nodes.

The solutions of Model III for each of the four land use density

limits are illustrated in Figures 73-76. Each figure depicts the

location of municipal water supplies and the status of development and

water quality for each element in the management area.

Figure 73 displays the optimal pattern of development under a land

used density restriction of 200 houses/km$^2$. The maximum potential

population from combining the three land use activities was 40,193 under

a global nitrate standard of 5 mg/l as N. Pre-existing

residential/commercial development precluded new development in elements

i,j = 9,3 - 13,5 - 13,6 - 13,7 - 14,3 and 14,4 because source densities

were higher than the specified desirable limit. Nondegradation

constraints for elements containing municipal water supplies (Long Pond

and Fresh Pond), discouraged development around and up-gradient from the

sources of town water. In virtually all other elements, new development

expanded up to 200 houses/km$^2$. The only binding water quality

constraints were the nondegradation constraints specified at elements

i,j = 8,4 - 9,6 - 11,7 - 12,3 and 13,4.

When the restriction on allowable residential/commercial density

was raised to 250 houses/km$^2$ the pattern of development was similar to
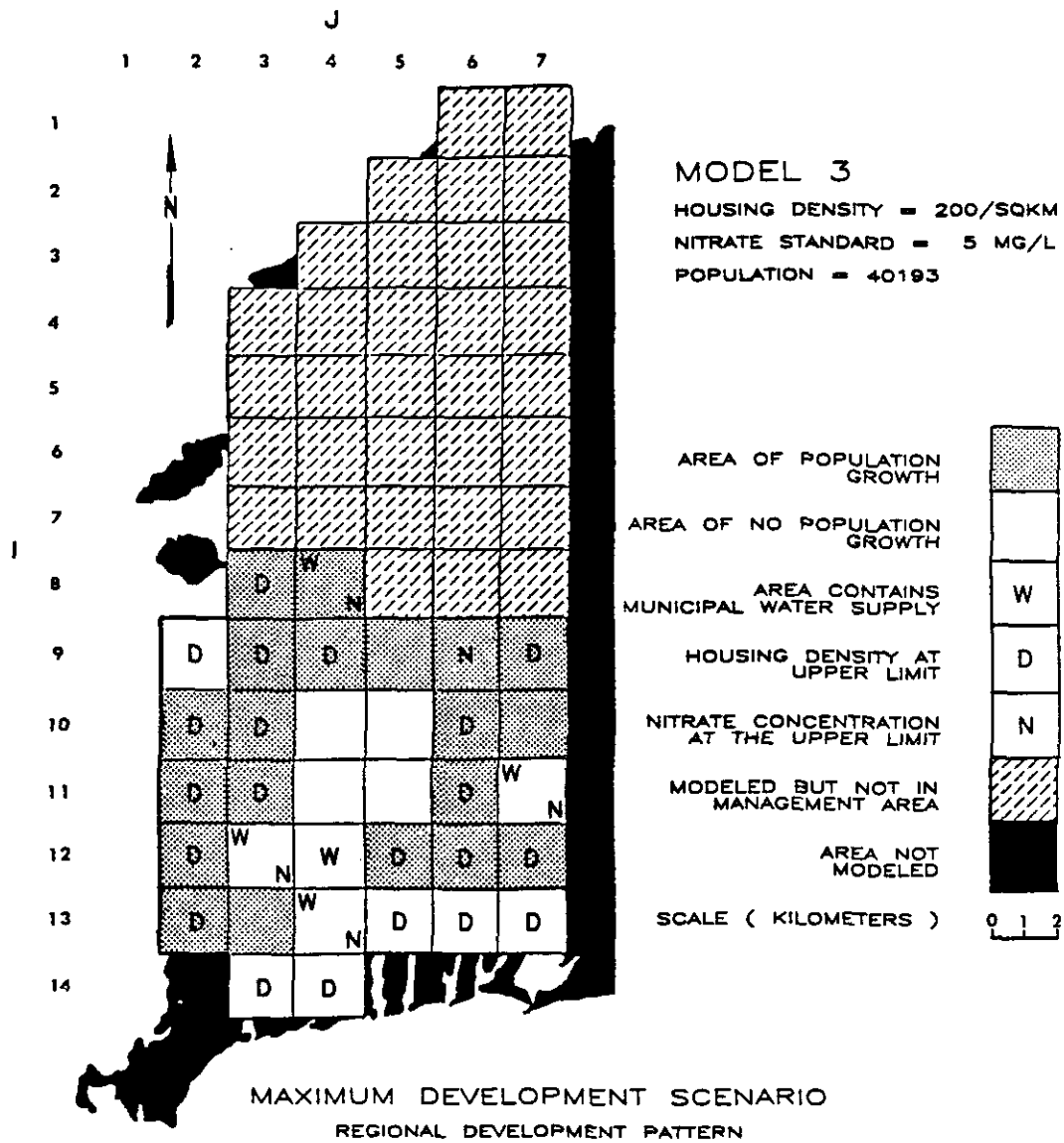
Figure 73. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 200 houses/sqkm and a global nitrate nitrogen standard of 5 mg/1).

MAXIMUM DEVELOPMENT SCENARIO
REGIONAL DEVELOPMENT PATTERN
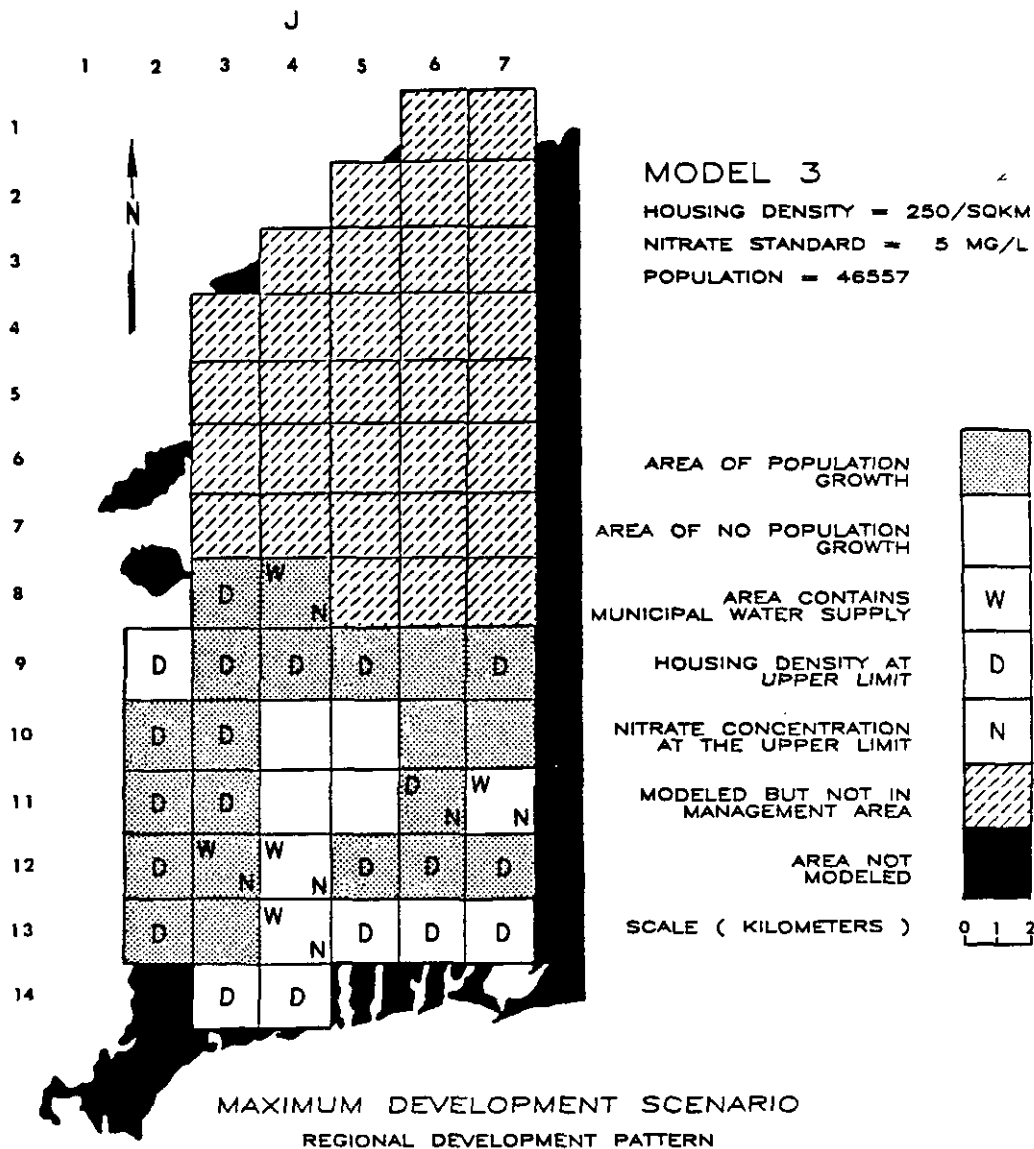
Figure 74.    Maximum residential/commercial land use development pattern
for Falmouth (for a specified land use density limit of 250
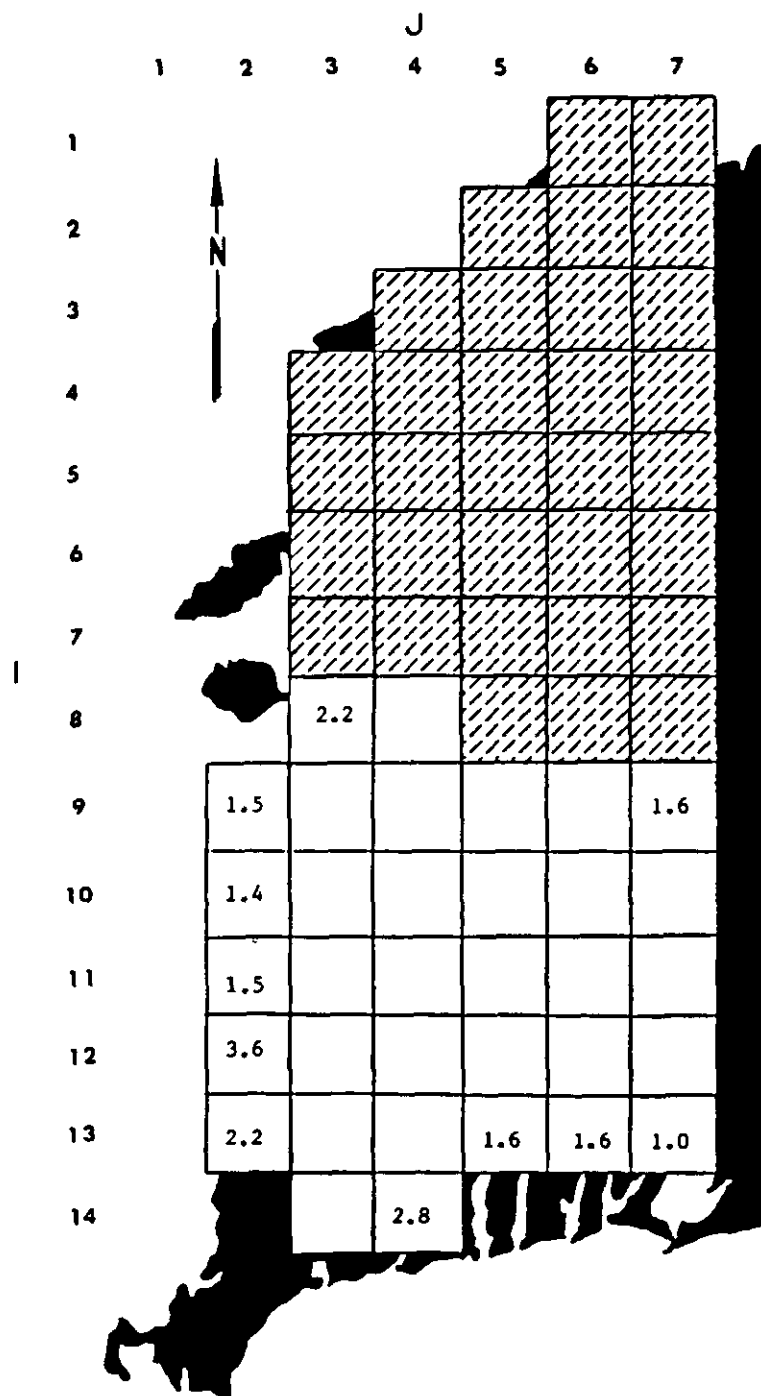houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).
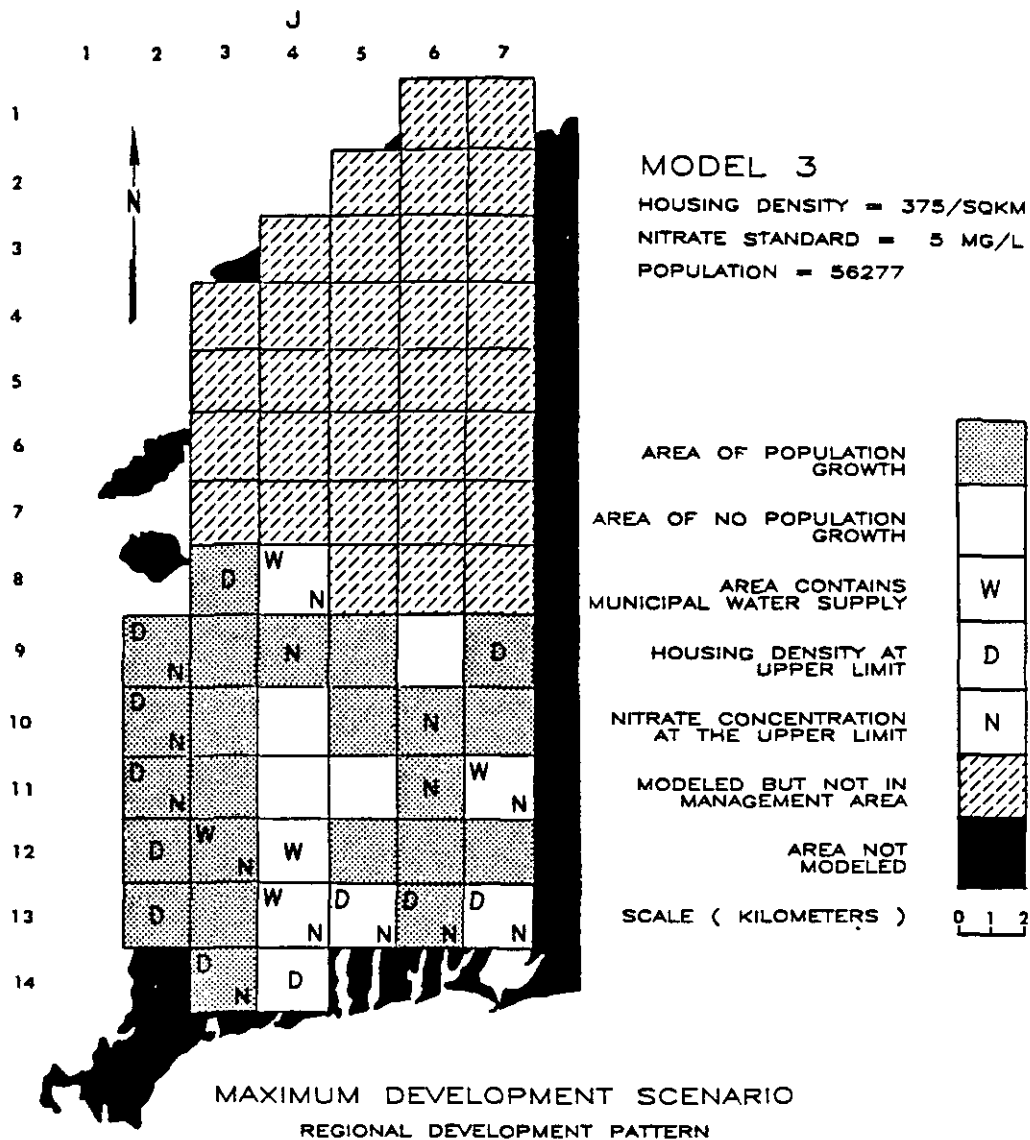
Figure 75.    Maximum residential/commercial land use development pattern
for Falmouth (for a specified land use density limit of 375
houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).
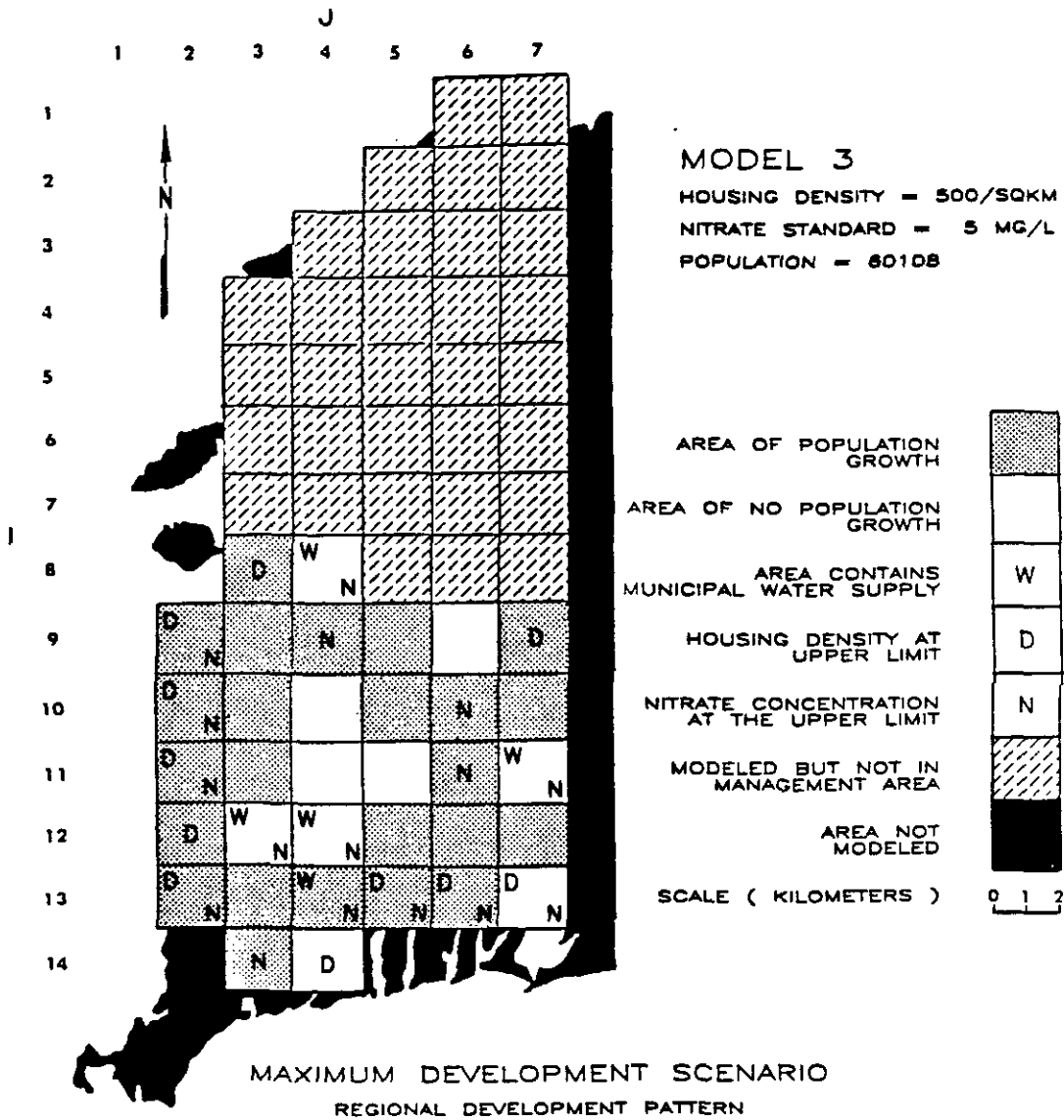
Figure 76. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

that achievable under a source density of 200 houses/km$^2$ (see Figure 74). Again, growth reached the maximum level allowed in most elements because elemental nitrate levels were not as yet approaching the global standard (5 mg/l as N).

Under a land use density restriction of 375 houses/km$^2$, the intensity and pattern of development changed to reflect the influence of binding global water quality constraints (see Figure 75). Maximum residential/commercial activity was situated along the coast, where most global nitrate nitrogen constraints were binding. Upgradient from the coast, development proceeded to levels which would allow down gradient coastal elements to meet global nitrate nitrogen standards. Nondegradation constraints were binding constraints for every element where applicable except in element i,j = 9,6. Here, predicted long term water quality was slightly less than the imposed nondegradation standard. As the density regulation was increased, land use activity began to expand in a few elements (i,j = 9,3 - 13,6 - 14,3) where more stringent source density limits had precluded growth because the concentration of pre-existing development was already unacceptably high.

A last model run was performed in which allowable levels of residential/commercial activity were increased to 500 houses/km$^2$. The solution, shown in Figure 76 indicated continued expansion of land use activities along the coast and in most coastal elements to the maximum allowable level. Development in the interior regions decreased to compensate for the additional coastal nitrate loads, in order that

nitrate concentrations along the coast would satisfy global water quality constraints. The nondegradation constraints were binding in all elements containing municipal water supplies. A five percent increase in population (117 people) was observed in element i,j = 13,4 where a nondegradation constraint was imposed to protect a municipal water supply. The increased population did not have an impact on water quality in the element. It appears that small changes in nitrate load can be tolerated without serious impact on the approximation of the nitrate concentration gradients in the continuity equations. This is due to the large discretization scales used in the model.

It was noted that the percentage of Falmouth area to receive additional development increased when more residential/commercial activity was allowed per unit area (see Figure 77). The observed increases were due to new development in areas where pre-existing land use activities had, under more stringent land use density regulations, precluded additional growth. Relaxing the source density limit increased the growth potential for Falmouth. Of the elements accepting new growth, the percentage in danger of violating the global nitrate nitrogen standard increased with every increase in the source density limit (see Figure 78). As the nitrate concentrations in groundwater increased with allowable source densities, the number of binding global water quality constraints became more prevalent. This in turn curtailed the number of nodes where potential development could approach the density limit (see Figure 79). The increased growth potential led to greater growth along the coastline, but for reasons of maintaining

PERCENT AREA OF FALMOUTH RECEIVING NEW DEVELOPMENT

HOUSING DENSITY LIMIT  ( HOUSES/SQKM )
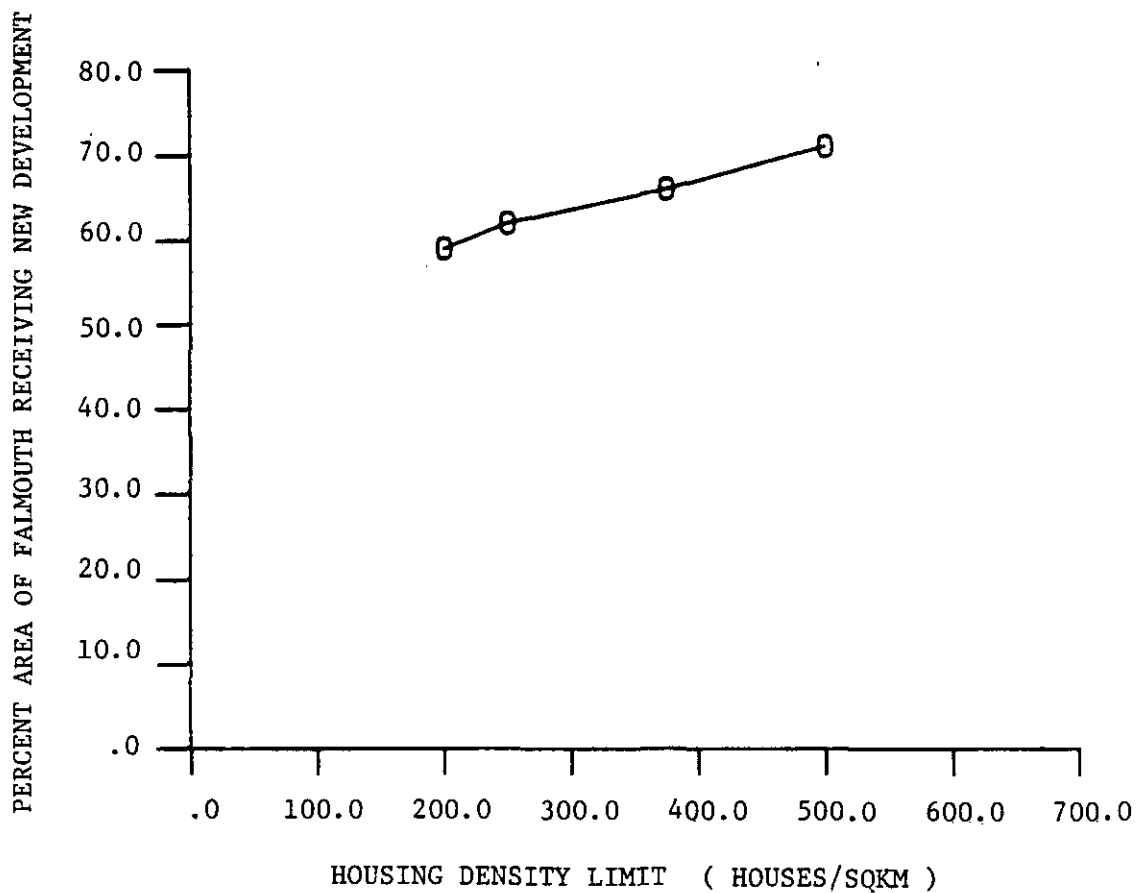
Figure 77.    The percent area of Falmouth to receive additional growth
             as a function of land use density limits (for maximum
             development of Falmouth under the constant global nitrate
             nitrogen standard of 5 mg/l).

Figure 78.   The percent new-growth area of Falmouth polluted to
             allowable nitrate levels as a function of land use density
             limits (for maximum development of Falmouth under the
             constant global nitrate nitrogen standard of 5 mg/l).
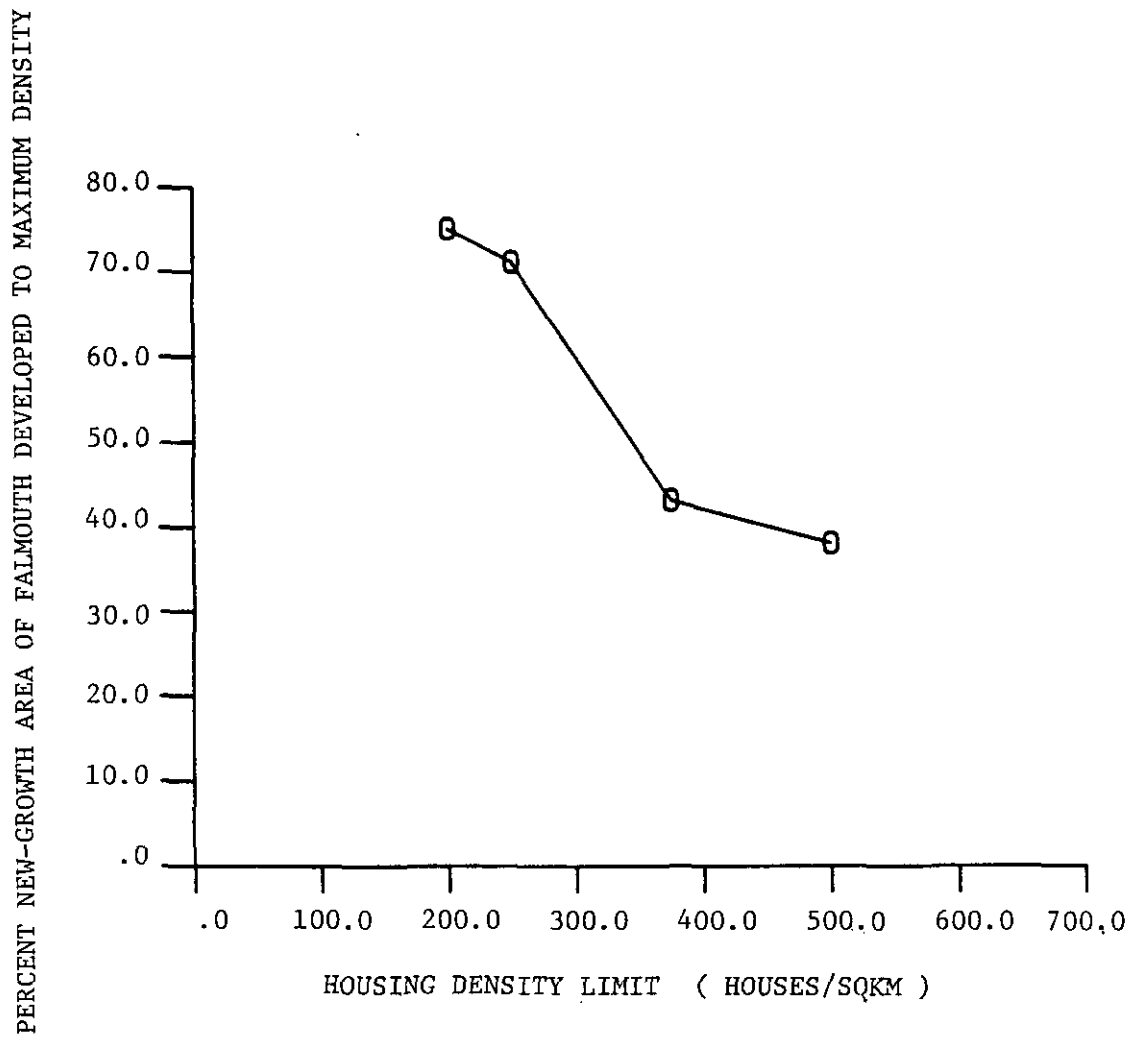
Figure 79.   The percent new-growth area of Falmouth to reach the
             allowable land use density limit as a function of land used
             density limits (for maximum development of Falmouth under
             the the constant global nitrate nitrogen standard of 5
             mg/l).

coastal groundwater quality this was at the cost of reduced growth potential for interior regions. New development, in elements where nondegradation constraints were imposed, was for the most part not observed. Specifically, for the nodes upgradient from Long Pond, no new development was possible without relaxing the nondegradation constraints at elements i,j = 12,3 - 12,4 - 13,4.

Beyond the total residential/commercial land use picture, the magnitude and patterns of the three component land use types were reviewed for possible relationships among them and the specified source density regulations. Figure 80 suggests that a distinct preference exist residential/commercial activities that are dependent on municipal water when the source density limits are stringent, while activities dependent on on-site water are more desirable with land use density restrictions allowing more development than 375 houses/km$^2$. A regional profile for municipal water use reveals that at most 38 percent of Falmouth selected the use of municipal water over the use of on-site wells. The management model selected residential/commercial surface activities that are dependent on onsite water wherever water quality constraints were restricting development in a region. Under opposite conditions, where development proceeded unhindered by global water quality constraints, the choice of water source is municipal. The model selects the land use type which contributes the lowest load when source density limits are sufficiently relaxed that groundwater quality constraints play a part in effectively defining the upper limit on allowable levels of residential/commercial land use activities.
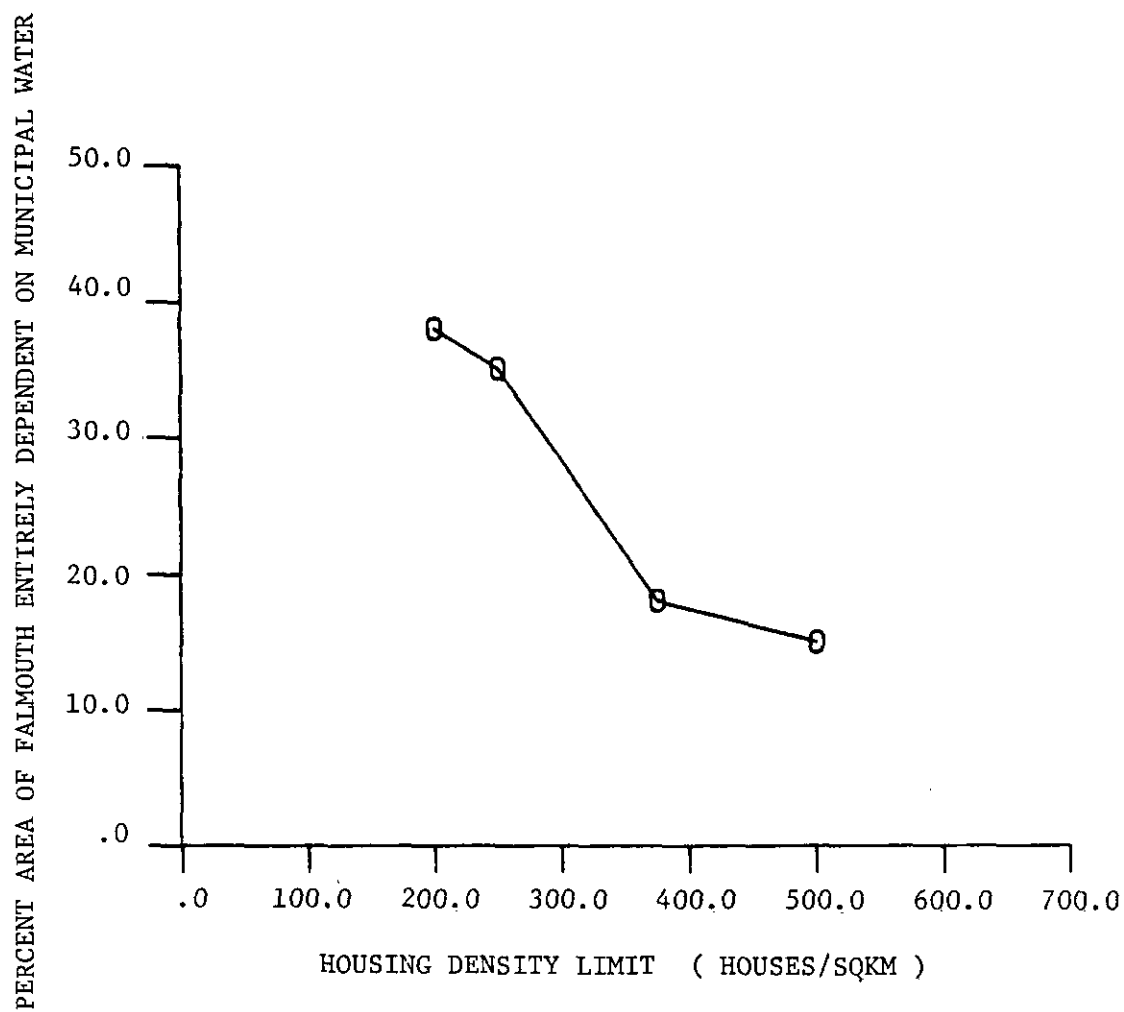
Figure 80.    Percent area of Falmouth entirely dependent on municipal
water as a function of land use density limits (for maximum
development of Falmouth under the constant global nitrate
nitrogen standard of 5 mg/l).

The patterns of the three residential/commercial land use

activities are illustrated in the series of Figures 81-84 which covers

the series of solutions of Model III under global source density

restrictions between 200-500 houses/km$^2$. As in previous figures (see

Figures 55-60), the shaded elements are those where municipal water was

the principal source of water. With the exception of element i,j = 11,2

and 11,3 options exist to develop residential/commercial activities

which depend on municipal water, on-site well water, or to convert

existing on-site well users to municipal water users. For element i,j =

14,2, on-site wells did not exist as a source of residential/commercial

water, in 1980; however, when additional development was feasible (when

the land use density regulation was elevated above 250 houses/km$^2$), on-

site well water use was introduced over increased municipal water usage.

This occurred because land use activities dependent on municipal water

were perceived by the model to deliver higher per capita nitrate loads,

and to add municipal water consumption would have led to a suboptimal

increase in elemental population under development conditions where

water quality changes were going to constrain development. The optimal

development in node i,j = 13,2 was always that which required town water

until relaxation of the land use density limit portended nitrate levels

which encouraged optimal development with on-site well water uses.

Development in element i,j = 12,2 was the same as with model runs

displayed above; land use activities needing on-site well water were

never permitted. For all other elements illustrated as having only town

water, residential/commercial use never increased in an element without

MODEL 3

HOUSING DENSITY = 200/SQKM
NITRATE STANDARD = 5 MG/L
POPULATION = 40193

MUNICIPAL WATER
AND SEPTIC TANKS

MUNICIPAL/INDIVIDUAL
WATER AND SEPTIC TANKS

AREA CONTAINS
MUNICIPAL WATER SUPPLY

NITRATE CONCENTRATION
AT THE UPPER LIMIT

MODELED BUT NOT IN
MANAGEMENT AREA

AREA NOT
MODELED

SCALE ( KILOMETERS )   0   1   2

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN
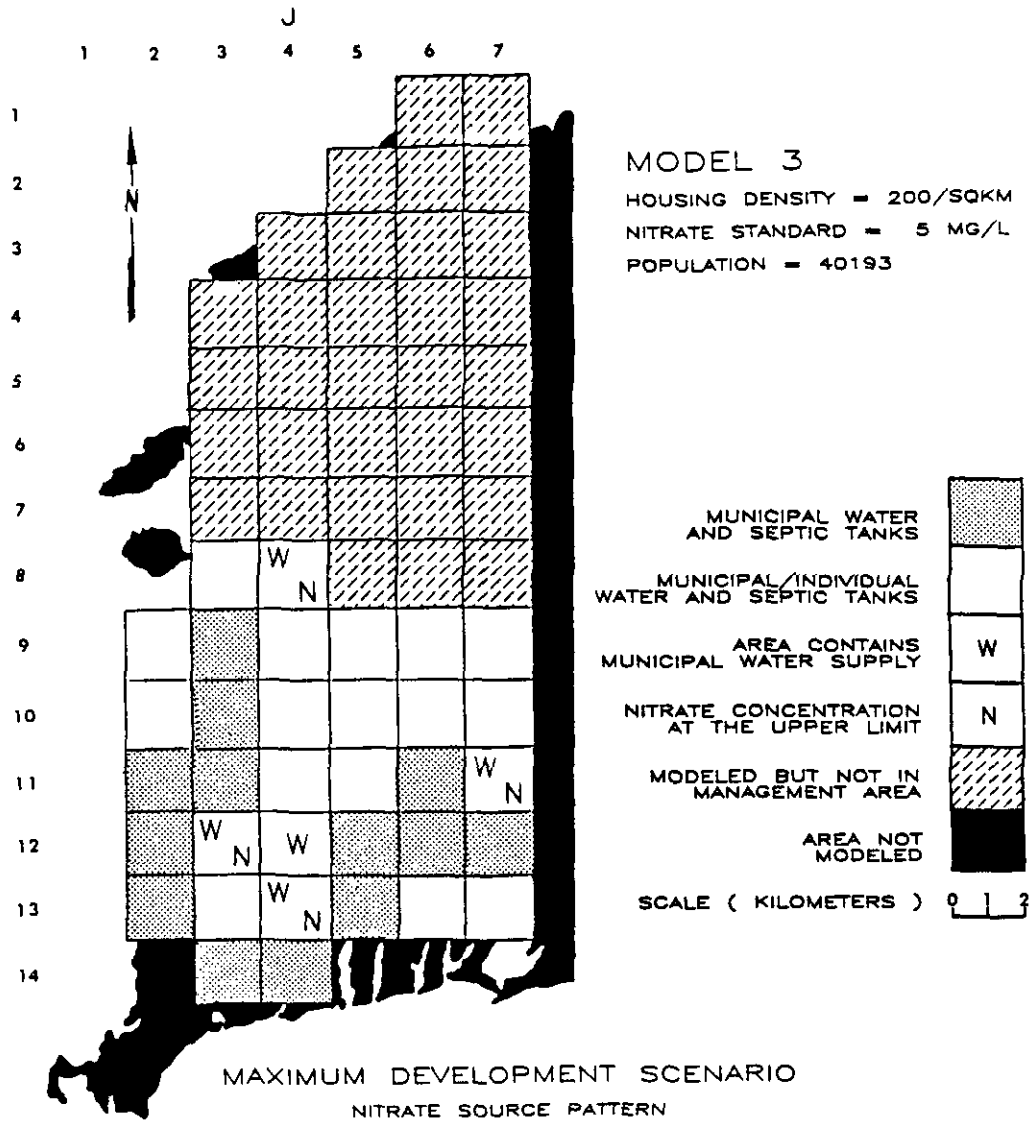
Figure 81.    Water usage patterns for residential/commercial land use in
Falmouth (for a specified land use density limit of 200
houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).
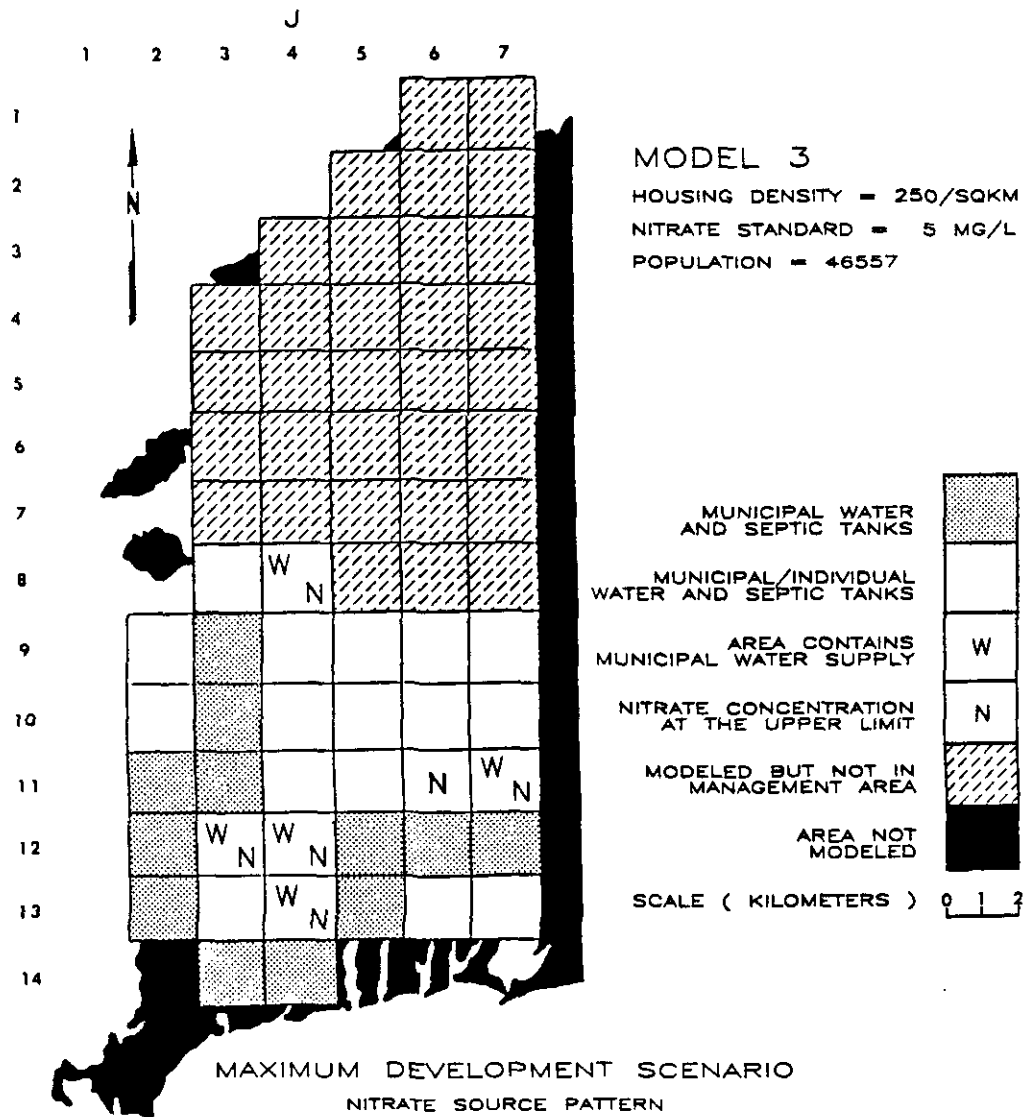
Figure 82. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

MODEL 3

HOUSING DENSITY = 375/SQKM
NITRATE STANDARD = 5 MG/L
POPULATION = 56277

MUNICIPAL WATER
AND SEPTIC TANKS

MUNICIPAL/INDIVIDUAL
WATER AND SEPTIC TANKS

AREA CONTAINS
MUNICIPAL WATER SUPPLY

NITRATE CONCENTRATION
AT THE UPPER LIMIT

MODELED BUT NOT IN
MANAGEMENT AREA

AREA NOT
MODELED

SCALE ( KILOMETERS )   0  1  2

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN

Figure 83.    Water usage patterns for residential/commercial land use in
              Falmouth (for a specified land use density limit of 375
              houses/sqkm and a global nitrate nitrogen standard of 5
              mg/l).

MODEL 3

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 5 MG/L
POPULATION = 60108

MUNICIPAL WATER
AND SEPTIC TANKS

MUNICIPAL/INDIVIDUAL
WATER AND SEPTIC TANKS

AREA CONTAINS
MUNICIPAL WATER SUPPLY

NITRATE CONCENTRATION
AT THE UPPER LIMIT

MODELED BUT NOT IN
MANAGEMENT AREA

AREA NOT
MODELED

SCALE ( KILOMETERS )

MAXIMUM DEVELOPMENT SCENARIO
NITRATE SOURCE PATTERN
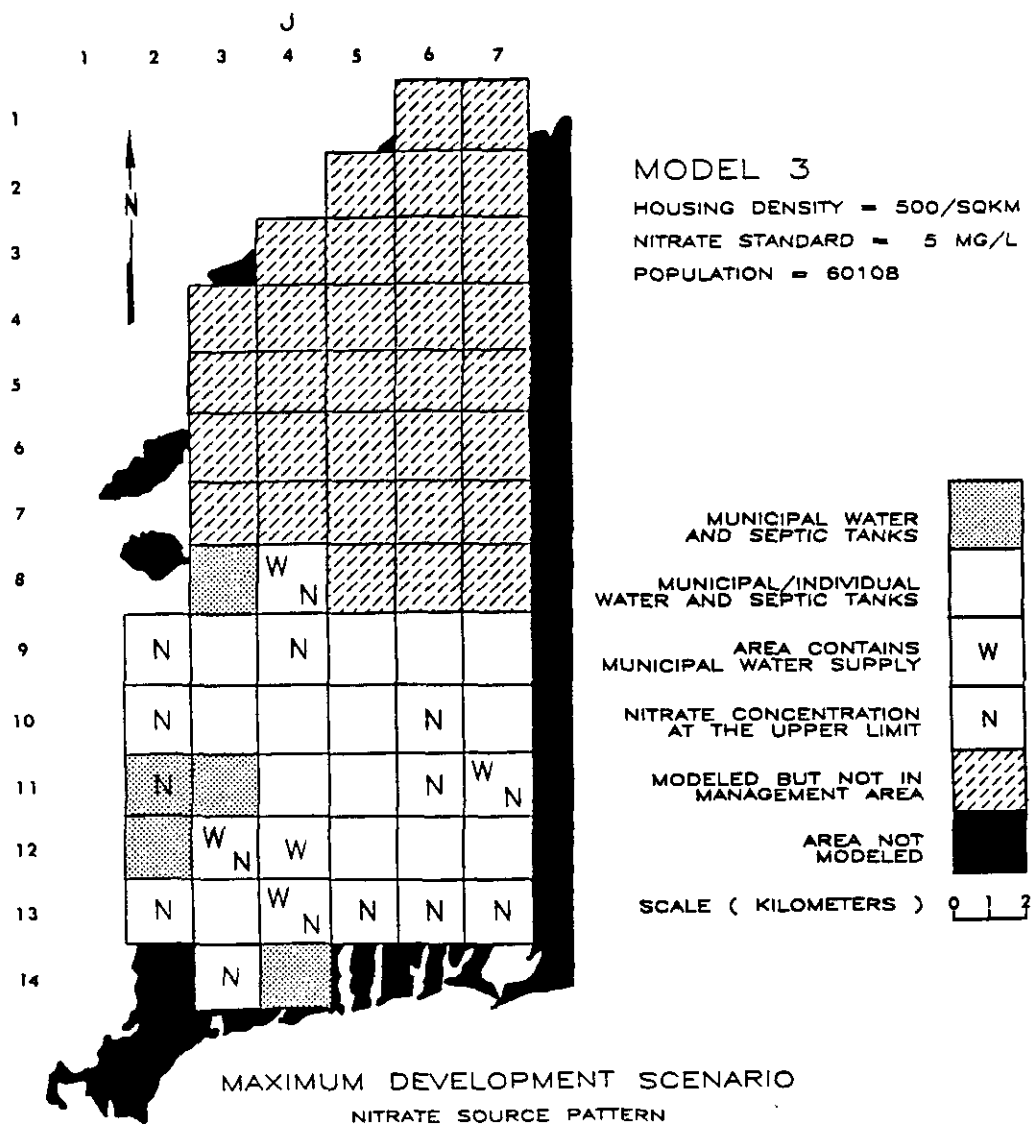
Figure 84.    Water usage patterns for residential/commercial land use in
Falmouth (for a specified land use density limit of 500
houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).

first connecting all elemental on-site well water users to the municipal water distribution system. It was also the case with multiple model runs with the various source density limits that wherever elemental development occurred, either on-site well water or municipal water usage increased but not both. Had the demand for town water ever exceeded the supply it could have been possible for all three land use types to have appeared simultaneously in any one element.

## 8.2.4.2 Effects of Land Use Density Constraints on Groundwater Protection

Groundwater protection through uniform source density controls can be an equitable and effective means of protecting groundwaters. Figure 85 shows the average steady-state nitrate nitrogen concentrations in Falmouth under maximum feasible development conditions for different specified regional land use density limits and a global nitrate nitrogen standard of 5 mg/l. Average steady-state nitrate concentrations increased from 2.1 mg/l from 1980 development to a maximum of 3.6 mg/l. Changes in the average concentration above a housing density of 375 houses/km$^2$ were small because additional development occurred along coastal nodes where the impact of subsurface loading on the average nitrate nitrogen concentration (recall Figure 41) is small. Because much of the high density development occurs along the coastline, and because much interior development is precluded by the nondegradation constraints, excellent water quality in nondeveloped areas compensates for poor water quality elsewhere. Of the area of Falmouth falling
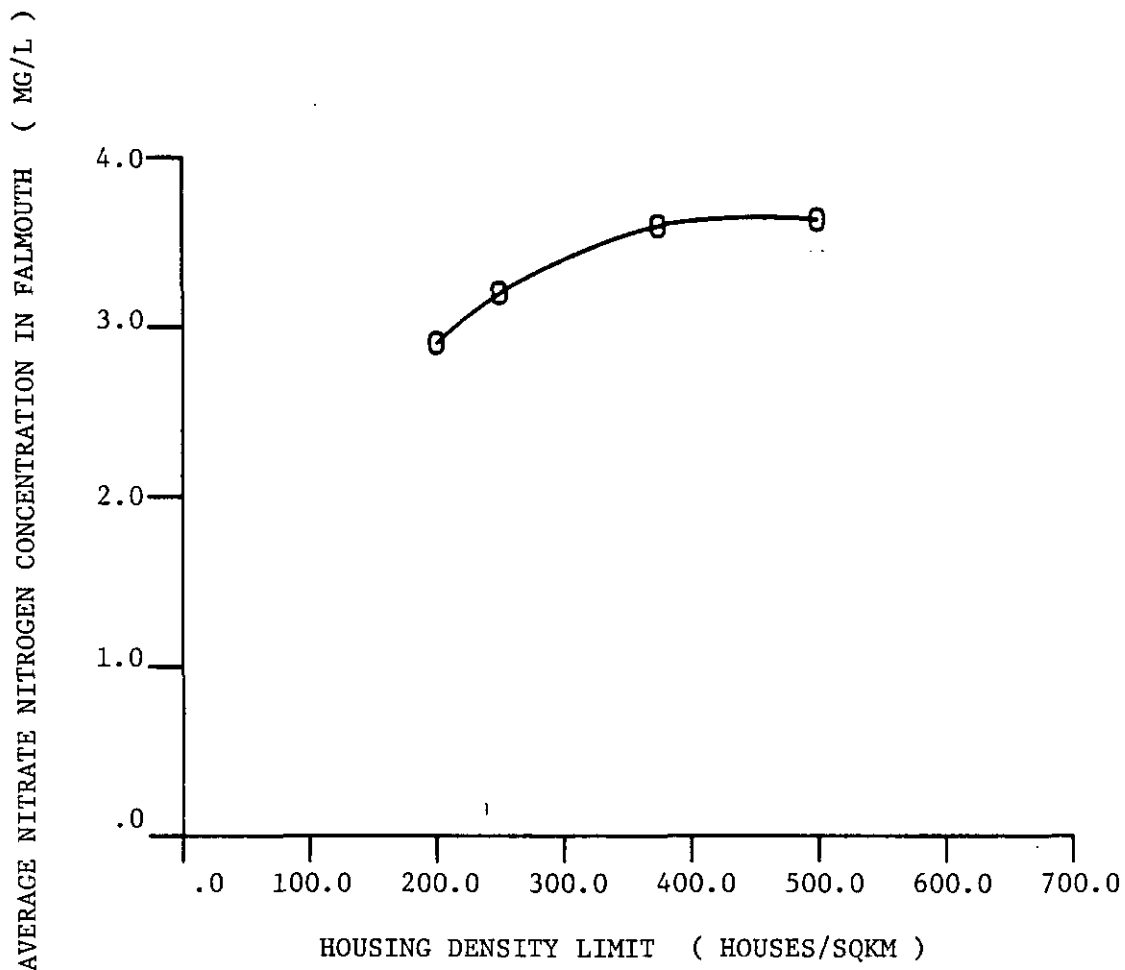
Figure 85.  Average nitrate nitrogen concentration in Falmouth as a
function land use density limits (for maximum development
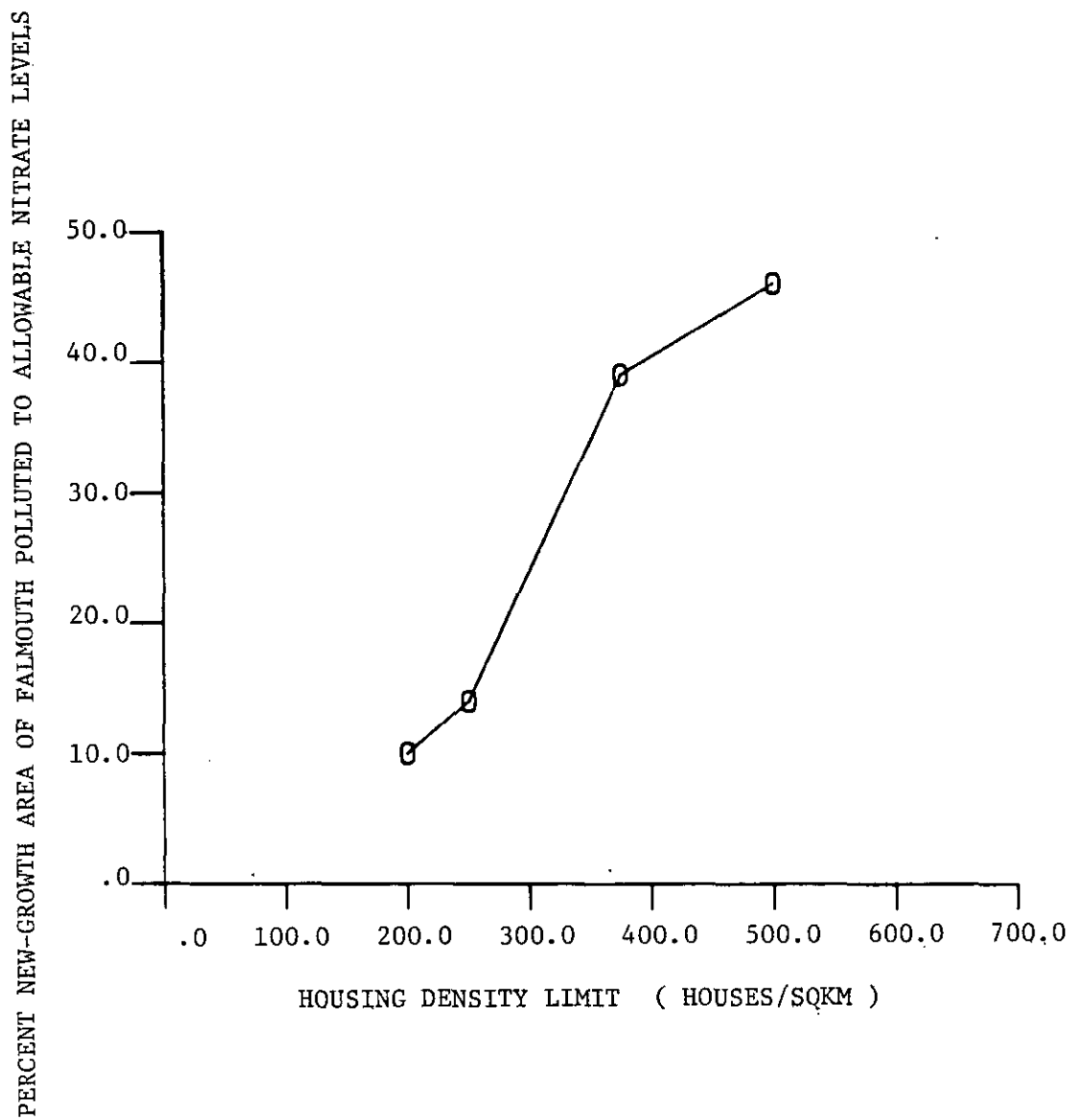scenarios under the constant global nitrate nitrogen
standard of 5 mg/l).

within the boundaries of the study area, the areal extent of groundwater which had degraded to allowable nitrate levels was 15 percent under a density regulation of 200 houses/$km^2$, but increased to 47 percent when 500 houses/$km^2$ were allowed (see Figure 86). The rapid increase in the percentage of the area reaching allowable nitrate limits was entirely in areas where global water quality constraints became solution binding constraints; hence, the water quality constraints were precluding additional growth where density constraints were not sufficient to prevent unacceptable groundwater contamination. The cost of a more stringent land use density limit is lower feasible population growth or lower total feasible land use development. When the residential/commercial density limit was relaxed the affect was to allow water quality constraints to play a larger role in determining the optimal pattern of development. By allowing more water quality constraints (beyond the nondegradation constraints) to establish the limits of development, the optimal pattern of growth becomes less uniform; recall that most development occurred along the coast which precluded equal development upgradient due to coastal water quality problems. Under the relaxed uniform density limits, Model III identified a pattern of development which could accommodate a larger population and satisfy water quality constraints, but would necessitate the enforcement of nonuniform development restrictions if the plan were ever implemented.

The steady-state nitrate nitrogen distributions generated from solving model III under each of the four land use density limits are
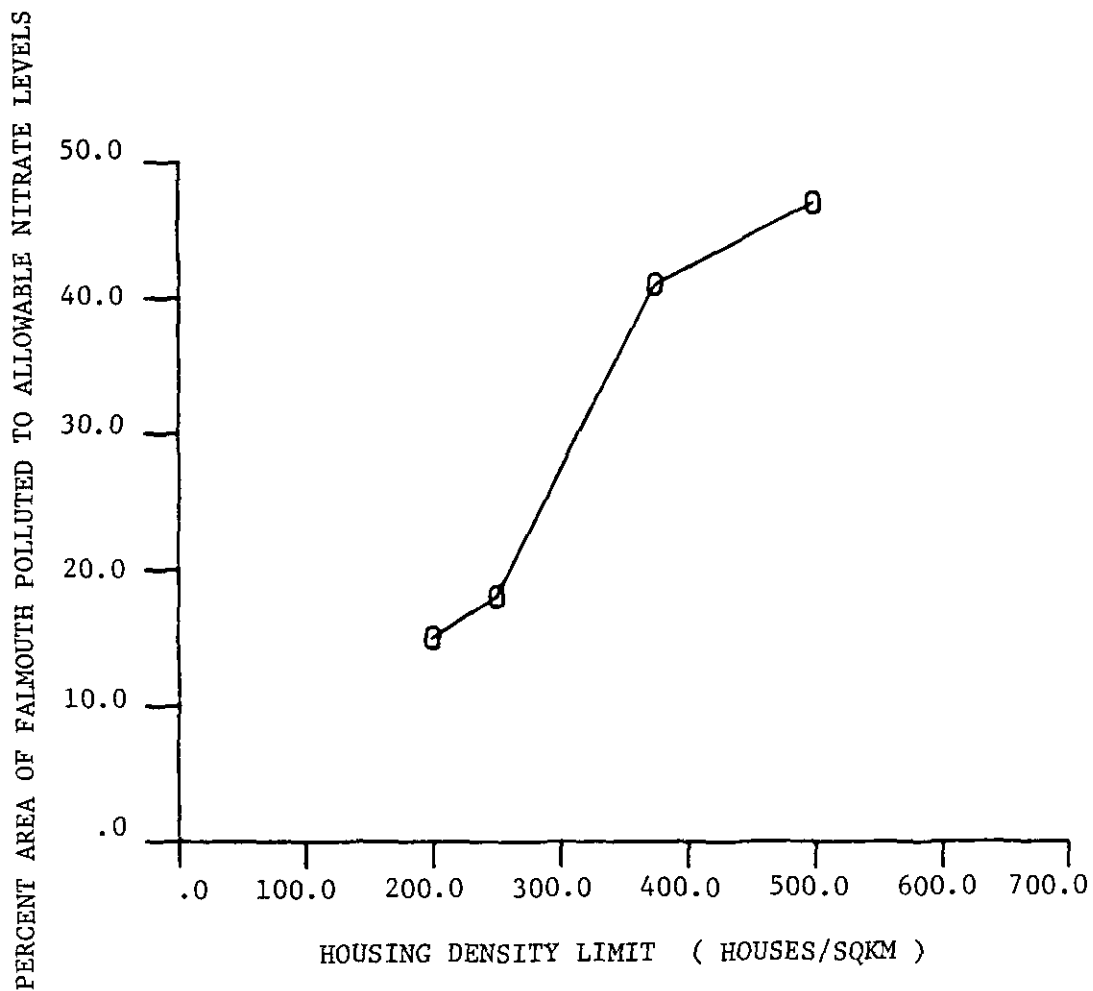
Figure 86. Percent area of Falmouth polluted to allowable nitrate levels as a function of land use density limits (for *maximum* development scenarios *under* the constant global nitrate nitrogen standard of 5 mg/l).

presented in Figures 87 through 90. Where density constraints and nondegradation constraints defined the solution space (Figures 87 and 88) stringent source density limits allowed equal degradation of groundwater between municipal water supplies. Higher source loads generated steeper gradients which developed supply wells and ponds. At density limits of 375 and 500 houses/$km^2$ (Figures 89 and 90) the pattern of nitrate contours emphasizes the effect of high development density on the coast with high nitrate contours protruding upgradient from the coastline. The differences between Figures 89 and 90 are small with the most visible difference being the smaller envelope of clean water around Long Pond in Figure 90.

The development tradeoffs associated with relaxing elemental land use density and water quality constraints were again investigated through the optimal values of the dual variables. Figure 91 presents the dual variables associated with the land use density constraints for the model solution where the density limit used was 250 houses/$km^2$. The values of these dual variables decreased with increases made in the source density limit (the right-hand-side of these constraints). The reduction in the magnitudes of the dual variables occurs with increases in the number of binding water quality constraints; hence the decreased marginal opportunity cost associated with relaxing elemental source density constraints could reflect a reduction in flexibility to shift development patterns in a manner which could yield a positive population gain.
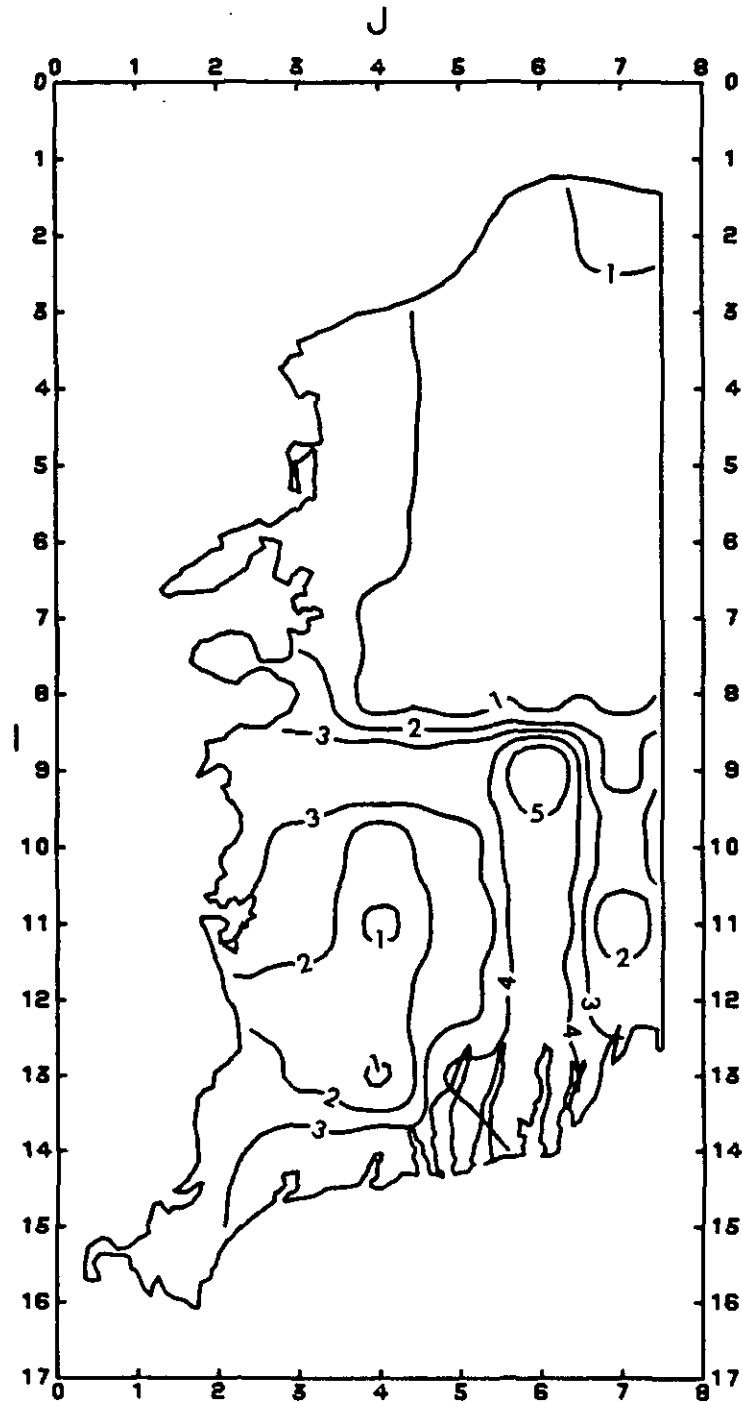
Figure 87. Predicted steady-state nitrate nitrogen concentration (mg/l) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 200 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

302
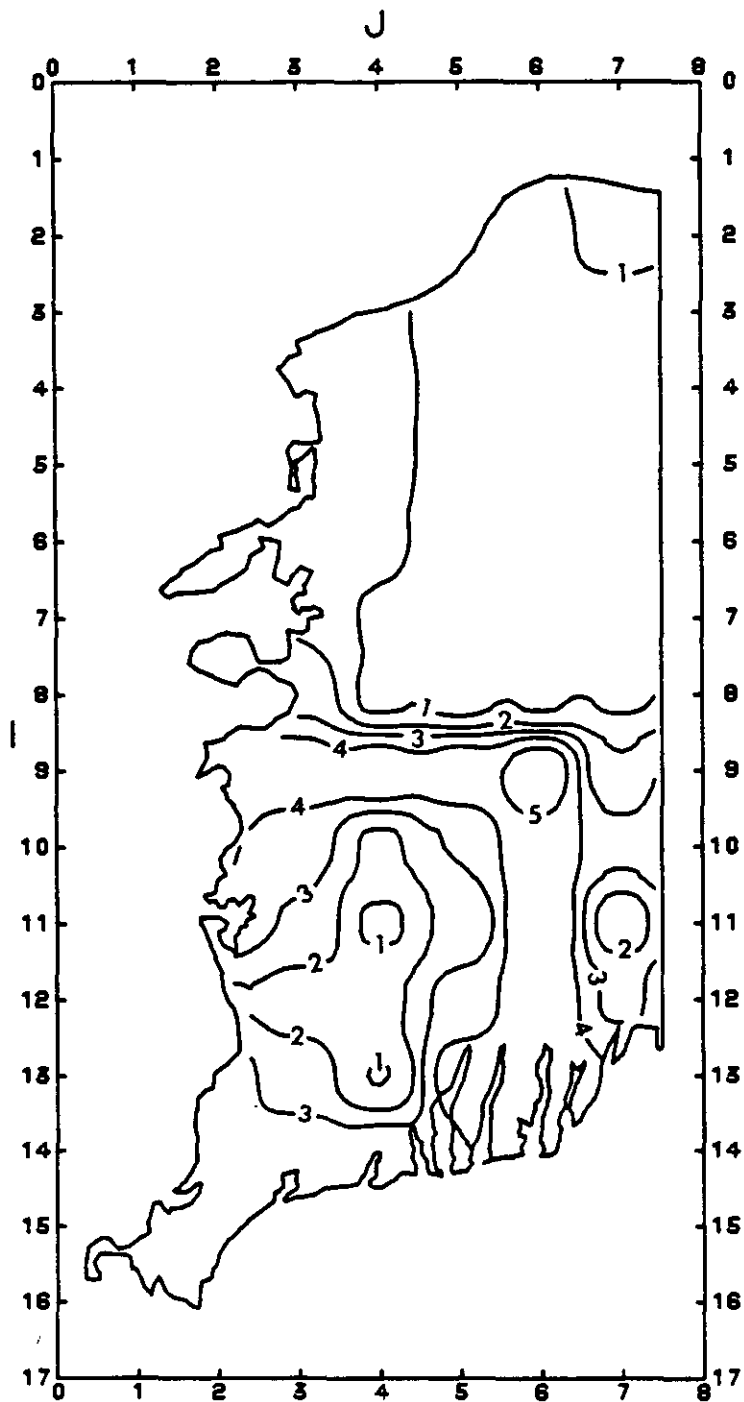


Figure 88.   Predicted steady-state nitrate nitrogen concentration
(mg/l) contours over Bourne and Falmouth (at maximum
feasible development of Falmouth for a specified land use
density limit of 250 houses/sqkm and a global nitrate
nitrogen standard of 5 mg/l).
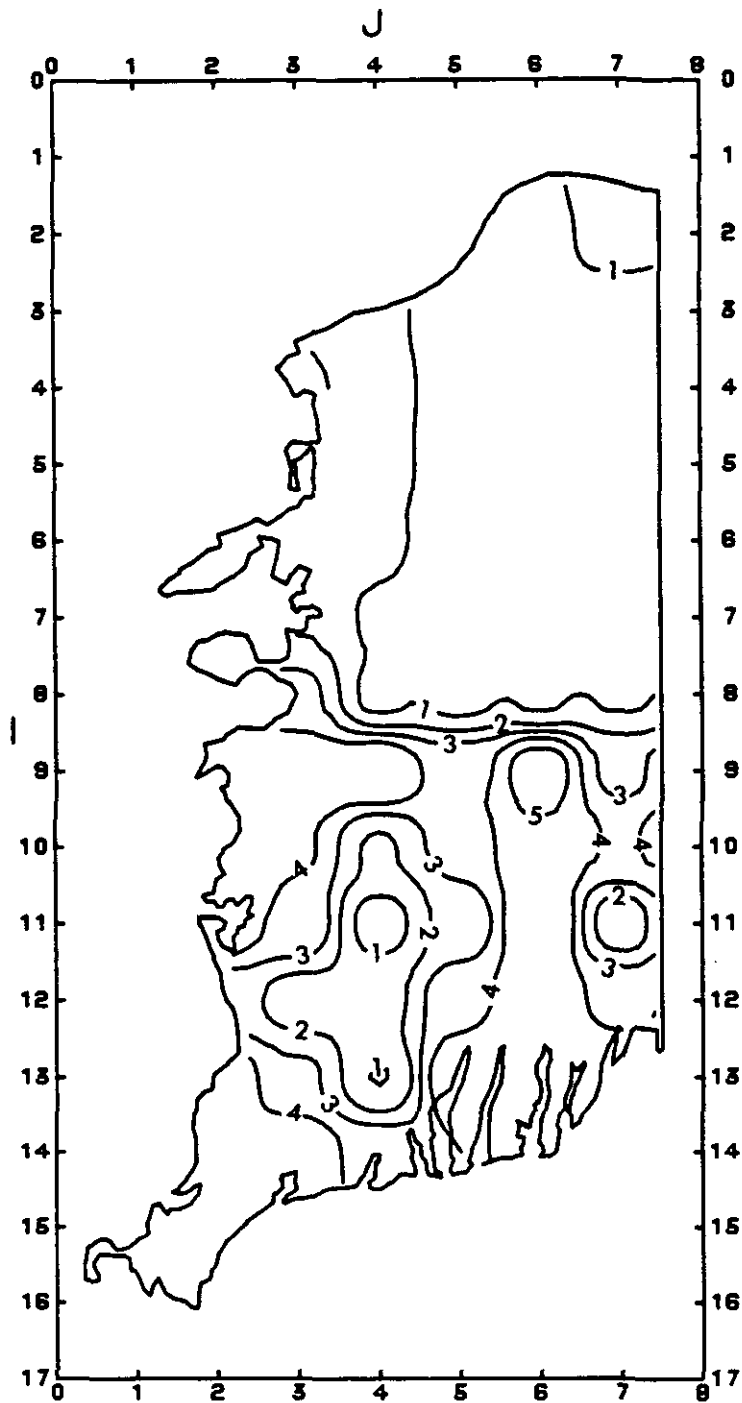
Figure 89.    Predicted steady-state nitrate nitrogen concentration
              (mg/l) contours over Bourne and Falmouth (at maximum
              feasible development of Falmouth for a specified land use
              density limit of 375 houses/sqkm and a global nitrate
              nitrogen standard of 5 mg/l).
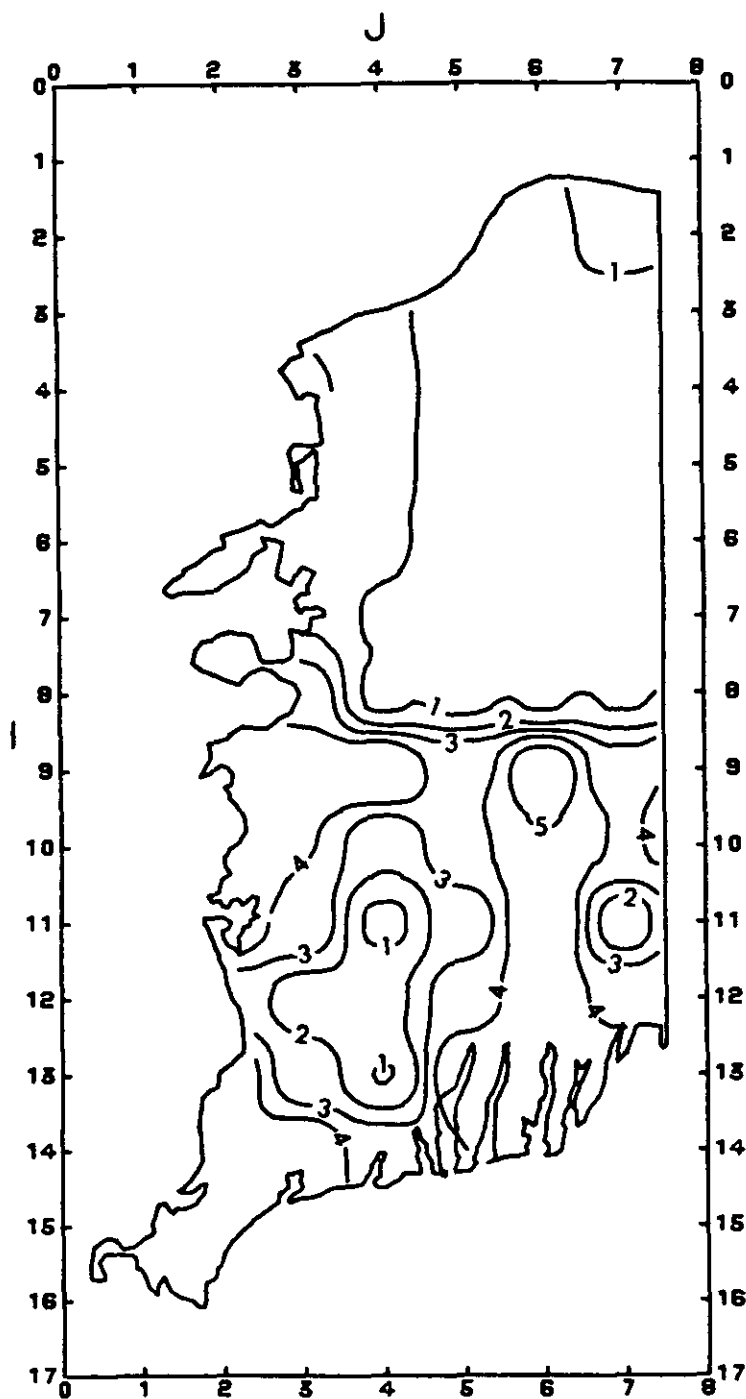
Figure 90. Predicted steady-state nitrate nitrogen concentration (mg/l) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

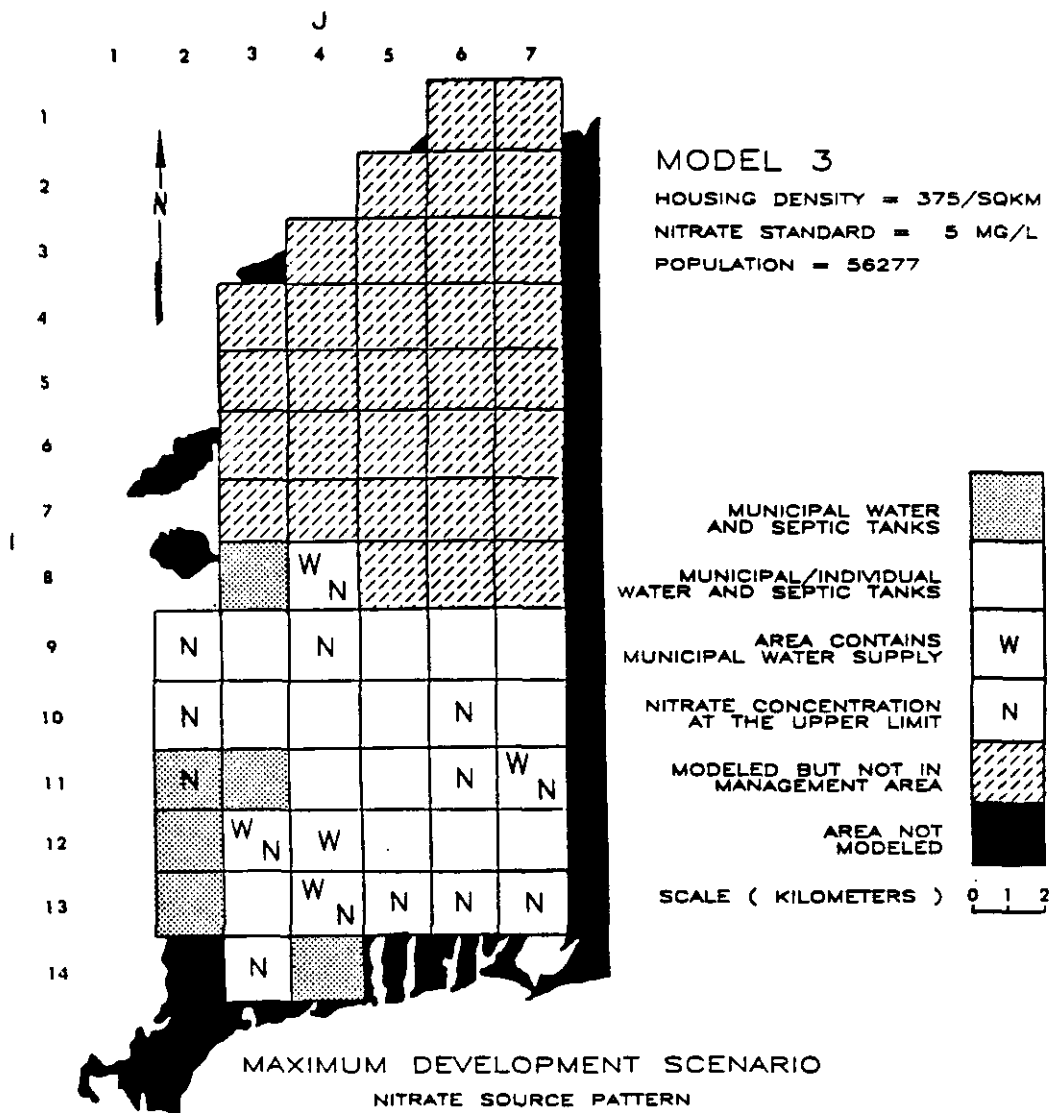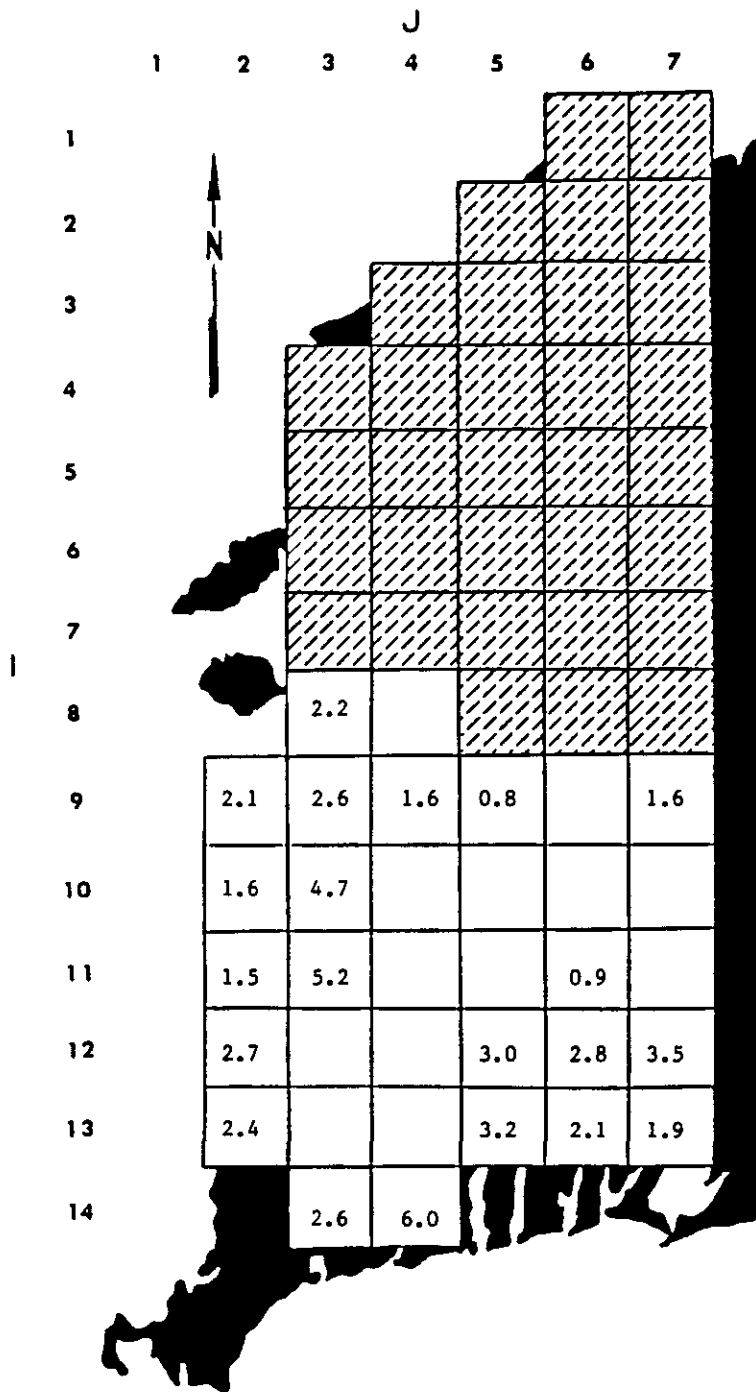Figure 91. Optimum values of dual variables associated with binding elemental land use density constraints (from the solution of Model III under a land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l). Numbers represent the additional population growth from allowing one more housing unit in appropriate elements.

The optimal values of dual variable associated with binding water quality constraints are displayed in Figure 92 from the solution where the applied density restriction was again 250 houses/km$^2$. For the water quality constraints which specified nondegradation of water quality at municipal water supplies, some of the dual variables decreased as more land use activity was allowed per unit area. The values of the dual variables express the marginal increase in the number of people who could be located in Falmouth if the associated nondegradation constraint was relaxed by 1 mg/l as nitrate nitrogen. The reduction in the value of the dual variable could indicate that Falmouth would have less to gain (measured in terms of population increases) by allowing more pollution in their water supplies if elsewhere development has proceeded to the extent of binding global water quality constraints (such as when the allowable source density limit is greater than 250 houses/km$^2$). This is equivalent to saying that the preserved area around a well becomes less important in providing additional room for development if growth elsewhere has forced changes in water quality elsewhere to the limits of acceptability.

The dual variables associated with the binding global water quality constraints did not appear to change as the nitrate concentrations in more regions approached the global standard. The stability may reflect that any new growth would occur only in the element where the water quality constraint was relaxed. It also reflects that to pollute an additional 1 mg/l nitrate nitrogen a specified number of people are required to add that nitrogen.
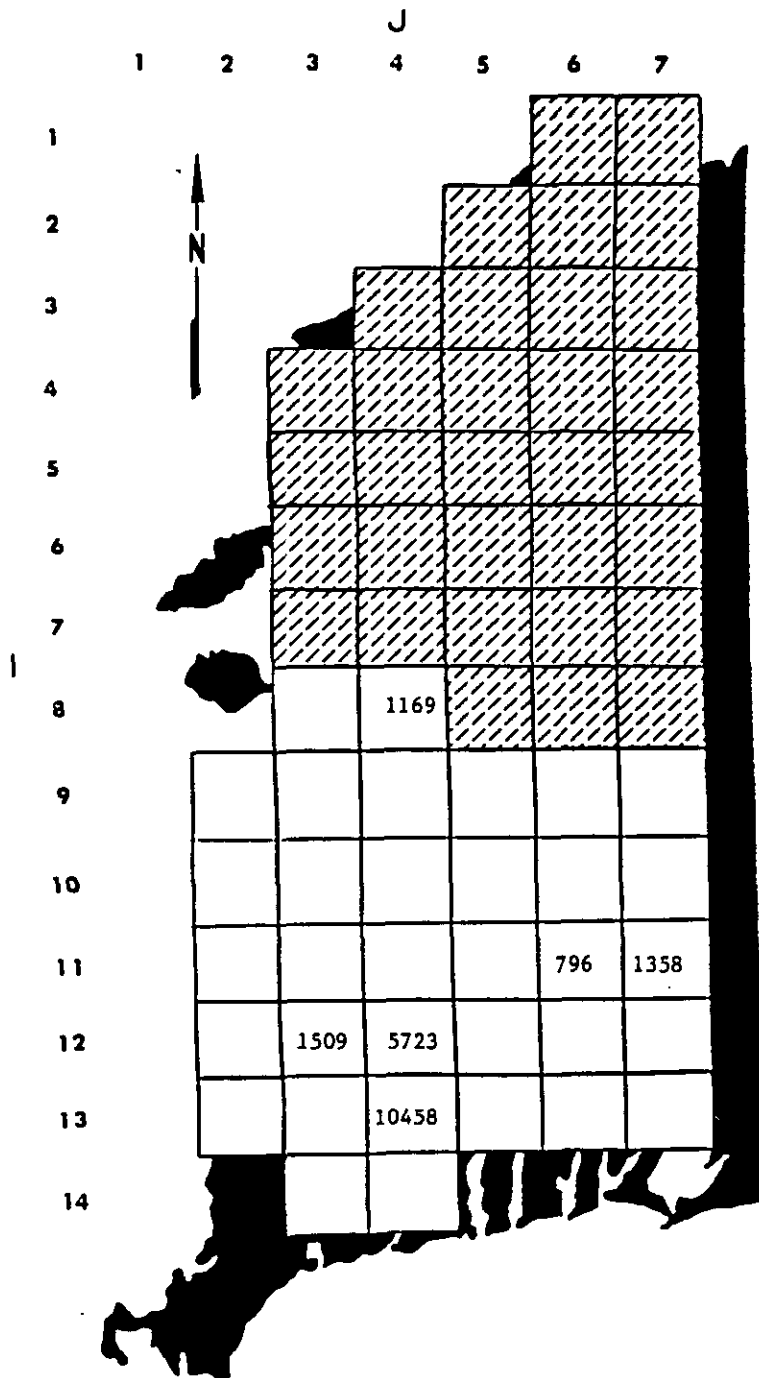
Figure 92. Optimum values of dual variables associated with binding elemental water quality constraints (from the solution of Model III under a land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l). Numbers represent the additional population growth from allowing one more mg/l of nitrate nitrogen in appropriate elemental groundwaters.

8.3 <u>Conclusions</u>

The application of Model III to Falmouth, Massachusetts illustrated the successful identification of optimal residential/commercial development patterns which incorporate existing development, accommodate maximum population growth, preserve water quality within standards, satisfy source density regulations, and operate within available resource limits. Optimal growth patterns varied with regard to type, location and density of the land use activities developed. Under conditions where global water quality constraints were nonbinding development approached maximum feasible uniformity. Alternatively, when global water quality constraints were effectively constraining development, they operated to restrict development in the interior while growth along the coast reached maximum allowable levels.

The nondegradation constraints defined zones where additional development was unacceptable. These constraints were a dominant factor in the design of optimal development patterns. Outside this 'zero growth zone' expanding development is determined by global water quality constraints and source density constraints. The dual variable associated the nondegradation constraints generally increased as the global nitrate nitrogen standard was increased; that is, the development opportunities associated with allowing further nitrate pollution in 'zero development' zones increased as the global nitrate nitrogen standard increased.

Relaxing the global water quality standard increased the real assimilative capacity of the aquifer and as a result land use activity

expanded to fill the increased capacity. For a given set of nondegradation constraints and a given source density limit there is a minimum global standard above which the optimal development pattern is no longer defined by binding global water quality constraints. For Falmouth this level was 8 mg/l nitrate nitrogen under a maximum source density limit of 500 houses/km$^2$, and approximately 5 mg/l for a density of 200 houses/km$^2$ (see Figure 93).

The imposition of global water quality constraints can lead to nonuniform development opportunity because preserving water quality in a two dimensional flow field may require the restriction of development in some areas to allow preferred growth in other areas. Proposed development scenarios which strive for uniform development opportunities must be sufficiently severe that global water quality constraints remain non-binding. Other causes of nonuniform development opportunity are preexisting sources (e.g., the Otis Plume and intense coastal development) and nondegradation constraints.

There exists a land use density for a specified groundwater quality standard above which the development changes from as uniform as possible to a nonuniform pattern where development opportunity is determined by global and land use density constraints. Use of stringent density constraints yields lower regional contaminant concentrations, more uniform development opportunities, but lower maximum feasible growth. Higher density limits generate, reduced average water quality, nonuniform development opportunity, but higher feasible population growth.
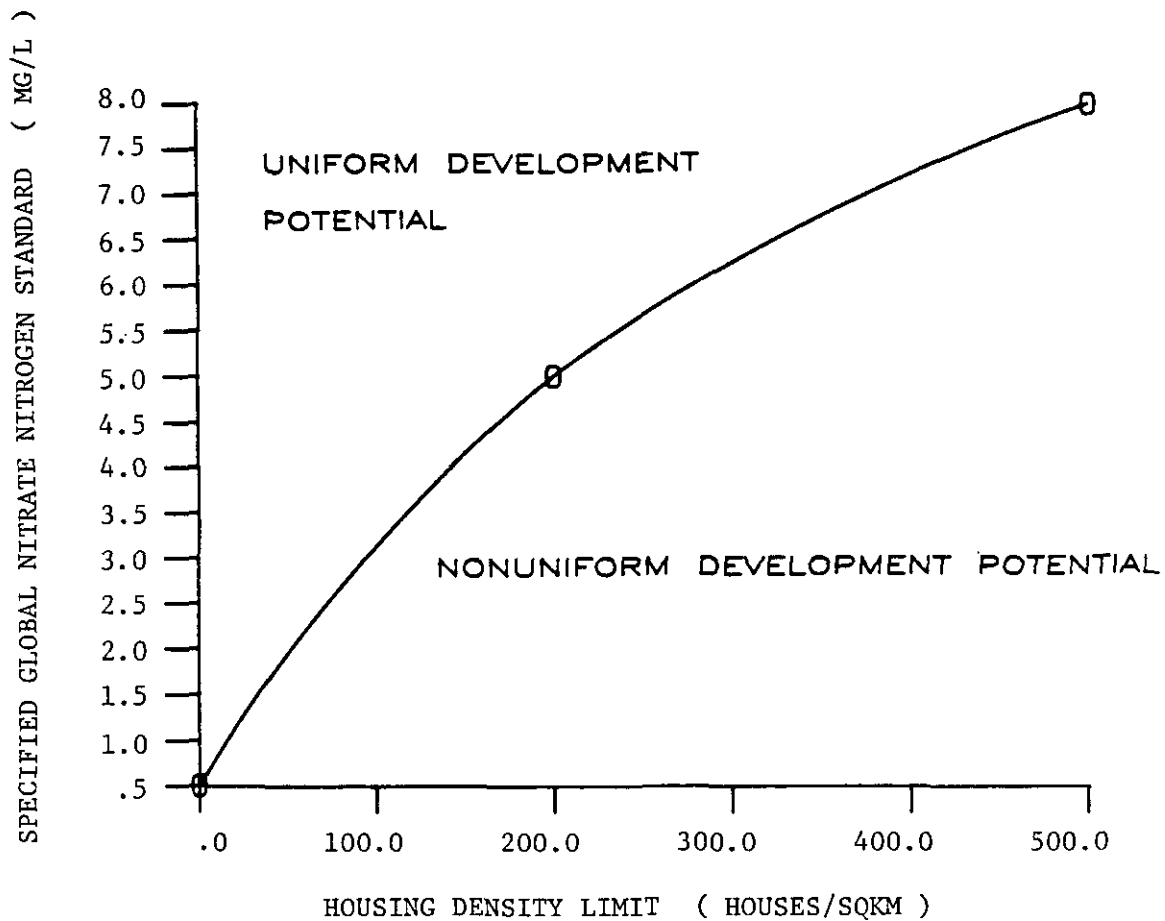
Figure 93. Regions of uniform and nonuniform maximum development opportunity in Falmouth for combinations of imposed global nitrate nitrogen standards and land use density limits.

Model III was not able to select between sources which varied in concentration of nitrate recharge flows. The model selected sources on the basis of their per capita mass loading rates and was not able to distinguish a dilute source (i.e., recharge from municipal water users) from a concentrated source (i.e., recharge from on-site well water users) because the linear continuity constraints could not incorporate the nonlinear dilution effects of disposing large volumes of diluted wastes as opposed to small volumes of concentrated waste waters to an aquifer. The errors associated with this modeling limitation were on the order of less than one percent. This is because recharge contributed from increased municipal water usage was generally small and situated in coastal nodes (or nodes near the coast) where the underlying groundwater flow operates as a buffer against significant water quality changes at the coasts.

CHAPTER 9


MODELING DEVELOPMENT SCENARIOS FOR MINIMIZING GROUNDWATER IMPACTS

FROM MULTIPLE LAND USE ACTIVITIES


The fourth and final nonpoint source groundwater pollution

management model is formulated to ascertain patterns to expand multiple

land use activities, such that the resultant groundwater quality impacts

are minimized.  The optimum pattern and combination of surface

activities incorporates the present pattern of land use development and

is identified from a specified population projection, stated development

restrictions around municipal water supplies, given restrictions on

available resources (i.e., land and water), imposed water quality

standards, and specified housing density regulations.  The specified

population projection is the anticipated development level at some

specified future time; it represents the minimum amount of growth which

must be included in the study region.

Model IV is applied to the same section of Western Cape Cod (the

town of Falmouth, Massachusetts) as Model III, and the general

formulation was adapted, as with Model III, to identify the regional

patterns of the three forms of residential/commercial land use evaluated

in Chapter 7.  The fourth model determines the combination of surface

activities required to accommodate a projected year-round population.

The optimal development pattern is identified from a large set of

feasible residential/commercial land use patterns which incorporate

present (1980) growth in Falmouth. Each development scenario of the feasible set satisfies constraints on desirable elemental nitrate nitrogen levels over the entire region, constraints on available water supplies, and constraints representing imposed regulations on specific land use activities.

Model results include: 1) contour plots of the minimum feasible steady-state regional groundwater nitrate distribution from the projected residential/commercial development; 2) the optimal population predictions for each element and for each class of residential/commercial land use; 3) maps illustrating optimum locations of growth; 4) maps showing the degree of development and the long term status of water quality in each element; and 5) figures depicting the optimum values of dual variables associated with constraints on source densities and water quality.

## 9.1  Formulation of Management Model IV

Model IV is a direct adaptation of Model III, with the same decision variables and the continuity constraints. Changes were made in the management constraint set and objective function. The formulation presented will evaluate the management of a combination of three composite land use activities known to contribute areal contamination of groundwaters; however, the formulation is general and the variety of surface activities considered, could be expanded to incorporate any number. Details on model components are presented below.

### 9.1.1 Objective Function

The optimum value of the objective function is the minimum average contaminant distribution in the management area that is feasible under the projected population scenario. The objective is a summation of all elemental contaminant variables in the study area.

$$\text{Minimize} \quad \frac{1}{\alpha} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} \cdot \rho_{i,j} \tag{86}$$

where $\rho_{i,j}$ = the value of one, if the element i,j is in both the region where contaminant transport is modeled and in the area where sources of groundwater pollution are being managed. It is equal to zero otherwise (such as 1 for Falmouth elements, and 0 for Bourne elements).

$\alpha \quad = \sum_{i=1}^{n} \sum_{j=1}^{m} \rho_{i,j}$, which equals all the elements in the management area.

### 9.1.2 Management Constraints

Most of the management constraints employed in Model III were also used in Model IV. Model modifications affected management constraints designed to incorporate present levels of land use activities into the optimal solution (see constraints 78 and 79). Rewritten below, the modifications restructured inequality constraints 78 and 79 into equality constraints for elements containing municipal supplies. These changes ensure that feasible values of recharge decision variables associated with the three land use types ($W_{i,j}$, $Z_{i,j}$, and $V_{i,j}$) reflect

existing and only existing levels of land use activities in these elements. The modified constraints are written as

$$D_{z_{i,j}} \cdot Z_{i,j} = P_{z_{i,j}} \qquad \forall \ i \text{ and } j \text{ where a municipal water supply is situated} \qquad (87)$$

$$D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} = P_{w_{i,j}} \qquad \forall \ i \text{ and } j \text{ where a municipal water supply is situated}$$

$$(88)$$

Constraints (87) and (88) preclude any increase in residential/commerical activity in elements containing municipal water supplies. All other elements continue to have management constraints like (78) and (79). For appropriate elements, constraint (87) sets values for the decision variables $Z_{i,j}$ to reflect permanent discrete populations responsible for these elemental recharge flows. Opportunities to switch from one land use activity producing recharge flow $W_{i,j}$ to another generating flow $V_{i,j}$ are feasible in elements containing municipal water supplies; however, constraint (88) ensures that the optimum long term combination of recharge flows $W_{i,j}$ and $V_{i,j}$ reflects a population equal to the original number of people ($P_{w_{i,j}}$) contributing to the elemental flows $W_{i,j}$.

Beyond the specific constraint modifications which restrict new development around municipal water supplies, Model IV contains a constraint which specifies a minimum level of residential/commercial

development (expressed as a minimum population level) to be accommodated
within the boundaries of the management area. The constraint operates
to force consideration of development levels which represent projections
of future growth. The formulation of this minimum development
constraint is a summation of all elemental subsurface recharge flows
generated from the three residential/commercial land use activities.
Each decision variable is addressed with an appropriate coefficient
which changes flows to population. The right-hand-side of the
constraint is the future population to be located in the management area
with a minimum impact on average regional groundwater quality. The
minimum population constraint is written as

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left[ D_{z_{i,j}} \cdot Z_{i,j} + D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \right] \geq P_e \qquad (89)$$

where $P_e$ = the projected population to be received in Falmouth

### 9.1.3 General Formulation of Model IV

The complete formulation of Model IV is

$$\text{Minimize} \quad \frac{1}{\alpha} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j} \cdot \rho_{i,j}$$

s.t.

Continuity Constraints:

$$[G]\{C\} - [c_w]\{W\} - [c_z]\{Z\} - [c_v]\{V\}$$

$$= [c_s]\{S\} + [c_u]\{U\} + c_q [I]\{Q\}$$

Management Constraints:

$$D_{z_{i,j}} \cdot Z_{i,j} \geq P_{z_{i,j}} \qquad \text{∀ i and j not containing municipal water supplies}$$

$$D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \geq P_{w_{i,j}} \qquad \text{∀ i and j not containing municipal water supplies}$$

$$D_{z_{i,j}} \cdot Z_{i,j} = P_{z_{i,j}} \qquad \text{∀ i and j containing municipal water supplies}$$

$$D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} = P_{w_{i,j}}$$

$$W_{i,j} \leq 0 \qquad \text{∀ i and j where appropriate}$$

$$Z_{i,j} \leq 0 \qquad \text{∀ i and j where appropriate}$$

$$V_{i,j} \leq 0 \qquad \text{∀ i and j where appropriate}$$

$$L_{z_{i,j}} \cdot Z_{i,j} + L_{w_{i,j}} \cdot W_{i,j} + L_{v_{i,j}} \cdot V_{i,j} \leq L_{s_{i,j}} \qquad \text{∀ i and j}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} R_{z_{i,j}} \cdot Z_{i,j} + R_{w_{i,j}} \cdot W_{i,j} + R_{v_{i,j}} \cdot V_{i,j} \leq R_s$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left[ D_{z_{i,j}} \cdot Z_{i,j} + D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \right] \geq P_e$$

Nonnegativity Constraints:

$$C_{i,j}, \ W_{i,j}, \ Z_{i,j} \ \text{and} \ V_{i,j} \geq 0 \qquad \text{∀ i and j}$$

## 9.2. Application of Model IV to Falmouth, Massachusetts

Model IV was used to investigate the management of nonpoint source nitrate contamination of groundwater in the town of Falmouth, Massachusetts (depicted in Figure 35). Nitrate transport was simulated

over the entire area west of the groundwater divide with the use of

continuity constraints and contaminant decision variables ($C_{i,j}$) in both

Bourne and Falmouth. The investigation of the optimum placement and

intensity of the three composite land use activities was carried out

using additional decision variables $Z_{i,j}$, $W_{i,j}$ and $V_{i,j}$, the optimal

values of which represent elemental recharge flows from three

residential/commercial land use activities in elements over Falmouth

alone.

The three land use activities contributing nonpoint source

groundwater nitrate pollution were; 1) recharge flows ($Z_{i,j}$ for all

elements i,j in Falmouth) from the combined domestic and commercial use

of municipal water, septic systems, and lawn fertilizers, 2) recharge

flows ($W_{i,j}$ for all elements i,j in Falmouth) produced from the combined

domestic and commercial use of on-site well water (as opposed to

municipal water), septic systems, and lawn fertilizers, and 3) recharge

from domestic and commercial activities which have abandoned on-site

wells for municipal water and therefore, produced recharge flows ($V_{i,j}$

for all elements i,j in Falmouth) from the combined domestic and

commercial use of municipal water, septic systems, and lawn fertilizers.

Other sources of groundwater nitrate contamination in Falmouth (i.e,

sewer exfiltration, natural recharge, and subsurface disposal of

secondary sewage) were treated as constant fluxes in the continuity

constraints (terms on the right-hand-side). Land use activity in Bourne

was not evaluated with the model; consequently, all sources of nonpoint

source groundwater nitrate contamination there were treated as constant nitrate fluxes in continuity constraints.

### 9.2.1 Data Requirements for Model IV

Model IV has the same underlying hydrologic and contaminant transport conditions as those used in Models I and II. The data requirements are the same as those identified for Model III; for details the reader is referred to Chapter 8. The only additional information required to construct Model IV, is the projected residential/commercial development level (expressed as projected population for the area) to be situated in an optimum pattern over the study area.

### 9.2.2 Constructing and Solving Management Model IV

A Fortran program was written to create a computer file containing the objective function and constraints of Model IV. The program embodied the groundwater flow model to define fluid velocity coefficients necessary for the construction of the continuity constraints. Data required by the Fortran program included 1) specified minimum development condition (the minimum population to be distributed throughout the area), 2) the global nitrate standard, 3) the residential housing density regulation, 4) the steady-state predicted nitrate concentrations from 1980 development (obtained from solving Model I or II), 5) the area and dimensions of the elements, 6) the elemental recharge rates of all sources, 7) the elemental pumpage rates, 8) the piecewise aquifer transmissivities, 9) the hydraulic and contaminant

320

model boundary conditions, and 10) the nitrate concentrations in all recharge flows.

The assembled model was a linear program containing 79 contaminant decision variables and 102 recharge decision variables. The problem constraint set included 66 continuity constraints, 68 constraints specifying present levels of land use activities, 36 constraints establishing maximum levels of land use activities, one constraint establishing a limit on available municipal water, one constraint specifying the minimum population level to be accommodated within Falmouth, and 34 water quality constraints (6 nondegradation and 28 global water quality). Thirteen imaginary contaminant decision variables and thirteen constraints were employed to ensure a no flux condition along the groundwater divide when the central difference molecules were used on boundary nodes. The total number of constraints was 219 and the total number of decision variables was 181. Several executions of the general model were performed where different minimum development projections were used while the global water quality standard and land use density limit were held constant.

### 9.2.3  Model IV Results

Model IV identified an optimum combination and intensity of elemental residential/commercial land use activities which have a minimum impact on average groundwater quality in Falmouth for a projected population level; this was achieved while simultaneously satisfying all other model constraints (i.e., water quality, present

development pattern, etc.). Multiple runs of Model IV were performed that span a range of projected population levels (35,000 - 50,000) for Falmouth. The various model runs were intended to identify for each project population the minimum groundwater nitrate distributions and the associated optimal development pattern.

### 9.2.3.1 Groundwater Protection

The protection of groundwater resources in Falmouth under the various optimal development plans was evaluated in terms of predicted groundwater nitrate changes under different projected population increases for the town.

Figure 94 shows the average steady-state nitrate nitrogen levels for Falmouth under a range of projected population levels. Average nitrate nitrogen concentrations increased from 2.1 mg/l (in 1980 for a population of 26,926) to 2.7 mg/l for a projected population of 50,000. Average groundwater nitrogen levels increased at one third the growth rate of population.

The ability to protect the average water quality in Falmouth against degradation from future development was founded in a general pattern of development identified by Model IV. Comparisons of average water quality in Falmouth from different development patterns accommodating the same population are shown in Figure 95. Higher average nitrate concentrations were identified with the management Model III where different land use density constraints were used to obtain maximum population and the associated average nitrate concentrations.
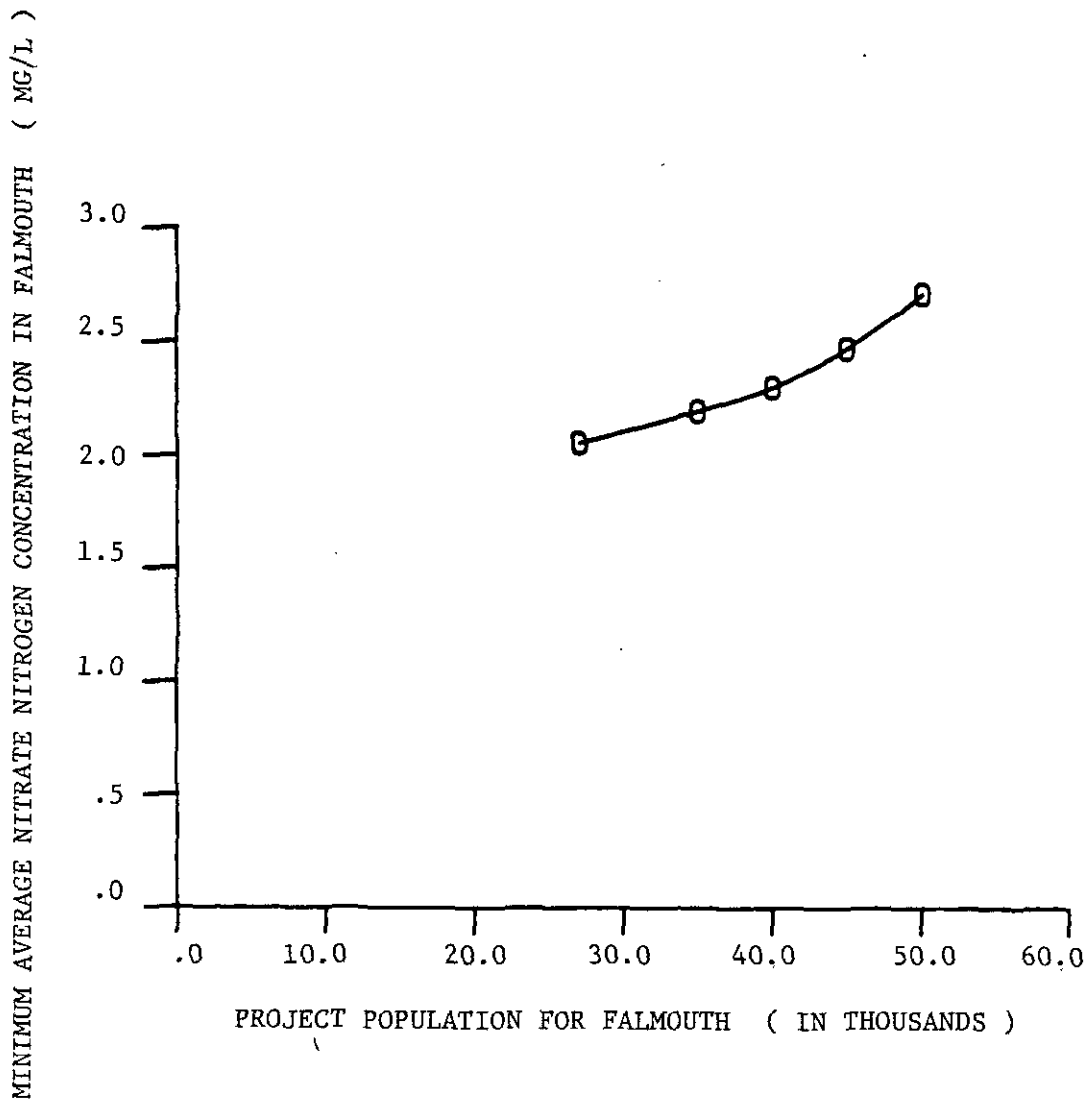
Figure 94.   Predicted minimum average nitrate nitrogen concentrations
for Falmouth as a function of projected population (for
development patterns of minimum groundwater impact under a
specified land use density limit of 500 houses/sqkm and a
global nitrate nitrogen standard of 5 mg/l).
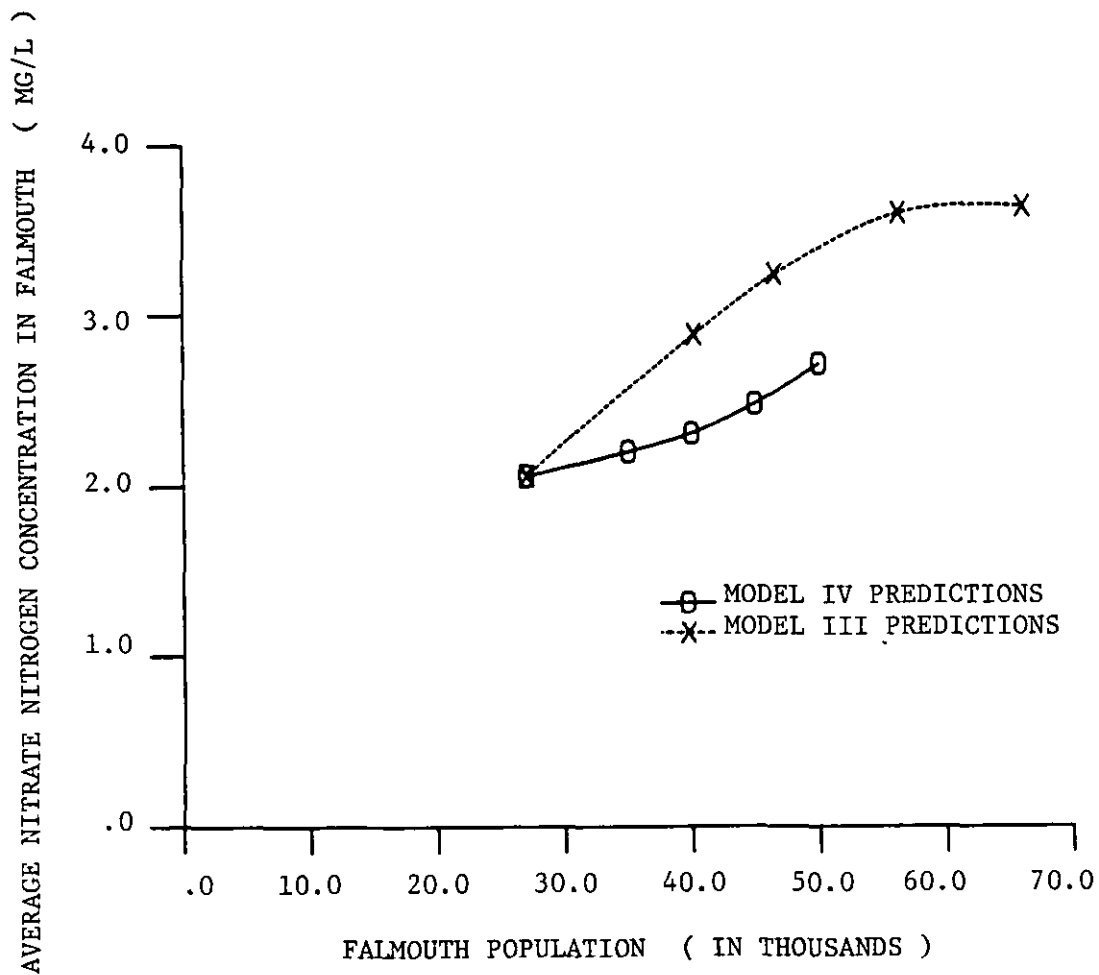
Figure 95.   Predicted average nitrate nitrogen concentrations in
             Falmouth as a function of population.  Model III
             predictions were obtained from varying the land use density
             limit from 200-500 houses/sqkm.  Model IV results are
             described in Figure 94.

For equivalent populations, the development pattern identified with Model III was more uniform and allowed more development in the interior regions than the development scenario mininum groundwater impact (as determined by Model IV). Development at interior elements (recharge zones) are known to have greater impacts on average regional groundwater quality compared to development along the coast (discharge zones). Consequently, for equivalent populations, a uniform development pattern (which forces interior growth) leads to higher average nitrate levels than a development pattern where population is situated near regional discharge zones. These results represent members of a set of potential development patterns which could accommodate equivalent populations. The results show that the different development patterns have different regional groundwater quality impacts; however, the set of development scenarios for any given population will be determined by the numerical values of the land use density constraints, the global nitrate nitrogen constraints, nondegradation constraints, the nature of the source, and the prevailing hydrologic conditions.

Several nondegradation constraints were not binding, that is, water quality was predicted to be better than the nondegradation standard. The predicted nitrate nitrogen concentrations were, however, close ($<$ .1 mg/l as N) to the nondegradation standard. Global water quality constraints were not important to the protection of Falmouth groundwater resources until the projected population reached 50,000. Further degradation of water quality at coastal nodes was precluded with binding global water quality constraints.

Figures 96-99 illustrate the minimum steady-state nitrate nitrogen distributions in Falmouth for development patterns accommodating projected residential/commercial development from 35,000 to 50,000 people. Water quality in elements containing Falmouth's major water supply (i,j = 12,3 - 12,4 and 13,4) is preserved below 2 mg/l as N even at population projections of 50,000 people. Increased development along the coast creates nitrate concentration contours which run parallel to the coastline. The peak nitrate level for the region appears in element i,j = 9,6 where Otis sewage treatment plant is located. At the highest population projection modeled (50,000), 29 percent of the region was in danger of violating water quality standards; this included six elements with nondegradation constraints and four binding global water quality constraints. In elements where new development was desirable, 22 percent of those elements were on the verge of exceeding allowable global nitrate nitrogen levels.

The groundwater protection tradeoffs associated with relaxing land use density and water quality constraints were investigated through the optimal values of the dual variables. For the binding constraints, the values of the dual variables were used to interpret marginal changes in the regional average groundwater nitrate levels in Falmouth for unit relaxations of the constraints. The values of the dual variables associated with binding land use density constraints are presented in Figure 100. These were obtained from the solution of Model IV where the housing density restriction was 500 household per square kilometer, the regional nitrate nitrogen standard was 5 mg/l and the projected
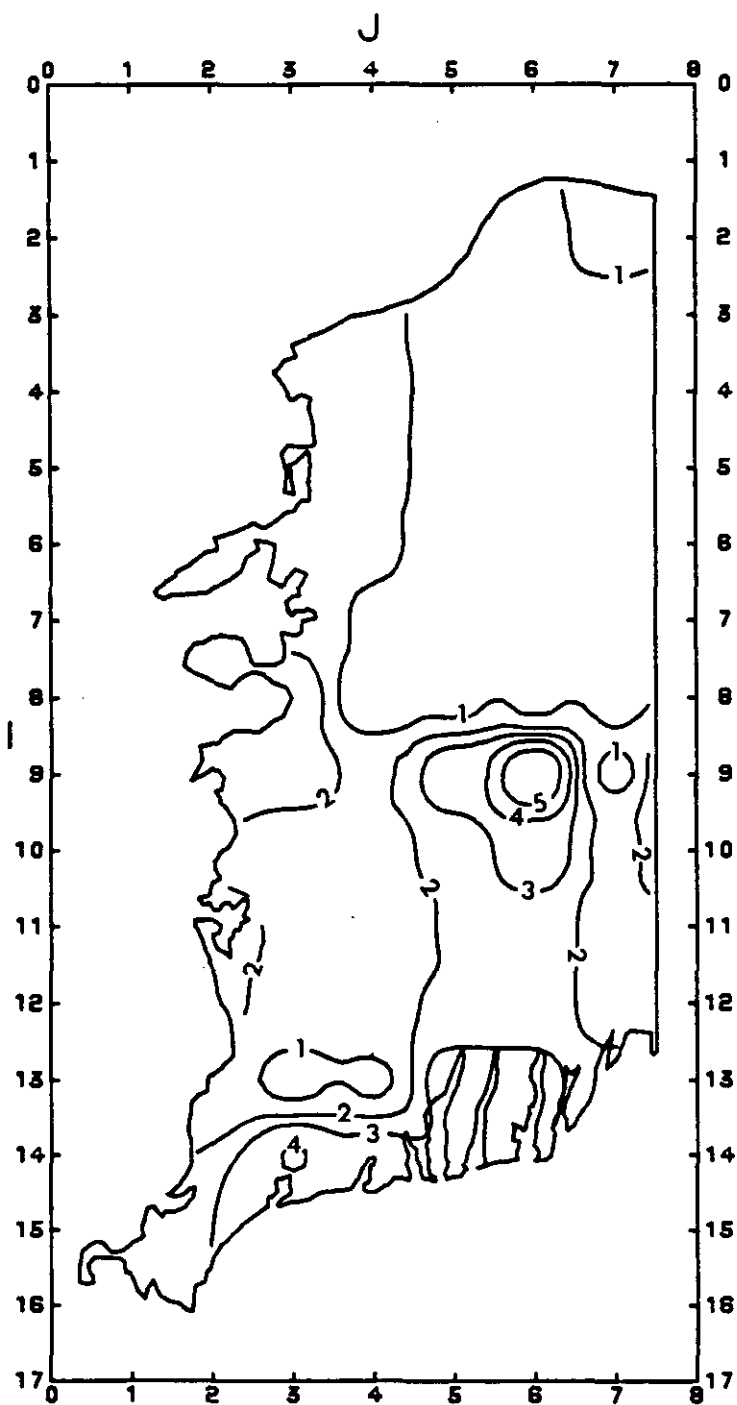
Figure 96.    Minimum steady-state nitrate nitrogen concentration (mg/l)
contours over Bourne and Falmouth predicted by Model IV
(for a projected Falmouth population of 35,000, a land use
density limit of 500 houses/sqkm and a global nitrate
nitrogen standard of 5 mg/l).

Figure 97.    Minimum steady-state nitrate nitrogen concentration (mg/1) contours over Bourne and Falmouth predicted by Model IV (for a projected Falmouth population of 40,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/1).
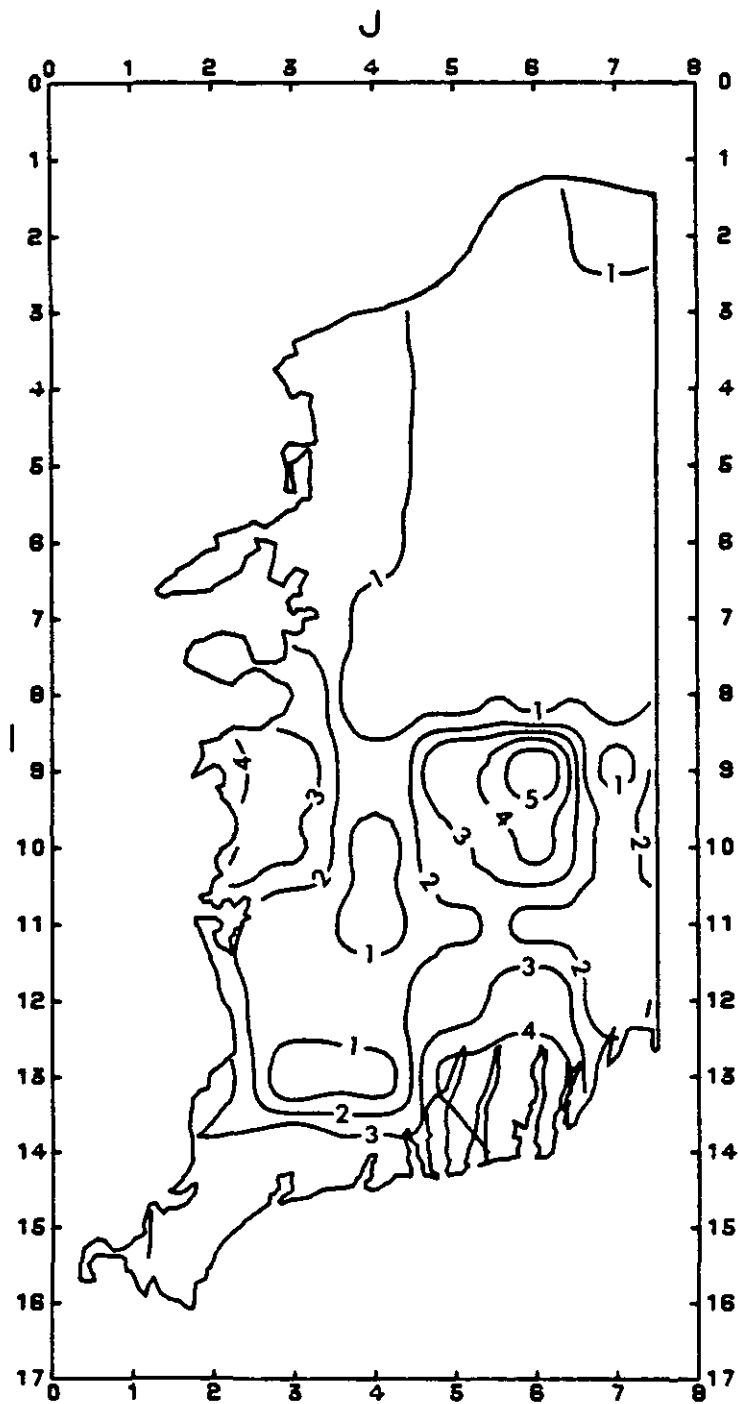
Figure 98.  Minimum steady-state nitrate nitrogen concentration (mg/1)
contours over Bourne and Falmouth predicted by Model IV
(for a projected Falmouth population of 45,000, a land use
density limit of 500 houses/sqkm and a global nitrate
nitrogen standard of 5 mg/1).
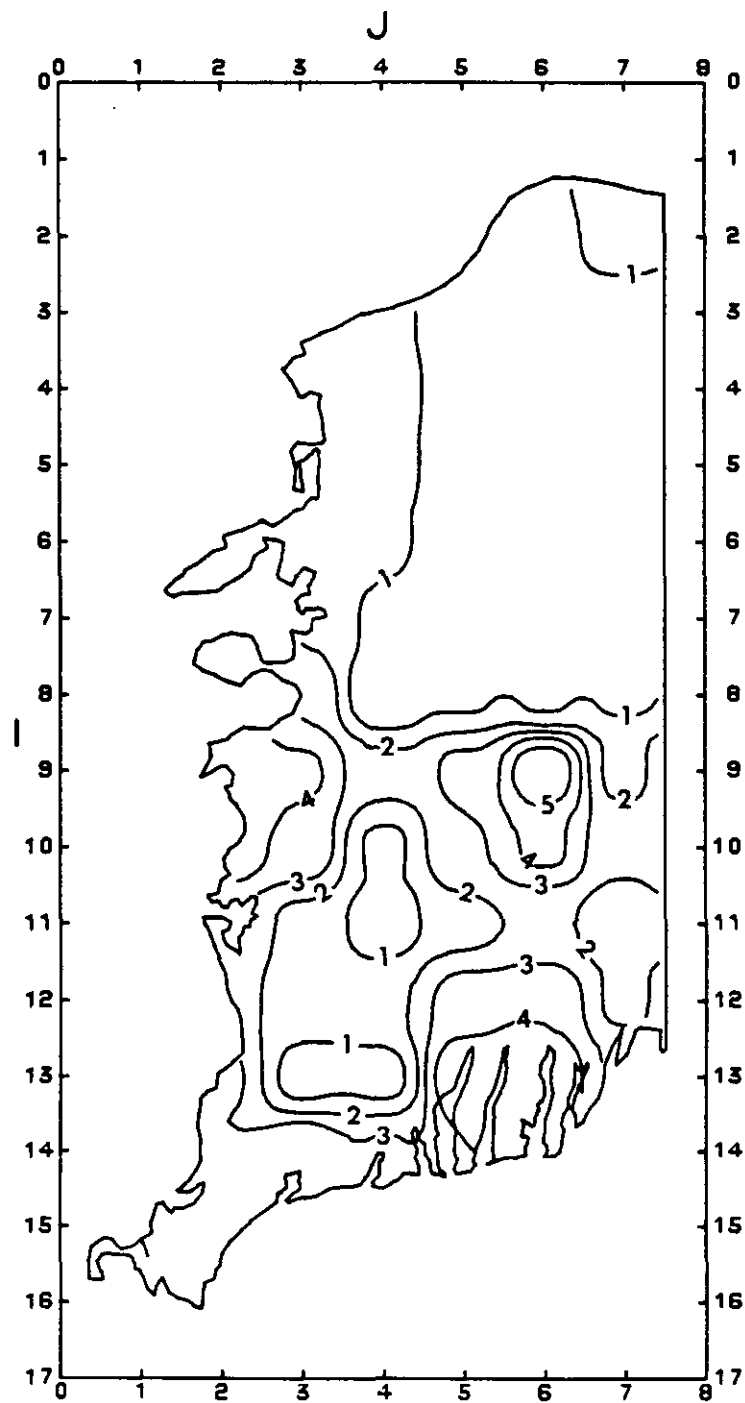
Figure 99.   Minimum steady-state nitrate nitrogen concentration (mg/l)
             contours over Bourne and Falmouth predicted by Model IV
             (for a projected Falmouth population of 50,000, a land use
             density limit of 500 houses/sqkm and a global nitrate
             nitrogen standard of 5 mg/l).
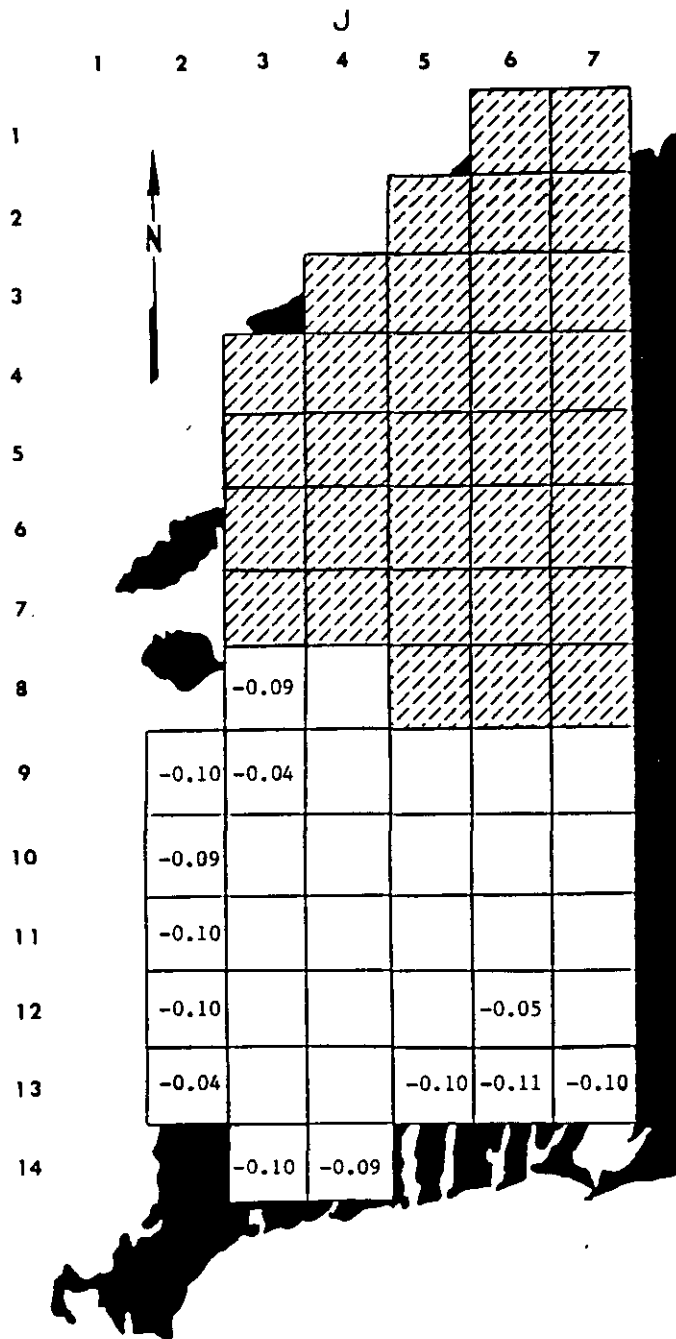
Figure 100. Optimum values of dual variables associated with binding elemental land use density constraints (from the solution of Model IV under a projected Falmouth population of 50,000, a land use density limit of 500 houses/sqkm, and a global nitrate nitrogen standard of 5 mg/1). Numbers represent the ug/1 decrease in average groundwater nitrate nitrogen over Falmouth from allowing one more housing unit in appropriate elements.

population of Falmouth was 50,000. The numbers appearing inside the elements represent the decrease in regional average nitrate nitrogen for groundwaters in Falmouth, if one more household could be added in those elements. Allowing more development along the coast would shift that development away from interior nodes. Since development in interior elements has greater impacts on average regional groundwater quality (due to its pollution of downgradient elements as well) this shift results in an overall decrease in regional average groundwater quality.

With every increase in projected growth more development is located at a greater distance inland from the coast. The average regional water quality impacts of development increase with the distance located inland from the coastline. Values of the dual variables associated with the land use density constraints increase as the projected population increases. The increase values of the duals reflects the growth in opportunity to reduce average nitrate levels by removing sources which were situated further inland with each successive increase in projected population.

The groundwater protection tradeoffs generated from relaxing the water quality constraints were shown in Figure 101. The numbers appearing in specific elements represent for those elements the marginal decrease in the regional average nitrate concentration ($\mu g/l$ as N) if their water quality constraints are relaxed by 1 mg/l as N. The municipal water supplies are viewed in the formulation as nitrate sinks. Relaxing the nondegradation constraints around municipal water supplies would permit more of the total projected development of Falmouth to be
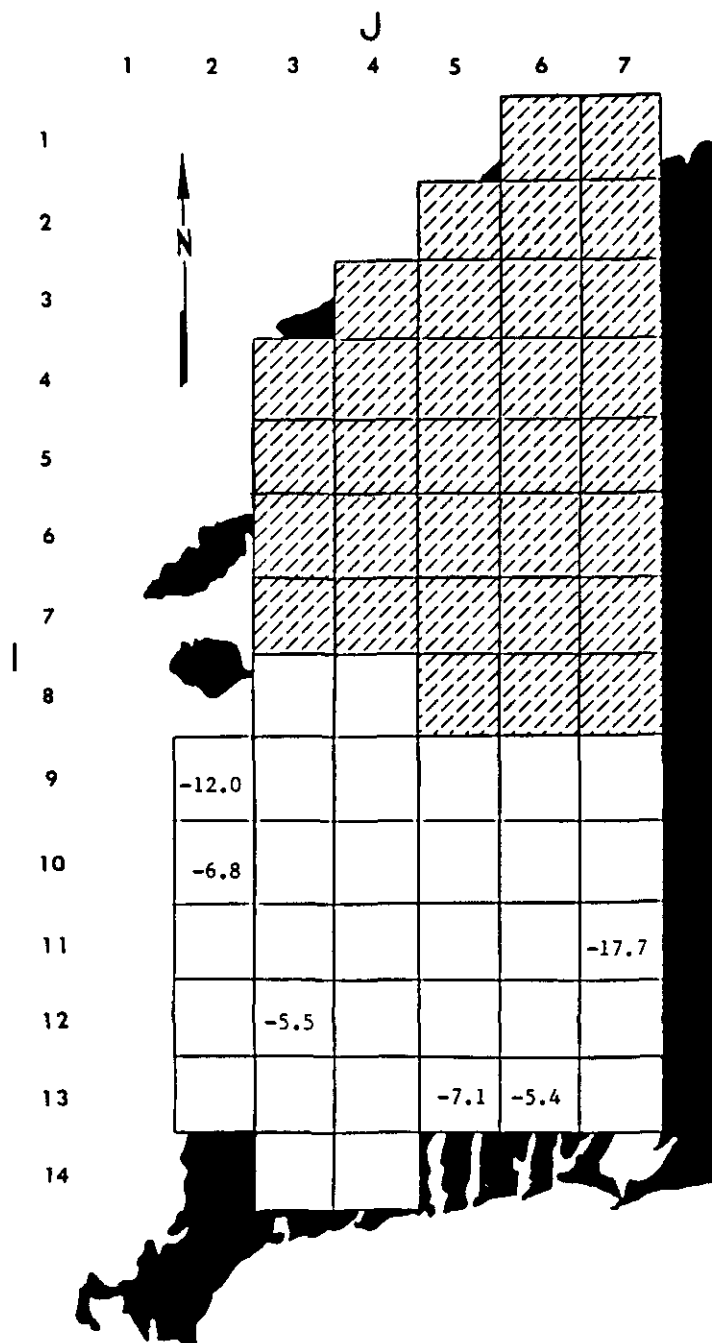
Figure 101. Optimum values of dual variables associated with binding
elemental water quality constraints (from the solution of
Model IV under a projected Falmouth population of 50,000, a
land use density limit of 500 houses/sqkm, and a global
nitrate nitrogen standard of 5 mg/l). Numbers represent
the ug/l decrease in average groundwater nitrate nitrogen
over Falmouth from allowing one more mg/l of nitrate
nitrogen in appropriate elemental groundwaters.

situated next to water supplies; hence, average nitrogen levels in the
groundwaters would decrease. Global water quality constraints were
binding along the coast. Increasing the standard from 5 to 6 mg/l as N
would allow a shift in development toward the coasts where it would
effect a lower impact on the regional average nitrate nitrogen levels.

## 9.2.3.2 Patterns of Land Use

The magnitudes and patterns of the three residential/commercial
land use activities were evaluated with land use development maps.
Figures 102-105 illustrate the patterns of development which produce the
lowest nitrate nitrogen levels in Falmouth for several population
projections. In general, growth increases from zero development in the
interior regions to maximum feasible levels along the coast.

As the population projection was increased, the percent of Falmouth
to receive added residential/commercial development increased from 29 to
52 percent for projections of 35,000 and 50,000 people, respectively
(see Figure 106). Among those elements to show growth, the percent to
achieve maximum allowable densities increased with population until the
global nitrate nitrogen constraint precluded additional development
along the coast. As shown in Figures 106 and 107, growth occurs over a
larger area when the projected population reaches 50,000, but a smaller
percent of the growth reaches maximum allowable limits after the
population exceeds 40,000.

The pattern of development in Falmouth was primarily determined by
binding density and nondegradation constraints for the range of

MINIMUM GROUNDWATER IMPACT SCENARIO
FOR PROJECTED POPULATION GROWTH
REGIONAL DEVELOPMENT PATTERN
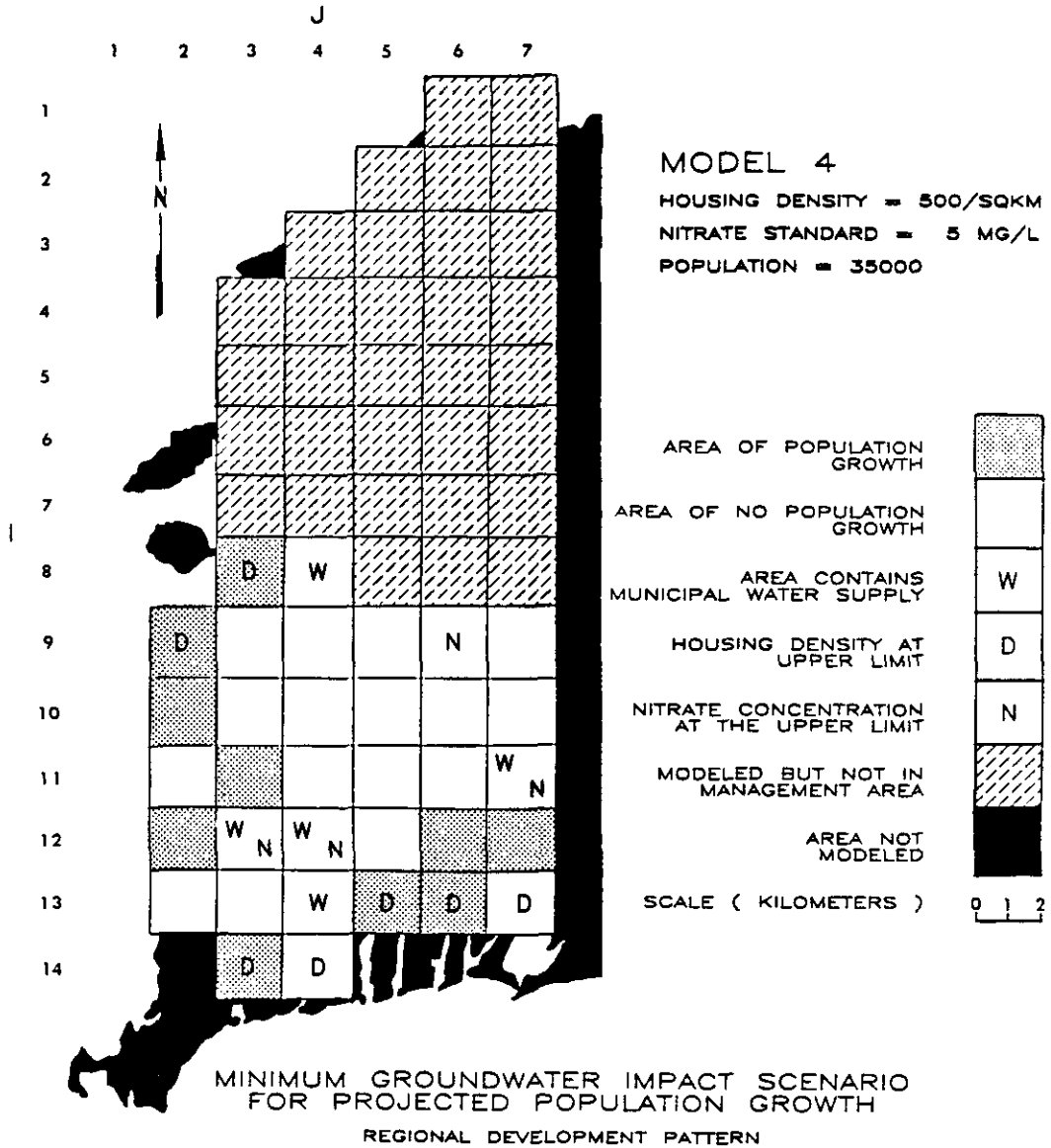
Figure 102. Falmouth residential/commercial land use development pattern for minimum groundwater quality impacts (for a projected population of 35,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).
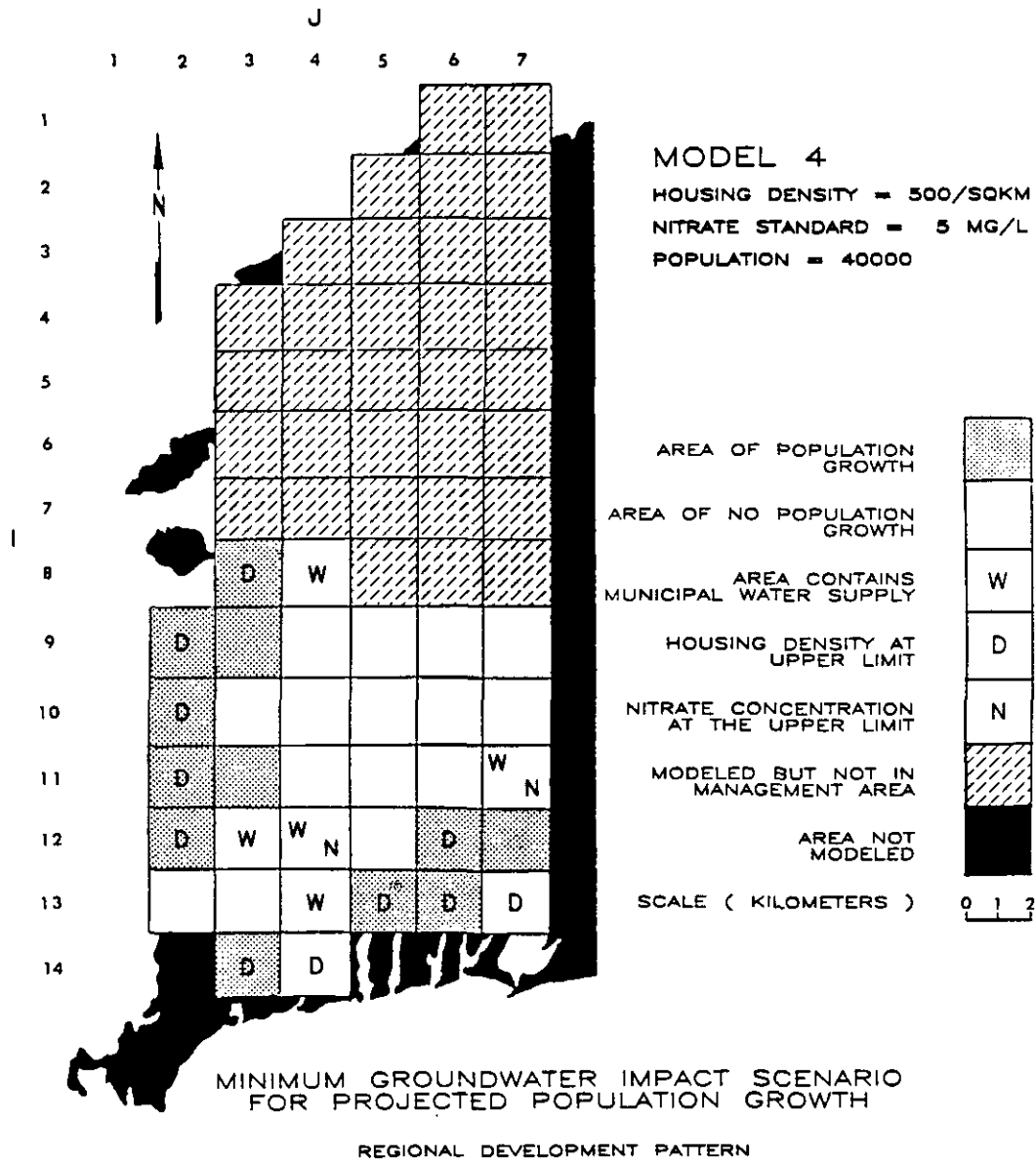
Figure 103. Falmouth residential/commercial land use development
pattern for minimum groundwater quality impacts (for a
projected population of 40,000, a land use density limit of
500 houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).

MODEL 4

HOUSING DENSITY = 500/SQKM
NITRATE STANDARD = 5 MG/L
POPULATION = 45000

AREA OF POPULATION GROWTH

AREA OF NO POPULATION GROWTH

AREA CONTAINS MUNICIPAL WATER SUPPLY

HOUSING DENSITY AT UPPER LIMIT

NITRATE CONCENTRATION AT THE UPPER LIMIT

MODELED BUT NOT IN MANAGEMENT AREA

AREA NOT MODELED

SCALE ( KILOMETERS )

MINIMUM GROUNDWATER IMPACT SCENARIO
FOR PROJECTED POPULATION GROWTH
REGIONAL DEVELOPMENT PATTERN
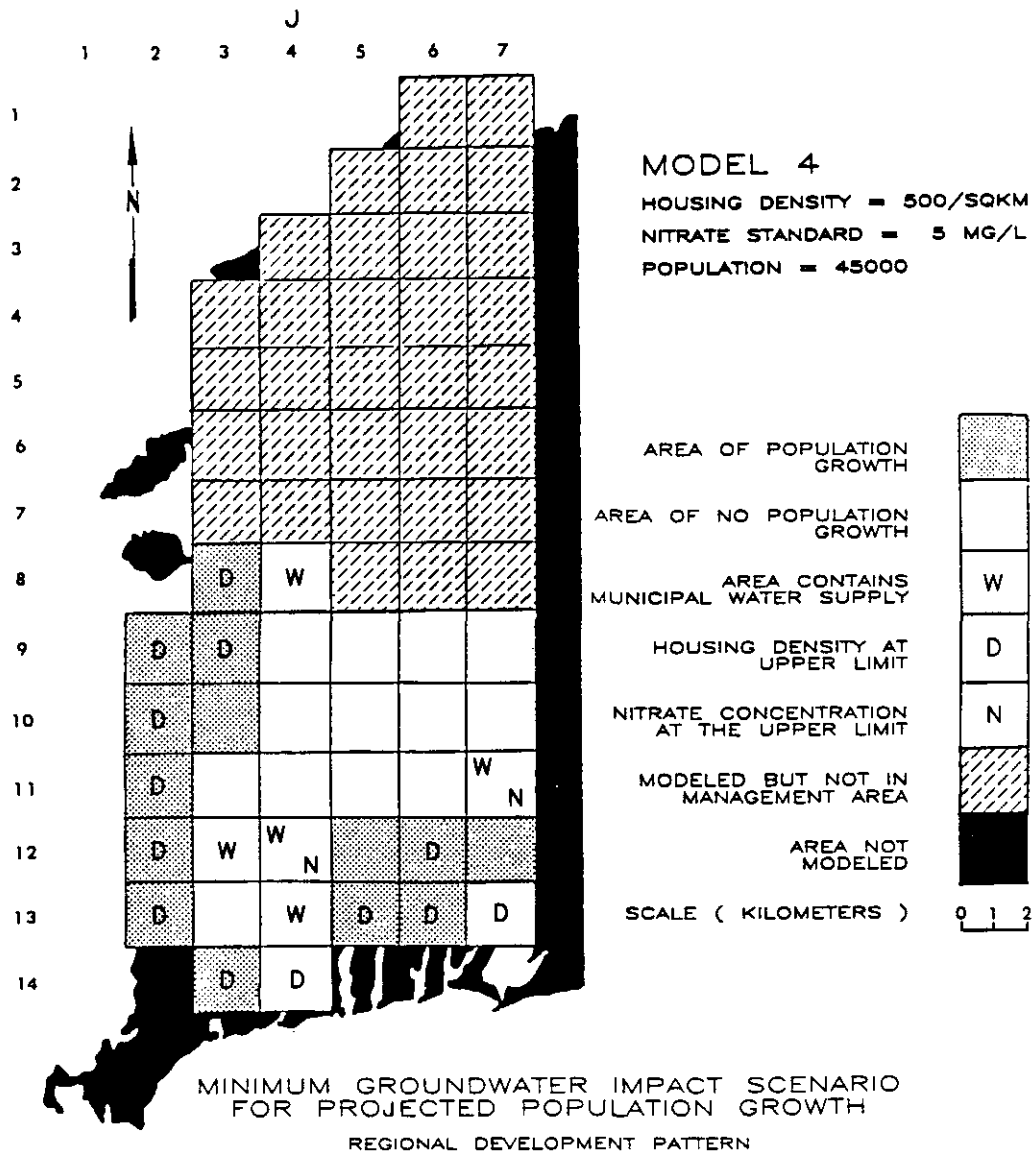
Figure 104. Falmouth residential/commercial land use development pattern for minimum groundwater quality impacts (for a projected population of 45,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).
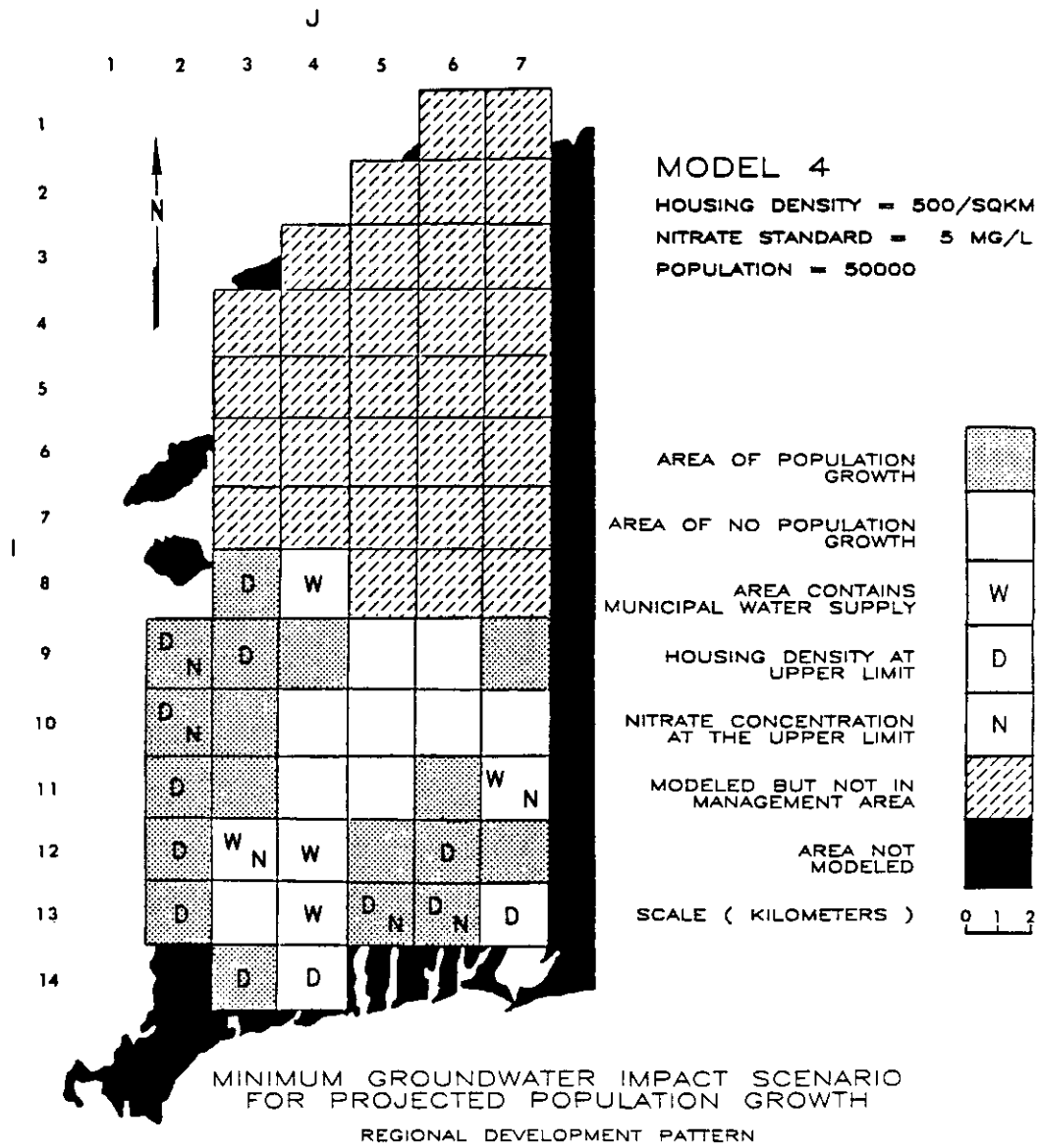
Figure 105. Falmouth residential/commercial land use development
pattern for minimum groundwater quality impacts (for a
projected population of 50,000, a land use density limit of
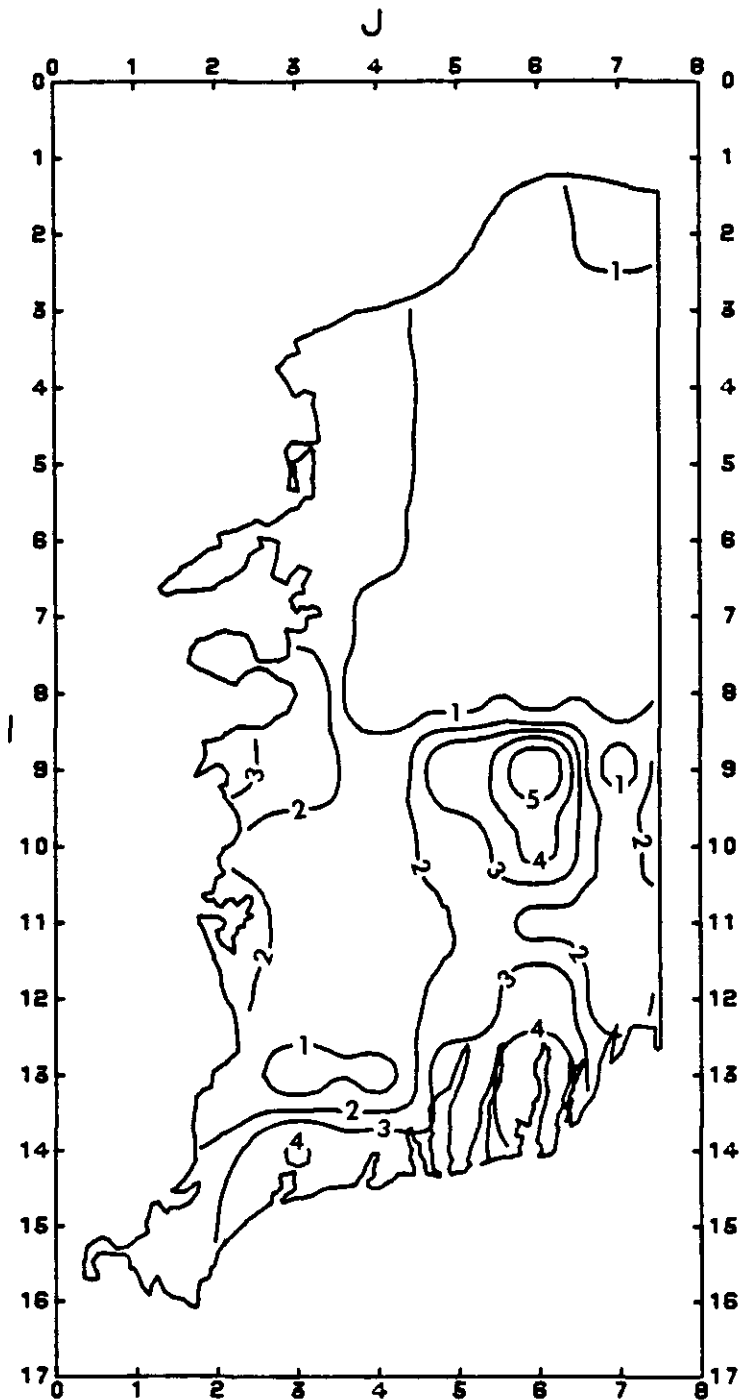500 houses/sqkm and a global nitrate nitrogen standard of 5
mg/l).
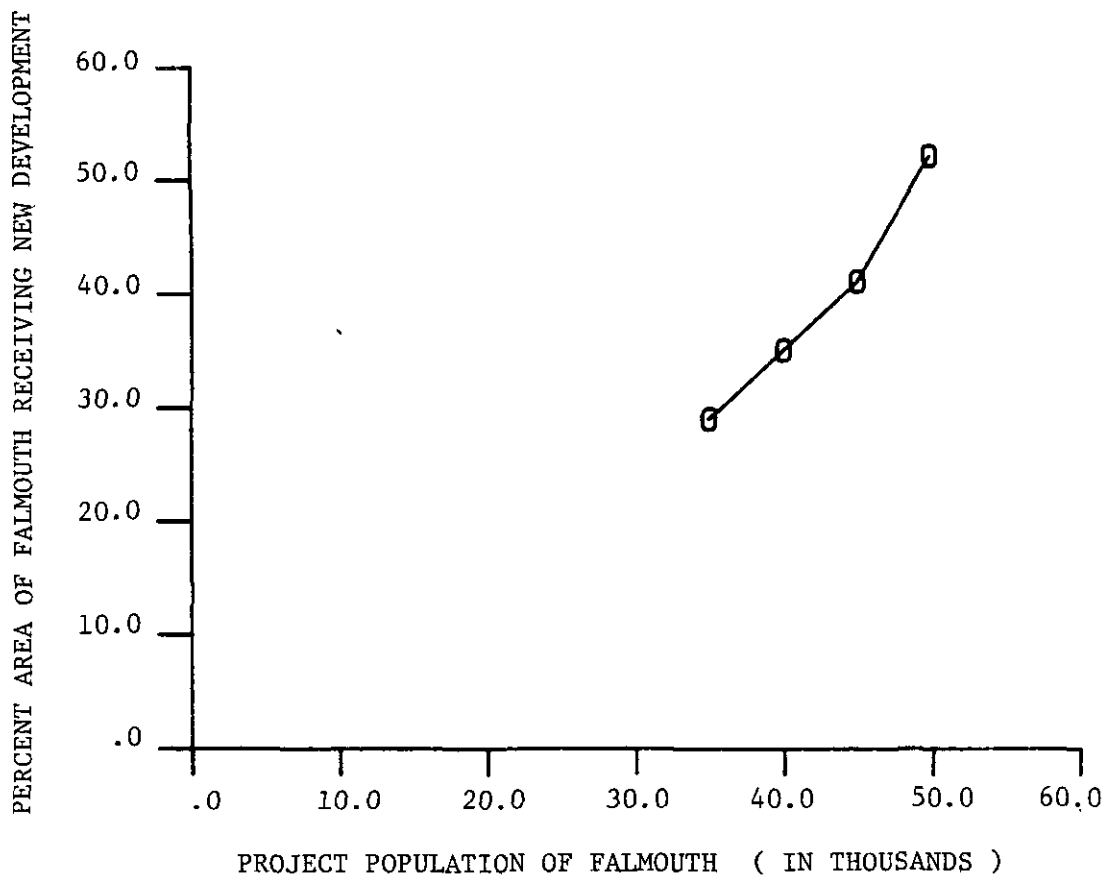
Figure 106. The percent area of Falmouth to receive additional growth
as a function of projected population (for the development
pattern of minimum groundwater impact under a land use
density limit of 500 houses/sqkm and a global nitrate
nitrogen standard of 5 mg/l).
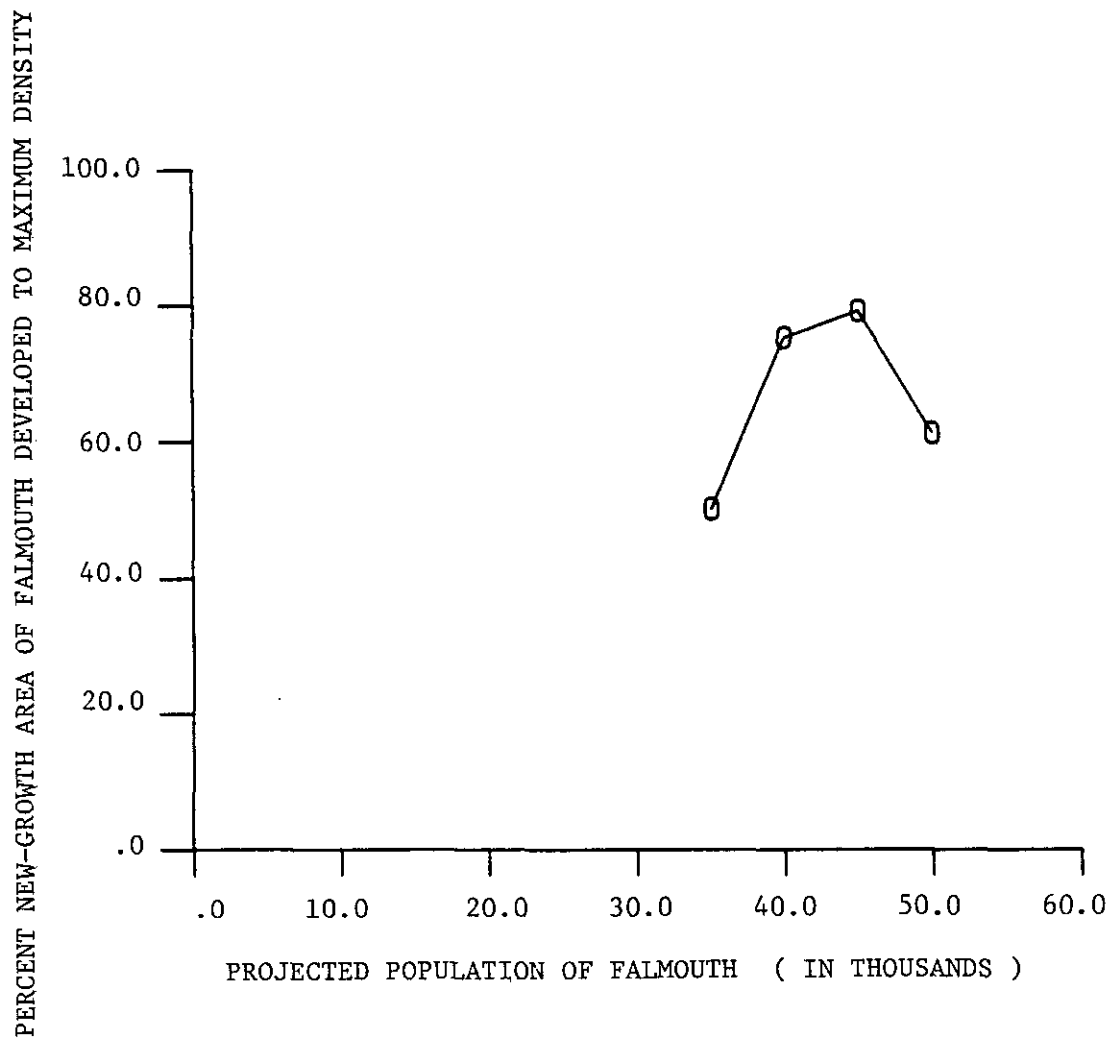
Figure 107. The percent new-growth area of Falmouth to reach the allowable land use density limit as a function of projected population (for the development pattern of minimum groundwater impact under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

projected populations evaluated with Model IV (see Figure 108). It was
not until the population exceeded 40,000 that development to the maximum
density in some elements was curtailed because of binding global water
quality constraints. During all the model runs an increased use of
municipal water was confined to only those nodes where use of septic
systems was prohibited (i,j = 11,2 and 11,3). Because municipal water
uses generate higher nitrate contaminant fluxes to the subsurface than
residential/commercial activities connected on-site well water, the
model consistently expanded on-site well water usage to keep per capita
nitrate loads at a minimum.

## 9.3 Conclusions

Application of Model IV to Falmouth, Massachusetts demonstrated the
identification of feasible development scenarios which can accommodate
specified population increases with minimal additional groundwater
degradation. The feasible development scenarios prevent additional
development in elements containing municipal supplies, but allow
development elsewhere. As long as groundwater quality remains within
global and nondegradation standards, present development is left intact,
and all source densities fall within zoning limits.

The optimal pattern of growth which leads to lower changes in
average groundwater quality is one that concentrates sources near the
discharge areas. The water quality advantages of coastal development
(over interior growth) were elucidated in Chapter 7 with results from
Model I. In the several model runs under small population projections
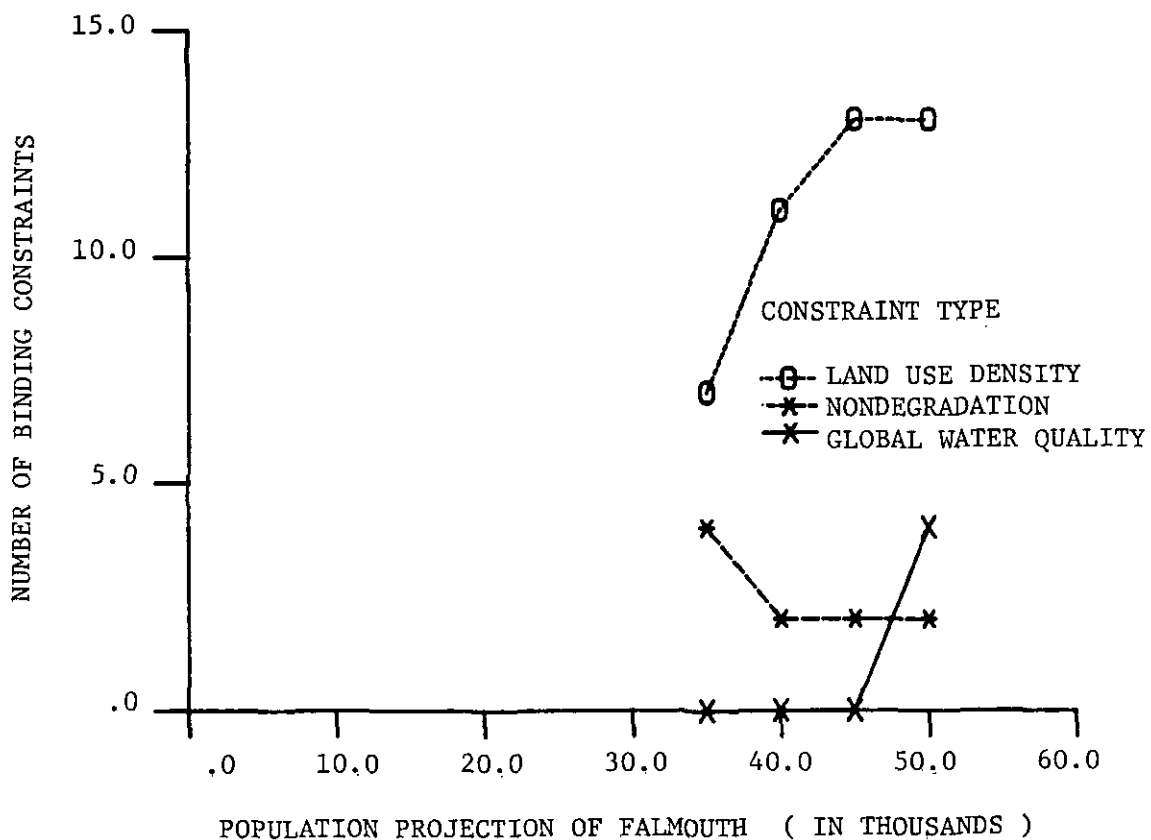
Figure 108.  Number of binding land use density, nondegradation, and global water quality constraints as a function of projected Falmouth population (for the development pattern of minimum groundwater impact under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).

development was curtailed primarily by density constraints. For higher development projections the minimum groundwater impact pattern for residential/commercial growth was determined by global water quality, nondegradation and land use density constraints.

Land use activities that necessitate conversions from on-site wells to town water or the expanded use of municipal water were all but avoided except in regions down gradient from the Falmouth sewage treatment plant; the exception included elements $i,j = 11,2$, and $11,3$ where use of municipal water was mandated for reasons of protecting health.

Comparisons were made of average nitrate nitrogen levels obtained from Models III and IV for equivalent development levels but different prevailing source density regulation. The results of the comparison point to a potential set of feasible development patterns which can accommodate the same population but have differing impacts on groundwater quality. Used in combination Models III and IV could identify many feasible development scenarios.

CHAPTER 10


CONCLUSIONS AND RECOMMENDATIONS


This research developed four models which could be used in the
evaluation of strategies for managing multiple land use activities so
that long term quality of groundwater could be protected from the
nonpoint source pollution associated with those activities.  All of the
management models were linear programs which included linear algebraic
equations from a numerical steady-state contaminant transport model as
part of the constraint set.  The nonpoint source groundwater pollution
management models were applied to a 'Sole Source Aquifer' underlying
towns of Bourne and Falmouth, Massachusetts.

Models I and II were used to delineate areas within the regional
groundwater flow system which are most critical to the preservation of
groundwater quality over the region and in specific sub-areas of the
region.  Model I revealed the relative importance of subareas within
Bourne and Falmouth which are critical to the preservation of regional
groundwater quality from dispersed nitrate pollution.  Model II
ascertained the comparable significance of protecting zones within a
recharge area surrounding a major municipal water supply.

Models III and IV were formulated to identify optimal patterns and
intensities of multiple land use activities.  Both Models will locate
multiple land use activities, set pollutant fluxes, and predict
groundwater impacts in a simultaneous fashion.  Model III determines the

maximum feasible development of a combination of land use activities. Model IV ascertains patterns to expand multiple land use activities, such that resultant groundwater impacts are minimized. Alternative development scenarios can be investigated by simple changes in model constraints. Models III and IV were used to evaluate the control of nitrate pollution from three similar residential/commercial land use activities through optimal development patterns which satisfy water quality standards and land use density limits. Land use development scenarios were determined from given development objectives (specific to each model) and constraints imposed on the land surface activities and their associated groundwater impacts (represented here by nitrate nitrogen concentrations). The combined results of several model runs produced, for various development scenarios, the resultant regional groundwater nitrate nitrogen distributions, population predictions, and development patterns for the three targeted residential/commercial land use activities.

In light of results obtained from the four models, several conclusions were drawn and recommendations for additional work were made.

10.1 Conclusions

1) The models developed in this work characterize where and to what extent future nonpoint source groundwater nitrate pollution should be controlled in order to preserve regional groundwater quality at specified levels.

2) Models III and IV identify regions where maintaining land use density limits and water quality standards is difficult if the optimum development pattern are pursued. Postoptimal analysis reveals the development and water quality tradeoffs of relaxing land use density limits and water quality standards.

3) The management model perceives differences between land use development alternatives as differences in unit mass loadings. Under water quality limited conditions the model selects land surface activities which generate the lowest contaminant loads.

4) Where nondegradation conditions exist growth is precluded in some areas. These zero-growth areas often extended upgradient from the protected waters.

3) Beyond the boundaries of the zero-growth zones, land use development was determined by a combination existing surface activities, land use density limits and global water quality constraints.

4) Beyond the boundaries of the zero-growth zones, the combination of land use density limits and global water quality standards can result in either uniform or nonuniform development opportunities.

5) Use of stringent density constraints can yield lower regional contaminant concentrations, more uniform development opportunities, but lower maximum feasible growth. Higher density limits generate higher average contaminant levels,

nonuniform development opportunity, and higher feasible
population growth.

6) For a given population, different combinations of water
quality standards and land use density restriction can lead to
different development patterns which in turn effect different
regional groundwater quality impacts.

7) For a given population, uniform development brought about
by stringent land use density limits can lead to higher
regional contaminant levels than nonuniform development
allowed with relaxed source density limits.

8) Development in discharge zones is preferable over growth in
recharge zones both from the perspective of maximum achievable
growth and from the perspective of preserving groundwater
resources.

## 10.2 Recommendations

1) Coupled with thorough geologic work, the above models should
be applied to better define the relative significance of
groundwater protection efforts in separate zones within
recharge areas around water supplies.

2) Clearly there are many feasible development scenarios for a
given population. Elucidating the noninferior set of
development patterns should be achieved with work that further
specifies development and water quality objectives.

3)  More land use activities should be included in the model.
    This would test the optimality of development scenarios
    identified with simple models which may not incorporate all
    the complexities of the planning process.

4)  Rarely do multiple land use activites produce one contaminant
    that affects goundwater quality.  Future research should
    address groundwater protection from nonpoint source pollution
    involving multiple contaminants.

APPENDIX A


Interpolated, Observed, and Simulated Groundwater Levels

on Western Cape Cod


Presented in this appendix are the average of observed elemental
water table elevations and interpolated water table elevations all of
which were obtained from references cited in Chapter 6.  In addition are
the water table elevations produced from the validated groundwater flow
model.  Columns A and B respectively contain the I and J coordinates of
elements in the discretized region of Western Cape Cod.  Column C lists
both water table elevations interpolated from maps and water table
elevations averaged from observation wells and ponds situated in the
area as defined by the numerical elements.  The last column, D, contains
water table elevations produced from the validated groundwater flow
model.  All water table elevations are expressed in meters above mean
sea level.

| A | B | C | D |
|---|---|---|---|
| 1 | 6 | .00 | .00 |
| 1 | 7 | .00 | .00 |
| 1 | 8 | .00 | .00 |
| 2 | 5 | .00 | .00 |
| 2 | 6 | 9.45 | 9.44 |
| 2 | 7 | 9.45 | 9.46 |
| 2 | 8 | 5.49 | 5.44 |
| 2 | 9 | .00 | .00 |
| 2 | 10 | .00 | .00 |
| 3 | 4 | .00 | .00 |
| 3 | 5 | 10.36 | 10.36 |
| 3 | 6 | 16.15 | 16.13 |
| 3 | 7 | 17.37 | 17.37 |
| 3 | 8 | 11.89 | 11.84 |
| 3 | 9 | 9.75 | 9.79 |
| 3 | 10 | 6.55 | 6.58 |
| 3 | 11 | .00 | .00 |
| 3 | 12 | .00 | .00 |
| 3 | 13 | .00 | .00 |
| 4 | 3 | .00 | .00 |
| 4 | 4 | 6.10 | 6.13 |
| 4 | 5 | 14.33 | 14.34, |
| 4 | 6 | 19.51 | 19.56 |
| 4 | 7 | 21.64 | 21.66 |
| 4 | 8 | 19.51 | 19.50 |
| 4 | 9 | 18.59 | 18.62 |
| 4 | 10 | 15.85 | 15.85 |
| 4 | 11 | 8.84 | 8.87 |
| 4 | 12 | 5.18 | 5.15 |
| 4 | 13 | 2.74 | 2.77 |
| 4 | 14 | .00 | .00 |
| 4 | 19 | .00 | .00 |
| 4 | 20 | .00 | .00 |
| 4 | 21 | .00 | .00 |
| 5 | 3 | .00 | .00 |
| 5 | 4 | 7.62 | 7.61 |
| 5 | 5 | 14.63 | 14.64 |
| 5 | 6 | 21.64 | 21.63 |
| 5 | 7 | 21.95 | 22.12 |
| 5 | 8 | 20.73 | 21.33 |
| 5 | 9 | 20.12 | 20.66 |
| 5 | 10 | 19.35 | 19.89 |
| 5 | 11 | 16.76 | 16.76 |
| 5 | 12 | 10.67 | 10.70 |
| 5 | 13 | 6.25 | 6.48 |

| A | B | C | D |
|---|---|---|---|
| 5 | 14 | 4.27 | 4.24 |
| 5 | 15 | .00 | .00 |
| 5 | 16 | .00 | .00 |
| 5 | 17 | .00 | .00 |
| 5 | 18 | .00 | .00 |
| 5 | 19 | 4.27 | 4.29 |
| 5 | 20 | 2.90 | 2.90 |
| 5 | 21 | 1.98 | 1.97 |
| 5 | 22 | .00 | .00 |
| 6 | 3 | .00 | .00 |
| 6 | 4 | 8.69 | 8.71 |
| 6 | 5 | 14.94 | 15.00 |
| 6 | 6 | 21.95 | 21.95 |
| 6 | 7 | 21.95 | 21.99 |
| 6 | 8 | 20.73 | 20.81 |
| 6 | 9 | 19.81 | 19.93 |
| 6 | 10 | 18.90 | 19.15 |
| 6 | 11 | 17.68 | 17.54 |
| 6 | 12 | 14.94 | 15.01 |
| 6 | 13 | 12.19 | 12.19 |
| 6 | 14 | 11.58 | 11.58 |
| 6 | 15 | 9.60 | 9.59 |
| 6 | 16 | 9.45 | 9.44 |
| 6 | 17 | 8.38 | 8.38 |
| 6 | 18 | 7.01 | 7.00 |
| 6 | 19 | 7.01 | 7.02 |
| 6 | 20 | 5.79 | 5.80 |
| 6 | 21 | 3.51 | 3.50 |
| 6 | 22 | .00 | .00 |
| 7 | 3 | .00 | .00 |
| 7 | 4 | 9.14 | 9.18 |
| 7 | 5 | 16.46 | 16.47 |
| 7 | 6 | 20.42 | 20.46 |
| 7 | 7 | 20.42 | 20.29 |
| 7 | 8 | 18.90 | 19.10 |
| 7 | 9 | 17.98 | 18.31 |
| 7 | 10 | 17.22 | 17.07 |
| 7 | 11 | 14.94 | 15.57 |
| 7 | 12 | 14.17 | 14.06 |
| 7 | 13 | 12.80 | 12.82 |
| 7 | 14 | 11.58 | 11.54 |
| 7 | 15 | 10.67 | 10.75 |
| 7 | 16 | 9.91 | 10.25 |
| 7 | 17 | 8.38 | 8.35 |
| 7 | 18 | 6.10 | 6.16 |

| A | B | C | D |
|---|---|---|---|
| 7 | 19 | 5.94 | 5.93 |
| 7 | 20 | 4.27 | 4.20 |
| 7 | 21 | 2.13 | 2.15 |
| 7 | 22 | .00 | .00 |
| 8 | 3 | .00 | .00 |
| 8 | 4 | 9.75 | 9.70 |
| 8 | 5 | 15.24 | 15.22 |
| 8 | 6 | 17.53 | 17.52 |
| 8 | 7 | 17.53 | 17.23 |
| 8 | 8 | 16.76 | 16.45 |
| 8 | 9 | 15.85 | 15.84 |
| 8 | 10 | 14.02 | 14.70 |
| 8 | 11 | 12.50 | 12.75 |
| 8 | 12 | 7.92 | 8.28 |
| 8 | 13 | 7.62 | 8.09 |
| 8 | 14 | 6.71 | 7.68 |
| 8 | 15 | 7.92 | 7.94 |
| 8 | 16 | 6.86 | 6.94 |
| 8 | 17 | 4.88 | 4.89 |
| 8 | 18 | .00 | .00 |
| 8 | 19 | 2.74 | 2.71 |
| 8 | 20 | 2.29 | 2.25 |
| 8 | 21 | .00 | .00 |
| 9 | 2 | .00 | .00 |
| 9 | 3 | 4.88 | 4.93 |
| 9 | 4 | 9.14 | 9.32 |
| 9 | 5 | 12.19 | 12.27 |
| 9 | 6 | 14.02 | 14.00 |
| 9 | 7 | 14.33 | 14.31 |
| 9 | 8 | 12.19 | 12.87 |
| 9 | 9 | 11.43 | 11.34 |
| 9 | 10 | 11.43 | 11.28 |
| 9 | 11 | 7.62 | 7.78 |
| 9 | 12 | .00 | .00 |
| 9 | 13 | 3.05 | 2.57 |
| 9 | 14 | .00 | .00 |
| 9 | 15 | .00 | .00 |
| 9 | 16 | .00 | .00 |
| 9 | 17 | .00 | .00 |
| 9 | 19 | .00 | .00 |
| 9 | 20 | .00 | .00 |
| 10 | 2 | .00 | .00 |
| 10 | 3 | 4.57 | 4.59 |
| 10 | 4 | 8.23 | 8.25 |
| 10 | 5 | 10.67 | 11.26 |

| A  | B  | C     | D     |
|----|----|-------|-------|
| 10 | 6  | 11.28 | 11.50 |
| 10 | 7  | 11.28 | 11.18 |
| 10 | 8  | 8.23  | 8.17  |
| 10 | 9  | .00   | .00   |
| 10 | 10 | 4.57  | 4.51  |
| 10 | 11 | .00   | .00   |
| 10 | 13 | .00   | .00   |
| 11 | 2  | .00   | .00   |
| 11 | 3  | 3.51  | 3.65  |
| 11 | 4  | 4.88  | 5.07  |
| 11 | 5  | 7.92  | 8.00  |
| 11 | 6  | 7.77  | 7.74  |
| 11 | 7  | 7.16  | 7.01  |
| 11 | 8  | 4.88  | 4.90  |
| 11 | 9  | 2.29  | 2.81  |
| 11 | 10 | .00   | .00   |
| 12 | 2  | .00   | .00   |
| 12 | 3  | 2.59  | 2.35  |
| 12 | 4  | 3.66  | 3.20  |
| 12 | 5  | 3.66  | 3.46  |
| 12 | 6  | 3.66  | 3.58  |
| 12 | 7  | 3.81  | 3.70  |
| 12 | 8  | 2.74  | 2.86  |
| 12 | 9  | 1.98  | 2.12  |
| 12 | 10 | .00   | .00   |
| 13 | 2  | .00   | .00   |
| 13 | 3  | 1.52  | 1.33  |
| 13 | 4  | 1.83  | 1.19  |
| 13 | 5  | .00   | .00   |
| 13 | 6  | .00   | .00   |
| 13 | 7  | .00   | .00   |
| 13 | 8  | .00   | .00   |
| 13 | 9  | .91   | .89   |
| 13 | 10 | .00   | .00   |
| 14 | 3  | .00   | .00   |
| 14 | 4  | .00   | .00   |
| 14 | 9  | .00   | .00   |

APPENDIX B


Characteristics of Western Cape Cod Aquifer


In this appendix are transmissivities, hydraulic conductivities,
and saturated thicknesses estimated at each element of the discretized
region of Western Cape Cod aquifer.  The transmissivities were obtained
from using elemental averaged observed water table elevations (see
Appendix A) and the validated inverse model.  Aquifer thicknesses were
obtained from Guswa and Le Blanc (1985).  Elemental hydraulic
conductivities were obtained from dividing saturated thicknesses into
discrete transmissivities.  Columns A and B respectively contain nodal
coordinates I and J.  Column C lists the elemental saturated thickness
in meters.  Elemental hydraulic conductivities are presented in column D
in units of meters per day.  Column E contains the predicted elemental
transmissivities expressed in square meters per day.

| A | B | C | D | E |
|---|---|---|---|---|
| 9 | 2 | 60. | 52. | 3131. |
| 10 | 2 | 61. | 40. | 2468. |
| 11 | 2 | 61. | 41. | 2516. |
| 12 | 2 | 62. | 44. | 2765. |
| 13 | 2 | 67. | 35. | 2314. |
| 4 | 3 | 24. | 101. | 2459. |
| 5 | 3 | 24. | 90. | 2193. |
| 6 | 3 | 30. | 61. | 1871. |
| 7 | 3 | 48. | 32. | 1509. |
| 8 | 3 | 53. | 13. | 696. |
| 9 | 3 | 62. | 59. | 3654. |
| 10 | 3 | 64. | 44. | 2777. |
| 11 | 3 | 63. | 73. | 4576. |
| 12 | 3 | 62. | 76. | 4686. |
| 13 | 3 | 65. | 66. | 4266. |
| 14 | 3 | 75. | 72. | 5368. |
| 3 | 4 | 38. | 21. | 789. |
| 4 | 4 | 43. | 31. | 1342. |
| 5 | 4 | 44. | 40. | 1749. |
| 6 | 4 | 48. | 36. | 1753. |
| 7 | 4 | 56. | 21. | 1174. |
| 8 | 4 | 60. | 24. | 1456. |
| 9 | 4 | 63. | 60. | 3768. |
| 10 | 4 | 65. | 22. | 1463. |
| 11 | 4 | 64. | 116. | 7412. |
| 12 | 4 | 64. | 71. | 4524. |
| 13 | 4 | 63. | 97. | 6066. |
| 14 | 4 | 75. | 105. | 7917. |
| 2 | 5 | 57. | 4. | 254. |
| 3 | 5 | 64. | 11. | 696. |
| 4 | 5 | 70. | 12. | 826. |
| 5 | 5 | 66. | 20. | 1330. |
| 6 | 5 | 69. | 26. | 1783. |
| 7 | 5 | 71. | 10. | 689. |
| 8 | 5 | 71. | 8. | 585. |
| 9 | 5 | 69. | 60. | 4152. |
| 10 | 5 | 69. | 57. | 3960. |
| 11 | 5 | 67. | 13. | 877. |
| 12 | 5 | 64. | 42. | 2711. |
| 13 | 5 | 69. | 69. | 4702. |
| 1 | 6 | 60. | 13. | 759. |
| 2 | 6 | 69. | 10. | 713. |
| 3 | 6 | 73. | 13. | 941. |
| 4 | 6 | 75. | 18. | 1383. |
| 5 | 6 | 79. | 6. | 449. |

| A | B | C | D | E |
|---|---|---|---|---|
| 6 | 6 | 79. | 5. | 411. |
| 7 | 6 | 80. | 10. | 833. |
| 8 | 6 | 77. | 20. | 1576. |
| 9 | 6 | 74. | 47. | 3463. |
| 10 | 6 | 71. | 54. | 3885. |
| 11 | 6 | 69. | 50. | 3429. |
| 12 | 6 | 75. | 79. | 5936. |
| 13 | 6 | 87. | 88. | 7684. |
| 1 | 7 | 61. | 8. | 504. |
| 2 | 7 | 69. | 9. | 590. |
| 3 | 7 | 74. | 5. | 364. |
| 4 | 7 | 78. | 11. | 873. |
| 5 | 7 | 80. | 49. | 3912. |
| 6 | 7 | 80. | 28. | 2234. |
| 7 | 7 | 80. | 14. | 1135. |
| 8 | 7 | 78. | 37. | 2843. |
| 9 | 7 | 77. | 49. | 3745. |
| 10 | 7 | 84. | 17. | 1447. |
| 11 | 7 | 93. | 26. | 2458. |
| 12 | 7 | 83. | 33. | 2764. |
| 13 | 7 | 37. | 75. | 2740. |
| 1 | 8 | 61. | 50. | 3026. |
| 2 | 8 | 65. | 34. | 2239. |
| 3 | 8 | 70. | 38. | 2704. |
| 4 | 8 | 77. | 10. | 770. |
| 5 | 8 | 79. | 96. | 7632. |
| 6 | 8 | 80. | 48. | 3829. |
| 7 | 8 | 80. | 74. | 5899. |
| 8 | 8 | 88. | 34. | 2981. |
| 9 | 8 | 96. | 110. | 10515. |
| 10 | 8 | 107. | 32. | 3370. |
| 11 | 8 | 115. | 52. | 5978. |
| 12 | 8 | 72. | 85. | 6132. |
| 13 | 8 | 37. | 162. | 5942. |
| 2 | 9 | 62. | 30. | 1862. |
| 3 | 9 | 69. | 13. | 904. |
| 4 | 9 | 77. | 10. | 801. |
| 5 | 9 | 79. | 76. | 6027. |
| 6 | 9 | 80. | 57. | 4563. |
| 7 | 9 | 86. | 92. | 7885. |
| 8 | 9 | 107. | 43. | 4645. |
| 9 | 9 | 117. | 23. | 2734. |
| 10 | 9 | 119. | 19. | 2268. |
| 11 | 9 | 100. | 93. | 9235. |
| 12 | 9 | 83. | 53. | 4418. |

| A | B | C | D | E |
|---|---|---|---|---|
| 13 | 9 | 37. | 109. | 4090. |
| 14 | 9 | 37. | 143. | 5249. |
| 2 | 10 | 69. | 20. | 1352. |
| 3 | 10 | 68. | 15. | 989. |
| 4 | 10 | 75. | 7. | 495. |
| 5 | 10 | 78. | 47. | 3652. |
| 6 | 10 | 79. | 63. | 4943. |
| 7 | 10 | 101. | 35. | 3540. |
| 8 | 10 | 118. | 93. | 10959. |
| 9 | 10 | 132. | 20. | 2632. |
| 10 | 10 | 143. | 10. | 1379. |
| 11 | 10 | 37. | 37. | 1354. |
| 12 | 10 | 37. | 90. | 3287. |
| 13 | 10 | 37. | 45. | 1660. |
| 3 | 11 | 66. | 20. | 1295. |
| 4 | 11 | 69. | 15. | 1024. |
| 5 | 11 | 77. | 6. | 435. |
| 6 | 11 | 78. | 20. | 1530. |
| 7 | 11 | 103. | 96. | 9967. |
| 8 | 11 | 57. | 72. | 4059. |
| 9 | 11 | 52. | 45. | 2329. |
| 10 | 11 | 44. | 13. | 575. |
| 3 | 12 | 71. | 39. | 2766. |
| 4 | 12 | 66. | 29. | 1868. |
| 5 | 12 | 71. | 14. | 1022. |
| 6 | 12 | 59. | 23. | 1388. |
| 7 | 12 | 58. | 14. | 829. |
| 8 | 12 | 52. | 84. | 4360. |
| 9 | 12 | 44. | 77. | 3413. |
| 3 | 13 | 77. | 70. | 5378. |
| 4 | 13 | 67. | 59. | 4000. |
| 5 | 13 | 67. | 68. | 4535. |
| 6 | 13 | 73. | 13. | 935. |
| 7 | 13 | 57. | 10. | 566. |
| 8 | 13 | 52. | 45. | 2329. |
| 9 | 13 | 47. | 26. | 1205. |
| 10 | 13 | 44. | 67. | 2951. |
| 4 | 14 | 72. | 63. | 4528. |
| 5 | 14 | 65. | 26. | 1663. |
| 6 | 14 | 72. | 5. | 382. |
| 7 | 14 | 56. | 14. | 805. |
| 8 | 14 | 51. | 91. | 4636. |
| 9 | 14 | 44. | 21. | 943. |
| 5 | 15 | 64. | 12. | 769. |
| 6 | 15 | 73. | 12. | 879. |

| A | B | C | D | E |
|---|---|---|---|---|
| 7 | 15 | 55. | 40. | 2185. |
| 8 | 15 | 52. | 18. | 935. |
| 9 | 15 | 44. | 23. | 1025. |
| 5 | 16 | 76. | 9. | 657. |
| 6 | 16 | 83. | 6. | 478. |
| 7 | 16 | 54. | 59. | 3186. |
| 8 | 16 | 51. | 21. | 1060. |
| 9 | 16 | 44. | 15. | 648. |
| 5 | 17 | 87. | 6. | 534. |
| 6 | 17 | 95. | 6. | 533. |
| 7 | 17 | 53. | 20. | 1078. |
| 8 | 17 | 49. | 22. | 1102. |
| 9 | 17 | 44. | 27. | 1192. |
| 5 | 18 | 98. | 6. | 630. |
| 6 | 18 | 104. | 8. | 783. |
| 7 | 18 | 50. | 29. | 1448. |
| 8 | 18 | 44. | 25. | 1124. |
| 4 | 19 | 102. | 5. | 481. |
| 5 | 19 | 105. | 13. | 1345. |
| 6 | 19 | 51. | 17. | 875. |
| 7 | 19 | 50. | 17. | 839. |
| 8 | 19 | 47. | 35. | 1654. |
| 9 | 19 | 44. | 59. | 2625. |
| 4 | 20 | 76. | 47. | 3575. |
| 5 | 20 | 88. | 26. | 2282. |
| 6 | 20 | 50. | 11. | 533. |
| 7 | 20 | 48. | 30. | 1456. |
| 8 | 20 | 46. | 42. | 1962. |
| 9 | 20 | 44. | 49. | 2162. |
| 4 | 21 | 51. | 65. | 3290. |
| 5 | 21 | 51. | 36. | 1840. |
| 6 | 21 | 48. | 13. | 637. |
| 7 | 21 | 46. | 57. | 2661. |
| 8 | 21 | 44. | 50. | 2204. |
| 5 | 22 | 48. | 25. | 1184. |
| 6 | 22 | 44. | 6. | 276. |
| 7 | 22 | 44. | 72. | 3182. |

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